BOOK ONE

THE DOCTRINE OF BEING

WITH WHAT MUST THE SCIENCE BEGIN?

It is only in recent times that thinkers have become aware of the difficulty of finding a beginning in philosophy, and the reason for this difficulty and also the possibility of resolving it has been much discussed. What philosophy begins with must be either *mediated* or *immediate*, and it is easy to show that it can be neither the one nor the other; thus either way of beginning is refuted.

The *principle* of a philosophy does, of course, also express a beginning, but not so much a subjective as an *objective* one, the beginning of *everything*. The principle is a particular determinate *content*—water, the one, *nous*, idea, substance, monad, etc. Or, if it refers to the nature of cognition and consequently is supposed to be only a criterion rather than an objective determination—thought, intuition, sensation, ego, subjectivity itself. Then here too it is the nature of the content which is the point of interest. The beginning as such, on the other hand, as something subjective in the sense of being a particular, inessential way of introducing the discourse, remains unconsidered, a matter of indifference, and so too the need to find an answer to the question, With what should the beginning be made? remains of no importance in face of the need for a principle in which alone the interest of the matter in hand seems to lie, the interest as to what is the *truth*, the *absolute ground*.

But the modern perplexity about a beginning proceeds from a further requirement of which those who are concerned with the dogmatic demonstration of a principle or who are sceptical about finding a subjective criterion against dogmatic philosophizing, are not yet aware, and which is completely denied by those who begin, like a shot from a pistol, from their inner revelation, from faith, intellectual intuition, etc., and who would be exempt from *method* and logic. If earlier abstract thought was interested in the principle only as content, but in the course of philosophical development has been impelled to pay attention to the other side, to the behaviour of the cognitive process, this implies that the *subjective* act has also been grasped as an *essential* moment of objective truth, and this brings with it the need to unite the method with the content, the form with the principle. Thus the principle ought also to be the beginning, and what is

the first for thought ought also to be the first in the *process* of thinking.

Here we have only to consider how the *logical* beginning appears; the two sides from which it can be taken have already been named, to wit, either as a mediated result or as a beginning proper, as an immediacy. This is not the place to deal with the question apparently so important in present-day thought, whether the knowledge of truth is an immediate knowledge having a pure beginning, a faith, or whether it is a mediated knowledge. In so far as this can be dealt with *preliminarily* it has been done elsewhere.[1] Here we need only quote from it this, that there is nothing, nothing in heaven or in nature or mind or anywhere else which does not equally contain both immediacy and mediation, so that these two determinations reveal themselves to be *unseparated* and inseparable and the opposition between them to be a nullity. But as regards the philosophical discussion of this, it is to be found in every logical proposition in which occur the determinations of immediacy and mediation and consequently also the discussion of their opposition and their truth. Inasmuch as this opposition, as related to thinking, to knowing, to cognition, acquires the more concrete form of immediate or mediated *knowledge*, it is the nature of cognition simply as such which is considered within the science of logic, while the more concrete form of cognition falls to be considered in the philosophy of spirit and in the phenomenology of spirit. But to want the nature of cognition clarified *prior* to the science is to demand that it be considered *outside* the science; *outside* the science this cannot be accomplished, at least not in a scientific manner and such a manner is alone here in place.

The beginning is *logical* in that it is to be made in the element of thought that is free and for itself, in *pure knowing*. It is *mediated* because pure knowing is the ultimate, absolute truth of *consciousness*. In the Introduction it was remarked that the phenomenology of spirit is the science of consciousness, the exposition of it, and that consciousness has for result the *Notion* of science, i.e. pure knowing. Logic, then, has for its presupposition the science of manifested spirit, which contains and demonstrates the necessity, and so the truth, of the standpoint occupied by pure

[1] In my *Encyclop. of the Phil. Sciences*, 3rd edition, *Preliminary Notion*, Section 61 et seq.

knowing and of its mediation. In this science of manifested spirit the beginning is made from empirical, *sensuous* consciousness and this is *immediate* knowledge in the strict sense of the word; in that work there is discussed the significance of this immediate knowledge. Other forms of consciousness such as belief in divine truths, inner experience, knowledge through inner revelation, etc., are very ill-fitted to be quoted as examples of immediate knowledge as a little reflection will show. In the work just mentioned immediate consciousness is also the first and that which is immediate in the science itself, and therefore the presupposition; but in logic, the presupposition is that which has proved itself to be the result of that phenomenological consideration—the Idea as pure knowledge. *Logic is pure science*, that is, pure knowledge in the entire range of its development. But in the said result, this Idea has determined itself to be the certainty which has become truth, the certainty which, on the one hand, no longer has the object over against it but has internalized it, knows it as its own self—and, on the other hand, has given up the knowledge of itself as of something confronting the object of which it is only the annihilation, has divested itself of this subjectivity and is at one with its self-alienation.

Now starting from this determination of pure knowledge, all that is needed to ensure that the beginning remains immanent in its scientific development is to consider, or rather, ridding oneself of all other reflections and opinions whatever, simply to take up, *what is there before us*.

Pure knowing as concentrated into this unity has sublated all reference to an other and to mediation; it is without any distinction and as thus distinctionless, ceases itself to be knowledge; what is present is only *simple immediacy*.

Simple immediacy is itself an expression of reflection and contains a reference to its distinction from what is mediated. This simple immediacy, therefore, in its true expression is *pure being*. Just as *pure* knowing is to mean knowing as such, quite abstractly, so too pure being is to mean nothing but *being* in general: being, and nothing else, without any further specification and filling.

Here the beginning is made with being which is represented as having come to be through mediation, a mediation which is also a sublating of itself; and there is presupposed pure knowing as

the outcome of finite knowing, of consciousness. But if no presupposition is to be made and the beginning itself is taken *immediately*, then its only determination is that it is to be the beginning of logic, of thought as such. All that is present is simply the resolve, which can also be regarded as arbitrary, that we propose to consider thought as such. Thus the beginning must be an *absolute*, or what is synonymous here, an *abstract* beginning; and so it *may not presuppose anything*, must not be mediated by anything nor have a ground; rather it is to be itself the ground of the entire science. Consequently, it must be purely and simply *an* immediacy, or rather merely *immediacy* itself. Just as it cannot possess any determination relatively to anything else, so too it cannot contain within itself any determination, any content; for any such would be a distinguishing and an interrelationship of distinct moments, and consequently a mediation. The beginning therefore is *pure being*.

To this simple exposition of what is only directly involved in the simplest of all things, the logical beginning, we may add the following further reflections; yet these cannot be meant to serve as elucidations and confirmations of that exposition—this is complete in itself—since they are occasioned by preconceived ideas and reflections and these, like all other preliminary prejudices, must be disposed of within the science itself where their treatment should be awaited with patience.

The insight that absolute truth must be a result, and conversely, that a result presupposes a prior truth which, however, because it is a first, objectively considered is unnecessary and from the subjective side is not known—this insight has recently given rise to the thought that philosophy can only begin with a *hypothetical* and *problematical* truth and therefore philosophizing can at first be only a quest. This view was much stressed by Reinhold in his later philosophical work and one must give it credit for the genuine interest on which it is based, an interest which concerns the speculative nature of the philosophical *beginning*. The detailed discussion of this view is at the same time an occasion for introducing a preliminary understanding of the meaning of progress in logic generally; for that view has a direct bearing on the advance; this it conceives to be such that progress in philosophy is rather a retrogression and a grounding or establishing by means of which we first obtain the result that what we began

with is not something merely arbitrarily assumed but is in fact the *truth*, and also the *primary truth*.

It must be admitted that it is an important consideration—one which will be found in more detail in the logic itself—that the advance is a *retreat into the ground*, to what is *primary* and *true*, on which depends and, in fact, from which originates, that with which the beginning is made. Thus consciousness on its onward path from the immediacy with which it began is led back to absolute knowledge as its innermost *truth*. This last, the ground, is then also that from which the first proceeds, that which at first appeared as an immediacy. This is true in still greater measure of absolute spirit which reveals itself as the concrete and final supreme truth of all being, and which at the *end* of the development is known as freely externalizing itself, abandoning itself to the shape of an *immediate being*—opening or unfolding itself [*sich entschliessend*] into the creation of a world which contains all that fell into the development which preceded that result and which through this reversal of its position relatively to its beginning is transformed into something dependent on the result as principle. The essential requirement for the science of logic is not so much that the beginning be a pure immediacy, but rather that the whole of the science be within itself a circle in which the first is also the last and the last is also the first.

We see therefore that, on the other hand, it is equally necessary to consider as *result* that into which the movement returns as into its *ground*. In this respect the first is equally the ground, and the last a derivative; since the movement starts from the first and by correct inferences arrives at the last as the ground, this latter is a result. Further, the *progress* from that which forms the beginning is to be regarded as only a further determination of it, hence that which forms the starting point of the development remains at the base of all that follows and does not vanish from it. The progress does not consist merely in the derivation of an other, or in the effected transition into a genuine other; and in so far as this transition does occur it is equally sublated again. Thus the beginning of philosophy is the foundation which is present and preserved throughout the entire subsequent development, remaining completely immanent in its further determinations.

Through this progress, then, the beginning loses the one-sidedness which attaches to it as something simply immediate and

abstract; it becomes something mediated, and hence the line of the scientific advance becomes a *circle.* It also follows that because that which forms the beginning is still undeveloped, devoid of content, it is not truly known in the beginning; it is the science of logic in its whole compass which first constitutes the completed knowledge of it with its developed content and first truly grounds that knowledge.

But because it is the *result* which appears as the absolute ground, this progress in knowing is not something provisional, or problematical and hypothetical; it must be determined by the nature of the subject matter itself and its content. The said beginning is neither an arbitrary and merely provisional assumption, nor is it something which appears to be arbitrarily and tentatively presupposed, but which is subsequently shown to have been properly made the beginning; not as is the case with the constructions one is directed to make in connection with the proof of a theorem in geometry, where it becomes apparent only afterwards in the proof that one took the right course in drawing just those lines and then, in the proof itself, in beginning with the comparison of those lines or angles; drawing such lines and comparing them are not an essential part of the proof itself.

Thus the *ground,* the *reason,* why the beginning is made with pure being in the pure science [of logic] is directly given in the science itself. This pure being is the unity into which pure knowing withdraws, or, if this itself is still to be distinguished as form from its unity, then being is also the content of pure knowing. It is when taken in this way that this *pure being,* this absolute immediacy has equally the character of something absolutely mediated. But it is equally essential that it be taken only in the one-sided character in which it is pure immediacy, *precisely because* here it is the beginning. If it were not this pure indeterminateness, if it were determinate, it would have been taken as something mediated, something already carried a stage further: what is determinate implies an other to a first. Therefore, it lies in the *very nature of a beginning* that it must be being and nothing else. To enter into philosophy, therefore, calls for no other preparations, no further reflections or points of connection.

We cannot really extract any further determination or *positive* content for the beginning from the fact that it is the beginning of philosophy. For here at the start, where the subject matter itself

is not yet to hand, philosophy is an empty word or some assumed, unjustified conception. Pure knowing yields only this negative determination, that the beginning is to be *abstract*. If pure being is taken as the *content* of pure knowing, then the latter must stand back from its content, allowing it to have free play and not determining it further. Or again, if pure being is to be considered as the unity into which knowing has collapsed at the extreme point of its union with the object, then knowing itself has vanished in that unity, leaving behind no difference from the unity and hence nothing by which the latter could be determined. Nor is there anything else present, any content which could be used to make the beginning more determinate.

But the determination of *being* so far adopted for the beginning could also be omitted, so that the only demand would be that a pure beginning be made. In that case, we have nothing but the *beginning* itself, and it remains to be seen what this is. This position could also be suggested for the benefit of those who, on the one hand, are dissatisfied for one reason or another with the beginning with being and still more so with the resulting transition of being into nothing, and, on the other hand, simply know no other way of beginning a science than by *presupposing some general idea*, which is then *analysed*, the result of such analysis yielding the first specific concept in the science. If we too were to observe this method, then we should be without a particular object, because the beginning, as the beginning of *thought*, is supposed to be quite abstract, quite general, wholly form without any content; thus we should have nothing at all beyond the general idea of a mere beginning as such. We have therefore only to see what is contained in such an idea.

As yet there is nothing and there is to become something. The beginning is not pure nothing, but a nothing from which something is to proceed; therefore being, too, is already contained in the beginning. The beginning, therefore, contains both, being and nothing, is the unity of being and nothing; or is non-being which is at the same time being, and being which is at the same time non-being.

Further, in the beginning, being and nothing are present as *distinguished* from each other; for the beginning points to something else—it is a non-being which carries a reference to being as to an other; that which begins, as yet *is* not, it is only on the

way to being. The being contained in the beginning is, therefore, a being which removes itself from non-being or sublates it as something opposed to it.

But again, that which begins already *is*, but equally, too, *is not* as yet. The opposites, being and non-being are therefore directly united in it, or, otherwise expressed, it is their ***undifferentiated unity***.

The analysis of the beginning would thus yield the notion of the unity of being and nothing—or, in a more reflected form, the unity of differentiatedness and non-differentiatedness, or the identity of identity and non-identity. This concept could be regarded as the first, purest, that is, most abstract definition of the absolute—as it would in fact be if we were at all concerned with the form of definitions and with the name of the absolute. In this sense, that abstract concept would be the first definition of this absolute and all further determinations and developments only more specific and richer definitions of it. But let those who are dissatisfied with *being* as a beginning because it passes over into nothing and so gives rise to the unity of being and nothing, let them see whether they find this beginning which begins with the general idea of a *beginning* and with its analysis (which, though of course correct, likewise leads to the unity of being and nothing), more satisfactory than the beginning with being.

But there is a still further observation to be made about this procedure. The said analysis presupposes as familiar the idea of a beginning, thus following the example of other sciences. These presuppose their subject-matter and take it for granted that everyone has roughly the same general idea of it and can find in it the same determinations as those indicated by the sciences which have obtained them in one way or another through analysis, comparison and other kinds of reasoning. But that which forms the absolute beginning must likewise be something otherwise known; now if it is something concrete and hence is variously determined within itself, then this *internal relation* is presupposed as something known; it is thus put forward as an *immediacy* which, however, it is not; for it is a relation only as a relation of distinct moments, and it therefore contains *mediation* within itself. Further, with a concrete object, the analysis and the ways in which it is determined are affected by contingency and arbitrariness. Which determinations are brought out depends on what each person just *finds* in

his own immediate, contingent idea. The relation contained in something concrete, in a synthetic unity, is *necessary* only in so far as it is not just given but is produced by the spontaneous return of the moments back into this unity—a movement which is the opposite of the analytical procedure, which is an activity belonging to the subject-thinker and external to the subject matter itself.

The foregoing shows quite clearly the reason why the beginning cannot be made with anything concrete, anything containing a relation *within itself*. For such presupposes an internal process of mediation and transition of which the concrete, now become simple, would be the result. But the beginning ought not itself to be already a first *and* an other; for anything which is in its own self a first *and* an other implies that an advance has already been made. Consequently, that which constitutes the beginning, the beginning itself, is to be taken as something unanalysable, taken in its simple, unfilled immediacy, and therefore *as being*, as the completely empty being.

If impatience with the consideration of the abstract beginning should provoke anyone to say that the beginning should be made not with the beginning, but straightway with the subject matter itself, well then, this subject matter is nothing else but the said empty being; for what this subject matter is, that will be explicated only in the development of the science and cannot be presupposed by it as known beforehand.

Whatever other form the beginning takes in the attempt to begin with something other than empty being, it will suffer from the defects already specified. Let those who are still dissatisfied with this beginning tackle the problem of avoiding these defects by beginning in some other way.

But we cannot leave entirely unmentioned an original beginning of philosophy which has recently become famous, the beginning with the *ego*. It came partly from the reflection that from the first truth the entire sequel must be derived, and partly from the requirement that the *first* truth must be something with which we are acquainted, and still more, something of which we are *immediately certain*. This beginning is, in general, not a contingent idea which can be differently constituted in different subjects. For the ego, this immediate consciousness of self, at first appears to be itself both an immediacy and also something much more

familiar to us than any other idea; anything else known belongs to the ego, it is true, but is still a content distinguished from it and therefore contingent; the ego, on the contrary, is the simple certainty of its own self. But the ego as such is *at the same time* also concrete, or rather, the ego is the most concrete of all things—the consciousness of itself as an infinitely manifold world. Before the ego, this concrete Being, can be made the beginning and ground of philosophy, it must be disrupted—this is the absolute act through which the ego purges itself of its content and becomes aware of itself as an abstract ego. Only this pure ego now is *not* immediate, is not the familiar, ordinary ego of our consciousness to which the science of logic could be directly linked for everyone. That act, strictly speaking, would be nothing else but the elevation to the standpoint of pure knowing where the distinction of subject and object has vanished. But as thus *immediately* demanded, this elevation is a subjective postulate; to prove itself a genuine demand, the progression of the concrete ego from immediate consciousness to pure knowing must have been indicated and exhibited through the necessity of the ego itself. Without this objective movement pure knowing, even in the shape of intellectual intuition, appears as an arbitrary standpoint, or even as one of the empirical *states* of consciousness with respect to which everything turns on whether or not it is found or can be produced in each and every individual. But inasmuch as this pure ego must be essential, pure knowing, and pure knowing is not *immediately* present in the individual consciousness but only as posited through the absolute act of the ego in raising itself to that stand-point, we lose the very advantage which is supposed to come from this beginning of philosophy, namely that it is something thoroughly familiar, something everyone finds in himself which can form the starting point for further reflection; that pure ego, on the contrary, in its abstract, essential nature, is something unknown to the ordinary consciousness, something it does not find therein. Instead, such a beginning brings with it the disadvantage of the illusion that whereas the thing under discussion is supposed to be something familiar, the ego of empirical self-consciousness, it is in fact something far removed from it. When pure knowing is characterized as ego, it acts as a perpetual reminder of the subjective ego whose limitations should be forgotten, and it fosters the idea that the propositions and relations

resulting from the further development of the ego are present and can already be found in the ordinary consciousness—for in fact it is this of which they are asserted. This confusion, far from clarifying the problem of a beginning, only adds to the difficulties involved and tends completely to mislead; among the uninitiated it has given rise to the crudest misunderstandings.

Further, as regards the *subjective* determinateness of the ego in general, it is true that pure knowing frees the ego from the restricted meaning imposed on it by the insuperable opposition of its object; but for this reason it would be *superfluous* at least to retain this subjective attitude and the determination of pure knowing as ego. This determination, however, not only introduces the disturbing ambiguity mentioned, but closely examined it also remains a subjective *ego*. The actual development of the science which starts from the ego shows that in that development the object has and retains the perennial character of an other for the ego, and that the ego which formed the starting-point is, therefore, still entangled in the world of appearance and is not the pure knowing which has in truth overcome the opposition of consciousness.

In this connection a further essential observation must be made, namely that although the ego could *in itself* or *in principle* [*an sich*] be characterized as pure knowing or as intellectual intuition and asserted as the beginning, we are not concerned in the science of logic with what is present only in *principle* or as something *inner*, but rather with the determinate reality *in thought* of what is inner and with the *determinateness* possessed by such an inner in this reality. But what, at the *beginning* of the science, is *actually present* of intellectual intuition—or of the eternal, the divine, the absolute, if its object be so named—cannot be anything else than a first, immediate, simple determination. Whatever richer name be given to it than is expressed by mere *being*, the consideration of such absolute must be restricted solely to the way in which it enters into our knowing as *thought* and is enunicated as such. True, intellectual intution is the forcible rejection of mediation and the ratiocinative, external reflection; but what it enunciates above and beyond simple immediacy is something concrete, something which contains within itself diverse determinations. However, as we have remarked, the enunciation and exposition of such concrete beginning is a process of mediation

which starts from *one* of the determinations and advances to the other, even though the latter returns to the first; it is a movement which at the same time may not be arbitrary or assertoric. Consequently, it is not the concrete something itself with which that exposition begins but only the simple immediacy from which the movement starts. And further, if something concrete is taken as the beginning, the conjunction of the determinations contained in it demand proof, and this is lacking.

If, therefore, in the expression of the absolute, or eternal, or God (and *God* has the absolutely undisputed right that the beginning be made with him)—if in the intuition or thought of these there is *implied more* than pure being—then this *more* must make its *appearance* in our knowing only as something *thought*, not as something imagined or figurately conceived; let what is present in intuition or figurate conception be as rich as it may, the determination which *first* emerges in knowing is simple, for only in what is simple is there nothing more than the pure beginning; only the immediate is simple, for only in the immediate has no advance yet been made from a *one* to an *other*. Consequently, whatever is intended to be expressed or implied beyond *being*, in the richer forms of representing the absolute or God, this is in the beginning only an empty word and only being; this simple determination which has no other meaning of any kind, this emptiness, is therefore simply as such the beginning of philosophy.

This insight is itself so simple that this beginning as such requires no preparation or further introduction; and, indeed, these preliminary, external reflections about it were not so much intended to lead up to it as rather to eliminate all preliminaries.

GENERAL DIVISION OF BEING

Being is determined, first, as against another in general;

Secondly, as immanently self-determining;

Thirdly, setting aside the preliminary character of this division, it is the abstract indeterminateness and immediacy in which it must be the beginning.

According to the first determination, *being* is classified as distinct from *essence*, for later in its development it proves to be in its totality only one sphere of the *Notion* and to this sphere as moment, it opposes another sphere.

According to the second determination, it is the sphere within which fall the determinations and the entire movement of its reflection. Here, *being* will posit itself in three determinations:

I as *determinateness* as such: *quality*
II as *sublated* determinateness: *magnitude*, *quantity*
III as *qualitatively* determined *quantity: measure.*

At this stage, this division is, as was remarked of these divisions generally in the *Introduction*, a preliminary statement; its determinations have first to arise from the movement of *being* itself and in so doing define and justify themselves. As regards the divergence of this classification from the usual presentation of the categories, namely, as *quantity*, *quality*, *relation* and *modality*—these moreover with Kant are supposed to be only titles for his categories though they are, in fact, themselves categories, only more general ones—this calls for no special comment here, as the entire exposition will show a complete divergence from the usual arrangement and significance of the categories.

This only perhaps can be remarked, that hitherto the determination of *quantity* has been made to precede *quality* and this—as is mostly the case—for no given reason. It has already been shown that the beginning is made with being *as such*, therefore, with qualitative being. It is easily seen from a comparison of quality with quantity that the former by its nature is first. For quantity is quality which has already become negative; *magnitude* is the determinateness which is no longer one with *being* but is already differentiated from it, sublated quality which has become indifferent. It includes the alterableness of being, although the

category itself, namely being, of which it is the determination, is not altered by it. The qualitative determinateness, on the other hand, is one with its being: it neither goes beyond it nor is internal to it, but is its immediate limitedness. Quality therefore, as the *immediate* determinateness, is primary and it is with it that the beginning must be made.

Measure is a *relation*, but not relation in general, for it is the specific relation between *quality* and *quantity*; the categories which Kant includes under relation will come up for consideration in quite another place. Measure can also, if one wishes, be regarded as a modality; but since with Kant modality is supposed no longer to constitute a determination of the content, but to concern only the relation of the content to thought, to the subjective element, it is a quite heterogeneous relation and is not pertinent here.

The third determination of *being* falls within the section Quality, for as abstract immediacy it reduces itself to a single determinateness in relation to its other determinatenesses within its sphere.

Section One: Determinateness (Quality)

Being is the indeterminate *immediate;* it is free from determinateness in relation to *essence* and also from any which it can possess within itself. This reflectionless *being* is *being* as it is immediately in its own self alone.

Because it is indeterminate being, it lacks all quality; but *in itself*, the character of indeterminateness attaches to it only in contrast to what is *determinate* or qualitative. But *determinate* being stands in contrast to being in general, so that the very indeterminateness of the latter constitutes its quality. It will therefore be shown that the *first* being is in itself determinate, and therefore, *secondly*, that it passes over into *determinate being*—is *determinate being*—but that this latter as finite being sublates itself and passes over into the infinite relation of being to its own self, that is, *thirdly*, into *being-for-self.*

CHAPTER I

BEING

A. BEING

Being, pure being, without any further determination. In its indeterminate immediacy it is equal only to itself. It is also not unequal relatively to an other; it has no diversity within itself nor any with a reference outwards. It would not be held fast in its purity if it contained any determination or content which could be distinguished in it or by which it could be distinguished from an other. It is pure indeterminateness and emptiness. There is *nothing* to be intuited in it, if one can speak here of intuiting; or, it is only this pure intuiting itself. Just as little is anything to be thought in it, or it is equally only this empty thinking. Being, the indeterminate immediate, is in fact *nothing,* and neither more nor less than *nothing.*

B. NOTHING

Nothing, pure nothing: it is simply equality with itself, complete emptiness, absence of all determination and content—undifferentiatedness in itself. In so far as intuiting or thinking can be mentioned here, it counts as a distinction whether something or *nothing* is intuited or thought. To intuit or think nothing has, therefore, a meaning; both are distinguished and thus nothing *is* (exists) in our intuiting or thinking; or rather it is empty intuition and thought itself, and the same empty intuition or thought as pure being. Nothing is, therefore, the same determination, or rather absence of determination, and thus altogether the same as, pure *being.*

C. BECOMING

1. UNITY OF BEING AND NOTHING

Pure being and *pure nothing* are, therefore, the same. What is the truth is neither being nor nothing, but that being—does not pass

over but has passed over—into nothing, and nothing into being. But it is equally true that they are not undistinguished from each other, that, on the contrary, they are not the same, that they are absolutely distinct, and yet that they are unseparated and inseparable and that each immediately *vanishes in its opposite*. Their truth is, therefore, this movement of the immediate vanishing of the one in the other: *becoming*, a movement in which both are distinguished, but by a difference which has equally immediately resolved itself.

Remark 1: *The Opposition of Being and Nothing in Ordinary Thinking*

Nothing is usually opposed to *something;* but the being of *something* is already determinate and is distinguished from another *something*; and so therefore the nothing which is opposed to the something is also the nothing of a particular something, a determinate nothing. Here, however, nothing is to be taken in its indeterminate simplicity. Should it be held more correct to oppose to being, *non-being* instead of nothing, there would be no objection to this so far as the result is conerned, for in *non-being* the relation to *being* is contained: both being and its negation are enunciated in a *single* term, nothing, as it is in becoming. But we are concerned first of all not with the form of opposition (with the form, that is, also of *relation*) but with the abstract, immediate negation: nothing, purely on its own account, negation devoid of any relations—what could also be expressed if one so wished merely by 'not'.

It was the *Eleatics*, above all Parmenides, who first enunicated the simple thought of *pure being* as the absolute and sole truth: *only being is, and nothing absolutely is not*, and in the surviving fragments of Parmenides this is enunicated with the pure enthusiasm of thought which has for the first time apprehended itself in its absolute abstraction. As we know, in the oriental systems, principally in Buddhism, *nothing*, the void, is the absolute principle. Against that simple and one-sided abstraction the deep-thinking Heraclitus brought forward the higher, total concept of *becoming* and said: *being* as little *is*, as nothing *is*, or, all *flows*, which means, all is a *becoming*. The popular, especially oriental proverbs, that all that exists has the germ of death in its very birth, that death, on the other hand, is the entrance into new life, express at

bottom the same union of being and nothing. But these expressions have a substratum in which the transition takes place; being and nothing are held apart in time, are conceived as alternating in it, but are not thought in their abstraction and consequently, too, not so that they are in themselves absolutely the same.

Ex nihilo nihil fit—is one of those propositions to which great importance was ascribed in metaphysics. In it is to be seen either only the empty tautology: nothing is nothing; or, if *becoming* is supposed to possess an actual meaning in it, then, since from *nothing* only *nothing becomes*, the proposition does not in fact contain *becoming*, for in it nothing remains nothing. Becoming implies that nothing does not remain nothing but passes into its other, into being. Later, especially Christian, metaphysics whilst rejecting the proposition that out of nothing comes nothing, asserted a transition from nothing into being; although it understood this proposition synthetically or merely imaginatively, yet even in the most imperfect union there is contained a point in which being and nothing coincide and their distinguishedness vanishes. The proposition: out of nothing comes nothing, nothing is just nothing, owes its peculiar importance to its opposition to *becoming* generally, and consequently also to its opposition to the creation of the world from nothing. Those who maintain the proposition: nothing is just nothing, and even grow heated in its defence, are unaware that in so doing they are subscribing to the abstract pantheism of the *Eleatics*, and also in principle to that of Spinoza. The philosophical view for which 'being is only being, nothing is only nothing', is a valid principle, merits the name of 'system of identity'; this abstract identity is the essence of pantheism.

If the result that being and nothing are the same seems startling or paradoxical in itself, there is nothing more to be said; rather should we wonder at this wondering which shows itself to be such a newcomer to philosophy and forgets that in this science there occur determinations quite different from those in ordinary consciousness and in so-called ordinary common sense—which is not exactly sound understanding but an understanding educated up to abstractions and to a belief, or rather a superstitious belief, in abstractions. It would not be difficult to demonstrate this unity of being and nothing in every example, in *every* actual thing or thought. The same must be said of *being* and *nothing*,

as was said above about immediacy and mediation (which latter contains a reference to an other, and hence to *negation*), that nowhere in heaven or on earth is there anything which does not contain within itself both being and nothing. Of course, since we are speaking here of a particular *actual something*, those determinations are no longer present in it in the complete untruth in which they are as being and nothing; they are in a more developed determination, and are grasped, for example, as positive and negative, the former being posited, reflected being, the latter posited, reflected nothing; the positive contains as its abstract basis being, and the negative, nothing. Thus in God himself, *quality* (energy, creation, power, and so forth), essentially involves the determination of the negative—they are the producing of an *other*. But an empirical elucidation by examples of the said assertion would be altogether superfluous here. Since the unity of being and nothing as the primary truth now forms once and for all the basis and element of all that follows, besides *becoming* itself, all further logical determinations: determinate being, quality, and generally all philosophical Notions, are examples of this unity. But self-styled sound common sense, if it rejects the unseparatedness of being and nothing, may be set the task of trying to discover an example in which the one is found separated from the other (*something* from *limit* or *limitation*, or, as just mentioned, the infinite, God, from energy or activity). Only the empty figments of thought, being and nothing themselves are these separated things and it is these that are preferred by 'sound common sense' to the truth, to the unseparatedness of both which is everywhere before us.

We cannot be expected to meet on all sides the perplexities which such a logical proposition produces in the ordinary consciousness, for they are inexhaustible. Only a few of them can be mentioned. One source among others of such perplexity is that the ordinary consciousness brings with it to such an abstract logical proposition, conceptions of something concrete, forgetting that what is in question is not such concrete something but only the pure abstractions of being and nothing and that these alone are to be held firmly in mind.

Being and non-being are the same, therefore it is the same whether this house is or is not, whether these hundred dollars are part of my fortune or not. This inference from, or application of,

the proposition completely alters its meaning. The proposition contains the pure abstractions of being and nothing; but the application converts them into a determinate being and a determinate nothing. But as we have said, the question here is not of determinate being. A determinate, a finite, being is one that is in relation to an other; it is a content standing in a necessary relation to another content, to the whole world. As regards the reciprocally determining context of the whole, metaphysics could make the—at bottom tautological—assertion that if a speck of dust were destroyed the whole universe would collapse. In the instances adduced against the proposition in question, something appears as not indifferent to whether it is or is not, not on account of being or non-being, but on account of its *content*, which brings it into relation with something else. If a specific content, any determinate being, is *presupposed*, then because it is *determinate*, it is in a manifold relationship with another content; it is not a matter of indifference to it whether a certain other content with which it is in relation is, or is not; for it is only through such relation that it essentially is what it is. The same is the case in the ordinary way of thinking (taking non-being in the more specific sense of such way of thinking as contrasted with actuality) in the context of which the being or the absence of a content, which, as determinate, is conceived as in relation to another, is not a matter of indifference.

This consideration involves what constitutes a cardinal factor in the Kantian criticism of the ontological proof of the existence of God, although here we are only interested in the distinction made in that proof between being and nothing generally, and *determinate* being or non-being. As we know, there was presupposed in that so-called proof the concept of a being possessing all realities, including therefore *existence*, which was likewise assumed as one of the realities. The main thesis of the Kantian criticism was that *existence* or being (these being taken here as synonymous) is not a *property* or a *real predicate*, that is to say, is not a concept of something which could be added to the *concept* of a thing. By this Kant means to say that being is not a determination of the content of a thing.[1] Therefore, he goes on to say, the possible does not contain more than the actual; a hundred actual dollars do not contain a whit more than a hundred possible

[1] Kant's *Critique of Pure Reason*, 2nd ed., pp. 628 sqq.

ones; that is, the content of the former has no other determination than has the content of the latter. If this content is considered as isolated, it is indeed a matter of indifference whether it is, or is not; it contains no distinction of being or non-being, this difference does not affect it at all. The hundred dollars do not diminish if they do not exist, or increase if they do. A difference must come only from elsewhere. 'On the other hand,' Kant reminds us, 'my fortune benefits more from a hundred actual dollars than from the mere concept of them or from their possibility. For in actuality, the *object* is not merely contained analytically in my concept, but is *added synthetically to my concept* (which is a determination of my state), although the hundred dollars in my thought are not themselves increased one whit by this being which they have apart from my concept.'

There are *presupposed* here two different states (to retain the Kantian expressions which are not free from a confused clumsiness): one, which Kant calls the concept (by which we must understand figurate conception), and another, the state of my fortune. For the one as for the other, my fortune and the figurate conception, a hundred dollars are a determination of a content or, as Kant expresses it, 'they are added to such a concept synthetically'; I as *possessor* of a hundred dollars or as not possessing them, or even I as *imagining* or not imagining them, is of course a different content. Stated more generally: the abstractions of being and nothing both cease to be abstractions if they acquire a determinate content; being is then reality, the determinate being of a hundred dollars; nothing is the negation, the determinate non-being of them. This determinate content itself, the hundred dollars, also grasped isolatedly in abstraction is unchanged the same in the one as it is in the other. But since, furthermore, being is taken as a state of my fortune, the hundred dollars stand in relation to this state, as regards which the determinateness which they are is not a matter of indifference; their being or non-being is only an *alteration*; they are transposed into the sphere of *determinate being*. When, therefore, it is urged against the unity of being and nothing that it is nevertheless not a matter of indifference whether anything (the hundred dollars) is, or is not, we practise the deception of converting the difference between whether I *have* or *have not* the hundred dollars into a difference between being and non-being—a deception based, as we have

shown, on the one-sided abstraction which ignores the determinate being present in such examples and holds fast merely to being and non-being, just as, conversely, the abstract being and nothing which should be apprehended is transformed into a definite being and nothing, into a determinate being. *Determinate being* is the first category to contain the real difference of being and nothing, namely, *something* and *other*. It is this real difference which is vaguely present in ordinary thinking, instead of abstract being and pure nothing and their only imagined difference.

As Kant expresses it, 'through its existence something enters into the context of the whole of experience, . . . we obtain thereby an additional object of *perception* without anything being added to our *concept* of the object'. As our explanation has shown, this means simply that something, through its existence, just because it is a determinate existence, is essentially in relationship with *others*, including also a percipient subject. The concept of the hundred dollars, says Kant, gains nothing by their being perceived. *Concept* here means the hundred dollars previously noted as thought *in isolation*. As thus isolated they are, it is true, an empirical content, but cut off, having no relationship with any other content and possessing no determinate character relatively to such; the form of identity-with-self strips them of any connection with an other, so that it is a matter of indifference whether they are perceived or not. But this so-called *concept* of the hundred dollars is a spurious concept; the form of simple self-relation does not belong to such a limited, finite content itself; it is a borrowed form attached to it by the subjective understanding; the being of the hundred dollars is not self-related but alterable and perishable.

The thinking or figurate conception which has before it only a specific, determinate being must be referred back to the previously-mentioned beginning of the science made by Parmenides who purified and elevated his own figurate conception, and so, too, that of posterity, to *pure thought*, to being as such and thereby created the element of the science. What is the first in the *science* had of necessity to show itself *historically* as the first. And we must regard the Eleatic *One* or *being* as the first step in the knowledge of thought; water and suchlike material principles are certainly *meant* to be the universal, but as material they are not pure thoughts; numbers are neither the first simple, nor the self-

communing thought, but the thought which is wholly external to itself.

The reference back from *particular finite* being to being as such in its wholly abstract universality is to be regarded not only as the very first theoretical demand but as the very first practical demand too. When for example a fuss is made about the hundred dollars, that it does make a difference to the state of my fortune whether I *have* them or *not*, still more whether *I* am or not, or whether something else is or is not, then—not to mention that there will be fortunes to which such possession of a hundred dollars will be a matter of indifference—we can remind ourselves that man has a duty to rise to that abstract universality of mood in which he is indeed indifferent to the existence or non-existence of the hundred dollars, whatever may be their quantitative relation to his fortune, just as it ought to be a matter of indifference to him whether he is or is not, that is, in finite life (for a state, a determinate being is meant), and so on—*si fractus illabatur orbis, impavidum ferient ruinae* was said by a Roman, and still more ought the Christian to possess this indifference.

There remains still to be noted the immediate connection between, on the one hand, the elevation above the hundred dollars and finite things generally, and on the other, the ontological proof and the Kantian criticism of it we have cited. This criticism, through its popular example, has made itself universally plausible: who does not know that a hundred actual dollars are different from a hundred merely possible ones? that they make a difference to the state of my fortune? Because this difference is so obvious with the hundred dollars, therefore the concept, that is, the specific nature of the content as an empty possibility, and being, are different from each other; *therefore* the Notion of God too is different from his being, and just as little as I can extract from the possibility of the hundred dollars their actuality, just as little can I extract from the Notion of God his existence; but the onotological proof is supposed to consist of this extraction of the existence of God from his Notion. Now though it is of course true that Notion is different from being, there is a still greater difference between God and the hundred dollars and other finite things. It is the *definition of finite things* that in them the Notion is different from being, that Notion and reality, soul and body, are separable and hence that they are perishable and mortal; the

abstract definition of God, on the other hand, is precisely that his Notion and his being are *unseparated* and *inseparable*. The genuine criticism of the categories and of reason is just this: to make intellect aware of this difference and to prevent it from applying to God the determinations and relationships of the finite.

Remark 2: *Defectiveness of the Expression 'Unity, Identity of Being and Nothing'*

Another contributory reason for the repugnance to the proposition about being and nothing must be mentioned; this is that the result of considering being and nothing, as expressed in the statement: being and nothing are one and the same, is incomplete. The emphasis is laid chiefly on their being one and the same, as in judgements generally, where it is the predicate that first states what the subject is. Consequently, the sense seems to be that the difference is denied, although at the same time it appears directly in the proposition; for this enunicates *both* determinations, being and nothing, and contains them as distinguished. At the same time, the intention cannot be that abstraction should be made from them and only the unity retained. Such a meaning would self-evidently be one-sided, because that from which abstraction is to be made is equally present and named in the proposition. Now in so far as the proposition: being and nothing are the same, asserts the identity of these determinations, but, in fact, equally contains them both as distinguished, the proposition is self-contradictory and cancels itself out. Bearing this in mind and looking at the proposition more closely, we find that it has a movement which involves the spontaneous vanishing of the proposition itself. But in thus vanishing, there takes place in it that which is to constitute its own peculiar content, namely, *becoming*.

The proposition thus *contains* the result, it is this *in its own self*. But the fact to which we must pay attention here is the defect that the result is not itself *expressed* in the proposition; it is an external reflection which discerns it therein. In this connection we must, at the outset, make this general observation, namely, that the proposition in the *form of a judgement* is not suited to express speculative truths; a familiarity with this fact is likely to remove many misunderstandings of speculative truths. Judgment is an *identical* relation between subject and predicate; in it we abstract from the fact that the subject has a number of determinatenesses

other than that of the predicate, and also that the predicate is more extensive than the subject. Now if the content is speculative, the *non-identical* aspect of subject and predicate is also an essential moment, but in the judgement this is not expressed. It is the form of simple judgement, when it is used to express speculative results, which is very often responsible for the paradoxical and bizarre light in which much of recent philosophy appears to those who are not familiar with speculative thought.

To help express the speculative truth, the deficiency is made good in the first place by adding the contrary proposition: being and nothing are not the same, which is also enunciated as above. But thus there arises the further defect that these propositions are not connected, and therefore exhibit their content only in the form of an antinomy whereas their content refers to one and the same thing, and the determinations which are expressed in the two propositions are supposed to be in complete union—a union which can only be stated as an *unrest* of *incompatibles*, as a *movement*. The commonest injustice done to a speculative content is to make it one-sided, that is, to give prominence only to one of the propositions into which it can be resolved. It cannot then be denied that this proposition is asserted; *but the statement is just as false as it is true*, for once one of the propositions is taken out of the speculative content, the other must at least be equally considered and stated. Particular mention must be made here of that, so to speak, unfortunate word, 'unity'. Unity, even more than identity, expresses a subjective reflection; it is taken especially as the relation which arises from *comparison*, from external reflection. When this reflection finds the same thing in two *different objects*, the resultant unity is such that there is presupposed the complete *indifference* to it of the objects themselves which are compared, so that this comparing and unity does not concern the objects themselves and is a procedure and a determining external to them. Unity, therefore, expresses wholly *abstract* sameness and sounds all the more blatantly paradoxical the more the terms of which it is asserted show themselves to be sheer opposites. So far then, it would be better to say only *unseparatedness* and *inseparability*, but then the affirmative aspect of the relation of the whole would not find expression.

Thus the whole true result which we have here before us is *becoming*, which is not merely the one-sided or abstract unity of

being and nothing. It consists rather in this movement, that pure being is immediate and simple, and for that very reason is equally pure nothing, that there *is* a difference between them, but a difference which no less sublates itself and is *not*. The result, therefore, equally asserts the difference of being and nothing, but as a merely fancied or imagined difference.

It is the common opinion that being is rather the sheer other of nothing and that nothing is clearer than their absolute difference, and nothing seems easier than to be able to state it. But it is equally easy to convince oneself that this is impossible, that it is *unsayable. Let those who insist that being and nothing are different tackle the problem of stating in what the difference consists.* If being and nothing had any determinateness by which they were distinguished from each other then, as has been observed, they would be determinate being and determinate nothing, not the pure being and pure nothing that here they still are. Their difference is therefore completely empty, each of them is in the same way indeterminate; the difference, then, exists not in themselves but in a third, in subjective *opinion*. Opinion, however, is a form of subjectivity which is not proper to an exposition of this kind. But the third in which being and nothing subsist must also present itself here, and it has done so; it is *becoming*. In this, being and nothing are distinct moments; becoming only *is*, in so far as they are distinguished. This third is an other than they; they subsist only in an other, which is equivalent to saying that they are not self-subsistent. Becoming is as much the subsistence of being as it is of non-being; or, their subsistence is only their being in a *one*. It is just this their subsistence that equally sublates their difference.

The challenge to distinguish between being and nothing also includes the challenge to say what, then, is being and what is nothing. Those who are reluctant to recognize either one or the other as only a *transition* of the one into the other, and who assert this or that about being and nothing, let them state *what* it is they are speaking of, that is, put forward a *definition* of being and nothing and demonstrate its correctness. Without having satisfied this first requirement of the ancient science whose logical rules they accept as valid and apply in other cases, all that they maintain about being and nothing amounts only to assertions which are scientifically worthless. If elsewhere it has been said

that existence, in so far as this at first is held to be synonymous with being, is the *complement* to *possibility*, then this presupposes another determination, possibility, and so being is not enunciated in its immediacy, but in fact as not self-subsistent, as conditioned. For being which is the outcome of *mediation* we shall reserve the term: *Existence.* But one *pictures* being to oneself, perhaps in the image of pure light as the clarity of undimmed seeing, and then nothing as pure night—and their distinction is linked with this very familiar sensuous difference. But, as a matter of fact, if this very seeing is more exactly imagined, one can readily perceive that in absolute clearness there is seen just as much, and as little, as in absolute darkness, that the one seeing is as good as the other, that pure seeing is a seeing of nothing. Pure light and pure darkness are two voids which are the same thing. Something can be distinguished only in determinate light or darkness (light is determined by darkness and so is darkened light, and darkness is determined by light, is illuminated darkness), and for this reason, that it is only darkened light and illuminated darkness which have within themselves the moment of difference and are, therefore, *determinate* being.

Remark 3: *The Isolating of These Abstractions*

The unity, whose moments, being and nothing, are inseparable, is at the same time different from them and is thus a third to them; this third in its own most characteristic form is *becoming. Transition* is the same as becoming except that in the former one tends to think of the two terms, from one of which transition is made to the other, as at rest, apart from each other, the transition taking place *between* them. Now wherever and in whatever form being and nothing are in question, this third must be present; for the two terms have no separate subsistence of their own but *are* only in becoming, in this third. But this third has many empirical shapes, which are set aside or ignored by abstraction in order to hold fast, each by itself, these its products, being and nothing, and to show them protected against transition. Such simple procedure of abstraction can be countered, equally simply, by calling to mind the empirical existence in which that abstraction is itself only a something having a determinate being. Or else it is some other form of reflection which is supposed to effect the separation of what is inseparable. Such determination carries

within itself its own opposite, and, without referring back and appealing to the nature of the thing itself, the determination of reflection can be refuted in its own self by taking it just as it presents itself and pointing out in it its own other. It would be lavour in vain to attempt to intercept all the shifts and turns of reflection and its arguments in order to cut off and render impossible to it all the evasions and digressions by which it conceals from itself its own self-contradiction. For this reason I, too, refrain from taking notice of many of the so-called objections and refutations which have been advanced against the proposition that neither being nor nothing truly *is*, but that their truth is only becoming. The intellectual training which alone can afford an insight into the nullity of such refutations, or rather spontaneously dispel such random fancies, is effected only by a critical knowledge of the forms of the understanding; but those who are most prolific with such objections straightway launch their reflections against the first propositions without first acquiring or having acquired, by a further study of logic, an awareness of the nature of these crude reflections.

We shall consider some of the results which appear when being and nothing are postulated in isolation from each other, each outside the sphere of the other, with the consequence that their transition is denied.

Parmenides held fast to *being* and was most consistent in affirming at the same time that *nothing* absolutely is not; only *being* is. As thus taken, entirely on its own, being is indeterminate, and has therefore no relation to an other; consequently, it seems that *from this beginning* no further *progress* can be made—that is, from this beginning itself—and that progress can only be achieved by linking it on to something extraneous, something *outside* it. Hence the progress made in affirming that being is the same as nothing appears as a second, absolute beginning—a transition which is independent of being and added to it from outside. If being had a determinateness, then it would not be the absolute beginning at all; it would then depend on an other and would not be immediate, would not be the beginning. But if it is indeterminate and hence a genuine beginning, then, too, it has nothing with which it could bridge the gap between itself and an other; it is at the same time the *end*. It is just as impossible for anything to break forth from it as to break into it; with Parmenides as with

Spinoza, there is no progress from being or absolute substance to the negative, to the finite. If, nevertheless, there is progress—which, as has been remarked, in the case of relationless, and so progress-less being can be accomplished only in an external manner—then this progress is a second, a fresh beginning. Thus Fichte's absolutely primary, unconditioned principle: **A = A**, is thesis; the second is antithesis. This latter is supposed to be *partly* conditioned, *partly* unconditioned (and so an internal contradiction). This is a progress by external reflection which, having negated the absolute with which it began—the antithesis is the negation of the first identity—straightway expressly converts its second *unconditioned* into a *conditioned*. But if there were any justification at all for the progress, that is, for sublating the first beginning, then this first would itself have to be of such a nature that an other could connect itself with it; and therefore it would have to be *determinate*. But neither *being*, nor even absolute substance, claims to be such: on the contrary. Being is the *immediate*, that which is still utterly *indeterminate*.

The most eloquent, perhaps forgotten, descriptions of the impossibility of advancing from an abstract first to something beyond it, and effecting a union of both, are made by Jacobi in support of his polemic against the Kantian *a priori* synthesis of self-consciousness in his *Treatise on the Undertaking of the Critical Philosophy to Bring Reason to Understanding*.[1] He states the problem thus: that there be demonstrated the originating or producing of a synthesis in a *pure* [unity], whether of consciousness, of space, or of time. 'Let space be one, time be one, consciousness be one . . . Now tell me how does any one of these three ones *purely* make itself into a manifold within itself . . . each is only a *one* and *no other;* a one and the same sort, a self-sameness without any distinction of one from the other; for these distinctions still slumber in the empty infinitude of the indeterminate from which each and everything determinate has yet to proceed! What brings *finitude* into those three infinities? What impregnates space and time *a priori* with number and measure and transforms them into a *pure manifold*? What brings *pure spontaneity* (ego) into oscillation? Whence does its pure vowel get its consonant, or rather how does its *soundless*, uninterrupted *sounding* interrupt itself and break off in order to gain at least a kind of 'self-sound'

[1] F. H. Jacobi, *Works*, vol. III.

(vowel), an *accent*?'[1] It is evident that Jacobi recognized very clearly the insubstantial nature, the *non ens*, of abstraction, whether so-called absolute (i.e. only abstract) space, or abstract time, or abstract pure consciousness, the ego; he remains fixed in such abstraction in order to maintain the impossibility of a transition to an other (the condition of a synthesis), and to the synthesis itself. The synthesis, which is the point of interest, must not be taken as a connection of determinations already *externally* there; the question is partly of the genesis of a second to a first, of a determinate to an indeterminate first principle, partly, however, of *immanent* synthesis, synthesis *a priori*—a self-subsistent, self-determined unity of distinct moments. *Becoming* is this immanent synthesis of being and nothing; but because synthesis suggests more than anything else the sense of an external bringing together of mutually external things already there, the name synthesis, synthetic unity, has rightly been dropped. Jacobi asks *how* does the pure vowel of the ego get its consonant, *what* brings determinateness into indeterminateness? The *what* would be easy to answer and has been answered by Kant in his own manner; but the question *how* means: in what peculiar manner, in what relationship, and so forth, and thus demands the statement of a particular category; but there can be no question here of a peculiar manner, of categories of the understanding. The very question *how* itself belongs to the bad habits of reflection, which demands comprehensibility, but at the same time presupposes its own fixed categories and consequently knows beforehand that it is armed against the answering of its own question. Neither has it with Jacobi the higher sense of a question concerning the *necessity* of the synthesis; for he remains, as has been said, fixed in the abstractions in order to maintain the impossibility of the synthesis. Especially graphic is his description of the procedure for reaching the abstraction of space. 'For a time I must try clean to forget that I ever saw, heard, touched or handled anything at all, myself expressly not excepted. Clean, clean, clean must I forget all movement, and precisely this *forgetting*, because it is hardest, I must make my greatest concern. Just as I have thought away everything in general, so I must also completely and entirely get rid of it, retaining nothing but the forcibly arrested intuition alone of infinite *immutable space*. I may not therefore *again think*

[1] F. H. Jacobi, *Works*, vol. III, p. 113.

into it my own self as something distinct from it and yet connected with it; I may not let myself be merely *surrounded* and *pervaded* by it: but I must wholly *pass over* into it, become one with it, transform myself into it; I must leave nothing over of myself but *this my intuition* itself, in order to contemplate it as a genuinely self-subsistent, independent, single and sole conception.'[1]

With this wholly abstract purity of continuity, that is, indeterminateness and vacuity of conception, it is indifferent whether this abstraction is called space, pure intuiting, or pure thinking; it is altogether the same as what the Indian calls Brahma, when for years on end, physically motionless and equally unmoved in sensation, conception, fantasy, desire and so on, looking only at the tip of his nose, he says inwardly only *Om, Om, Om*, or else nothing at all. This dull, empty consciousness, understood as consciousness, is—*being*.

In this void, Jacobi now continues, he experiences the opposite of what Kant assures him he should experience; he does not find himself to be a *many* and *manifold*, but rather a one devoid of all plurality and variety; indeed, 'I myself am the *impossibility*, the *annihilation* of all that is manifold and plural—*cannot* from my pure, absolutely simple, immutable being *produce again* or spook into myself even the least bit of anything . . . Thus all separatedness and juxtaposition, and all manifoldness and plurality based thereon, are revealed (in this purity) as a *sheer impossibility*.'[2]

This impossibility amounts to nothing else than the tautology: I hold fast to abstract unity and shut out all plurality and manifoldness, confine myself to the differenceless and the indeterminate and shut my eyes to all that is differentiated and determinate. The Kantian *a priori* synthesis of self-consciousness, that is, the function of this unity to differentiate itself and in this differentiation to preserve itself, is attenuated by Jacobi into the same abstraction. That 'synthesis *in itself*', the 'original act of judgement', he converts one-sidedly into 'the *copula in itself*—an "is, is, is", without beginning or end and without what, who or which. This repetition of repetition *ad infinitum* is the sole business, function and product of the absolutely pure synthesis; it is itself empty, pure, absolute repetition itself.' Though, in fact, since there is no breaking off, that is, no negation or distinguishing in it, it is not a repetition but merely undifferentiated, simple being.

[1] F. H. Jacobi, *Works*, vol. III, p. 147. [2] *Ibid.*, p. 149.

But, then, is this still a synthesis if Jacobi omits precisely that which makes the unity a synthetic unity?

In the first place, it must be said that when Jacobi thus fixes himself in absolute or abstract space, time and consciousness, he places and fixes himself in this way in something which is *empirically* false; *there is*, that is, there is empirically present, no such space and time which is not spatially and temporally limited, or whose continuity is not filled by manifoldly limited determinate being and change, so that these limits and changes belong, unseparated and inseparable, to the nature of spatiality and temporality; similarly, consciousness is filled with determinate sensation, conception, desire and so on; it does not exist separated from some particular content. The empirical *transition*, moreover, is self-evident; consciousness can of course make empty space, empty time, and even empty consciousness itself or pure being, its object and content, but it does not stop at that; it goes beyond it or rather presses forward out of such a vacuity to a better content, that is, to a content which in some way or other is more concrete, and which to that extent is better and truer however bad it may be in other respects; just such a content is in general synthetic, this word being taken in its more general sense. Thus Parmenides has to reckon with illusion and opinion, the opposite of being and truth; Spinoza likewise, with attributes, modes, extension, movement, understanding, will, and so on. The synthesis contains and demonstrates the falsity of those abstractions; in it they are in unity with their other, not, therefore, as independently self-subsistent, not as absolute, but purely as relative.

The demonstration of the empirical nullity of empty space, and so forth, is not, however, what we are concerned with. Consciousness by making abstraction can, of course, fill itself with such indeterminates also and the abstractions thus held fast are the *thoughts* of pure space, pure time, pure consciousness, or pure being. It is the thought of pure space, etc.—that is, pure space, etc., *in its own self*—that is to be demonstrated as null: that it is as such already its own opposite, that its opposite has already penetrated into it, that it is already by itself the accomplished coming-forth-from-itself, a determinateness.

But this is found immediately in them. They are, as Jacobi profusely describes them, results of abstraction; they are expressly

determined as *indeterminate* and this—to go back to its simplest form—is being. But it is this very *indeterminateness* which constitutes its determinateness; for indeterminateness is opposed to determinateness; hence as so opposed it is itself determinate or the negative, and the pure, quite abstract negative. It is this indeterminateness or abstract negation which thus has being present within it, which reflection, both outer and inner, enunciates when it equates it with nothing, declares it to be an empty product of thought, to be nothing. Or it can be expressed thus: because being is devoid of all determination whatsoever, it is not the (affirmative) determinateness which it is; it is not being but nothing.

In the pure reflection of the beginning as it is made in this logic with being as such, the transition is still concealed; because *being* is posited only as immediate, therefore *nothing* emerges in it only immediately. But all the subsequent determinations, like determinate being which immediately follows, are more concrete; in determinate being there is already *posited* that which contains and produces the contradiction of those abstractions and therefore their transition. When being is taken in this simplicity and immediacy, the recollection that it is the result of complete abstraction, and so for that reason alone is abstract negativity, nothing, is left behind, outside the science, which, within its own self, from *essence* onwards will expressly exhibit the said one-sided *immediacy* as a mediated immediacy where being is *posited* as *existence* and the mediating agent of this being is *posited* as *ground*.

In the light of such recollection, the transition from being into nothing can be represented, or, as it is said, *explained* and *made intelligible*, as something even easy and trivial; of course the being which is made the beginning of the science is *nothing*, for abstraction can be made from everything, and if abstraction is made from everything then *nothing* is left over. But, it may be continued, the beginning is thus not an affirmative, not being, but just nothing, and nothing is then also the *end*, at least as much as immediate being, and even more so. The shortest way is to let such reasoning take its course and then wait and see what is the nature of its boasted results. That *nothing* would be the result of such reasoning and that now the beginning should be made with nothing (as in Chinese philosophy), need not cause

us to lift a finger, for before we could do so this nothing would no less have converted itself into being (see Section B above). But further, this abstraction from everything (which 'everything' nevertheless is an affirmative *being*) having been presupposed, then it must be understood more exactly; the result of making abstraction from all that is, is first of all abstract being, *being* as such; just as in the cosmological proof of the existence of God from the contingent being of the world, in which proof we rise above such contingent being, *being* is still taken up with us in our ascent and is determined as *infinite being*. Of course, one *can* also abstract from this pure being, being *can* be thrown in with the all from which abstraction has already been made; then nothing remains. Now if we want to forget the *thinking* of *nothing*, that is, its conversion into being, or are ignorant of it, we *can* proceed in the style of 'one *can*'; we can for example (God be praised!) also abstract from nothing (for the creation of the world, too, is an abstraction from nothing), and then what remains is not nothing, for it is just from this that we have made abstraction; we have in fact arrived at being again. This, 'one *can*', gives an external play of abstraction, in which the abstracting itself is only the one-sided activity of the negative. It is directly implied in this very form of 'one *can*', that for it being is just as indifferent as nothing, and that with the vanishing of either of them there is equally an arising of the other; but it is equally a matter of indifference whether one starts from the doing of nothing, or from nothing; for the former, that is the mere abstracting, has neither more nor less of truth in it than mere nothing has.

The dialectic employed by Plato in treating of the One in the *Parmenides* is also to be regarded rather as a dialectic of external reflection. Being and the One are both Eleatic forms which are the same thing. But they are also to be distinguished; and it is thus that Plato takes them in that dialogue. After removing from the One the various determinations of whole and parts, of being-within-itself, of being-in-another, etc., of shape, time, etc., he reaches the result that being does not belong to the One, for being belongs to any particular something only in one of these modes.[1] Plato next deals with the proposition: *the One is*, and we should refer to Plato himself to see how, starting from this proposition, he accomplishes the transition to the *non-being* of

[1] p. 141e., vol. III, ed. Steph.

the One. He does it by *comparing* the two determinations of the proposition put forward: *the One is*; it contains the One *and* being, and 'the One is' contains more than when we only say: the One. It is through their being *different* that the moment of negation contained in the proposition is demonstrated. It is evident that this course has a presupposition and is an external reflection.

Here the way in which the One is connected with being is such that being, which is supposed to be held fast abstractly *by itself*, is demonstrated in the simplest way and without any effort of thought, to be in a union which implies the contrary of what is supposed to be maintained. Being, taken as it is immediately, belongs to *a subject*, is something enunciated, has an empirical *existence* in general and stands therefore in the field of limitation and the negative. In whatever phrases or turns of speech understanding may express itself in attacking the unity of being and nothing and appealing to what immediately confronts us, it will find just in this very experience nothing but *determinate* being, being with a limitation or negation—that very unity which it rejects. The assertion of immediate being thus reduces to an empirical existence, and it cannot reject the *demonstration* of this because it is to the immediacy which is outside of thought that it wants to cling.

The same is the case with *nothing*, only contrariwise, and this reflection on it is familiar and has been made often enough. Nothing, taken in its immediacy, shows itself as affirmative, as *being*; for according to its nature it is the same as being. Nothing is thought of, imagined, spoken of, and therefore it *is*; in the thinking, imagining, speaking and so on, nothing has its being. But, further, this being is also distinguished from it; it is therefore said that although nothing is in thought or imagination, yet for that very reason it is not *nothing* that *is*, being does not belong to nothing as such, but only thought or imagination is this being. With this distinguishing it is equally not to be denied that nothing stands in *relationship* to a being; but in the relation, even though it contains the difference, there is present a unity with being. In whatever way nothing is enunciated or indicated, it shows itself connected with, or if you like in contact with a being, unseparated from a being, that is to say in a *determinate* being.

But when the presence of nothing in a determinate being is thus demonstrated, there still lingers on the thought of this

difference of it from being, namely that the determinate being of nothing does not at all pertain to nothing itself, that nothing does not possess an independent being of its own, is not being as such. Nothing, it is said, is only the absence of being, darkness thus only the *absence* of light, cold only absence of heat, and so on. And darkness only has meaning in relation to the eye, in external comparison with the positive factor, light, and similarly cold is only something in our sensation; on the other hand, light and heat, like being, are objective, active realities on their own account, and are of quite another quality and dignity than this negative, than nothing. One can often find it put forward as a weighty reflection and an important piece of information that darkness is *only absence* of light, cold *only absence* of heat. About this acute reflection in this field of empirical objects, it can be empirically observed that darkness does in fact show itself active in light, determining it to colour and thereby imparting visibility to it, since, as was said above, just as little is seen in pure light as in pure darkness. Visibility, however, is effected in the eye, and the supposed negative has just as much a share in this as the light which is credited with being the real, positive factor; similarly, cold makes its presence known in water, in our sensations etc., and if we deny it so-called objective reality it is not a whit the worse for our doing so. But a further objection would be that here, too, as before, it is a negative with a determinate content that is spoken of, the argument is not confined to pure nothing, to which being, regarded as an empty abstraction, is neither inferior nor superior. But cold, darkness, and similar determinate negations are to be taken directly as they are by themselves and we shall then see what we have thereby effected in respect of their universal determination which has led them to be introduced here. They are supposed to be not just nothing but the nothing of light, heat, etc., of something determinate, of a content; thus they are a determinate, a contentful, nothing if one may so speak. But, as will subsequently appear, a determinateness is itself a negation, and so they are negative nothings; but a negative nothing is an affirmative something. The conversion of nothing through its determinateness (which previously appeared as a *determinate* being in a subject thinker, or in some other form) into an affirmative, appears to the consciousness which is fixed in the abstraction of the understanding as the acme of paradox;

the insight that the negation of the negation is something positive, simple as it is, or rather because of its very simplicity, appears as a triviality to which haughty understanding need pay no heed, although the correctness of the insight is admitted—and the insight is not only correct, but, because of the universality of such determinations, it has its infinite extension and universal application, so that it were indeed well to pay attention to it.

A further remark can be made about the determination of the transition of being and nothing into each other, namely that it is to be understood as it is without any further elaboration of the transition by reflection. It is immediate and quite abstract because the transient moments are themselves abstract, that is, because the determinateness of either moment by means of which they passed over into each other is not yet posited in the other; nothing is not yet *posited* in being, although it is true that being is *essentially* nothing, and *vice versa*. It is therefore inadmissible to employ more developed forms of mediation here and to hold being and nothing in any kind of relationship—the transition in question is not yet a relation. Thus is it impermissible to say: nothing is the *ground* of being, or being is the *ground* of nothing or nothing is the *cause* of being, and so forth; or, transition into nothing can only occur *under the condition* that something *is*, or into being only *under the condition* of non-being. The kind of connexion cannot be further determined without the connected *sides* being further determined at the same time. The connexion of ground and consequent, etc., has no longer merely being and nothing as the sides which it connects, but expressly being which is a ground, and something which, although merely posited and not self-subsistent, is yet not the abstract nothing.

Remark 4: Incomprehensibility of the Beginning

What has been said indicates the nature of the dialectic against the beginning of the world and also its end, by which the *eternity* of matter was supposed to be proved, that is, the dialectic against *becoming*, coming-to-be or ceasing-to-be, in general. The Kantian antinomy relative to the finitude or infinity of the world in space and time will be considered more closely under the Notion of quantitative infinity. This simple, ordinary dialectic rests on holding fast to the opposition of being and nothing. It is proved

in the following manner that a beginning of the world, or of anything, is impossible:

It is impossible for anything to begin, either in so far as it is, or in so far as it is not; for in so far as it is, it is not just beginning, and in so far as it is not, then also it does not begin. If the world, or anything, is supposed to have begun, then it must have begun in nothing, but in nothing—or nothing—is no beginning; for a beginning includes within itself a being, but nothing does not contain any being. Nothing is only nothing. In a ground, a cause, and so on, if nothing is so determined, there is contained an affirmation, a being. For the same reason, too, something cannot cease to be; for then being would have to contain nothing, but being is only being, not the contrary of itself.

It is obvious that in this proof nothing is brought forward against becoming, or beginning and ceasing, against this *unity* of being and nothing, except an assertoric denial of them and an ascription of truth to being and nothing, each in separation from the other. Nevertheless this dialectic is at least more consistent than ordinary reflective thought which accepts as perfect truth that being and nothing only *are* in separation from each other, yet on the other hand acknowledges beginning and ceasing to be equally genuine determinations; but in these it does in fact assume the unseparatedness of being and nothing.

With the absolute separateness of being from nothing presupposed, then of course—as we so often hear—beginning or becoming is something *incomprehensible*; for a presupposition is made which annuls the beginning or the becoming which yet is *again* admitted, and this contradiction thus posed and at the same time made impossible of solution, is called *incomprehensible*.

The foregoing dialectic is the same, too, as that which understanding employs the notion of *infinitesimal* magnitudes given by higher analysis. A more detailed treatment of this notion will be given later. These magnitudes have been defined as such that they *are* in their vanishing, not *before* their vanishing, for then they are finite magnitudes, or *after* their vanishing, for then they are nothing. Against this pure notion it is objected and reiterated that such magnitudes are *either* something *or* nothing; that there is no *intermediate state* between being and non-being ('state' is here an unsuitable, barbarous expression). Here too, the absolute separation of being and nothing is assumed. But against this it

has been shown that being and nothing are, in fact, the same, or to use the same language as that just quoted, that *there is nothing which is not an intermediate state between being and nothing*. It is to the adoption of the said determination, which understanding opposes, that mathematics owes its most brilliant successes.

This style of reasoning which makes and clings to the false presupposition of the absolute separateness of being and nonbeing is to be named not dialectic but sophistry. For sophistry is an argument proceeding from a baseless presupposition which is uncritically and unthinkingly adopted; but we call dialectic the higher movement of reason in which such seemingly utterly separate terms pass over into each other spontaneously, through that which they are, a movement in which the presupposition sublates itself. It is the dialectical immanent nature of being and nothing themselves to manifest their unity, that is, becoming, as their truth.

2. MOMENTS OF BECOMING: COMING-TO-BE AND CEASING-TO-BE

Becoming is the unseparatedness of being and nothing, not the unity which abstracts from being and nothing; but as the unity of *being* and *nothing* it is this *determinate* unity in which there *is* both being and nothing. But in so far as being and nothing, each unseparated from its other, *is*, each *is not*. They *are* therefore in this unity but only as vanishing, sublated moments. They sink from their initially imagined *self-subsistence* to the status of *moments*, which are still *distinct* but at the same time are sublated.

Grasped as thus distinguished, each moment is in this *distinguishedness* as a unity with the *other*. Becoming therefore contains being and nothing as *two* such unities, *each* of which is itself a unity of being and nothing; the one is being as immediate and as relation to nothing, and the other is nothing as immediate and as relation to being; the determinations are of unequal values in these unities.

Becoming is in this way in a double determination. In one of them, *nothing* is immediate, that is, the determination starts from nothing which relates itself to being, or in other words changes into it; in the other, *being* is immediate, that is, the determination

starts from being which changes into nothing: the former is coming-to-be and the latter is ceasing-to-be.

Both are the same, *becoming*, and although they differ so in direction they interpenetrate and paralyse each other. The one is *ceasing-to-be*: being passes over into nothing, but nothing is equally the opposite of itself, transition into being, coming-to-be. This coming-to-be is the other direction: nothing passes over into being, but being equally sublates itself and is rather transition into nothing, is ceasing-to-be. They are not reciprocally sublated —the one does not sublate the other externally—but each sublates itself in itself and is in its own self the opposite of itself.

3. SUBLATION OF BECOMING

The resultant equilibrium of coming-to-be and ceasing-to-be is in the first place *becoming* itself. But this equally settles into a stable unity. Being and nothing are in this unity only as vanishing moments; yet becoming as such *is* only through their distinguishedness. Their vanishing, therefore, is the vanishing of becoming or the vanishing of the vanishing itself. Becoming is an unstable unrest which settles into a stable result.

This could also be expressed thus: becoming is the vanishing of being in nothing and of nothing in being and the vanishing of being and nothing generally; but at the same time it rests on the distinction between them. It is therefore inherently self-contradictory, because the determinations it unites within itself are opposed to each other; but such a union destroys itself.

This result is the vanishedness of becoming, but it is not *nothing*; as such it would only be a relapse into one of the already sublated determinations, not the resultant of *nothing and being*. It is the unity of being and nothing which has settled into a stable oneness. But this stable oneness is being, yet no longer as a determination on its own but as a determination of the whole.

Becoming, as this transition into the unity of being and nothing, a unity which is in the form of being or has the form of the one-sided *immediate* unity of these moments, is *determinate being*.

Remark: The Expression 'To Sublate'

To sublate, and the *sublated* (that which exists ideally as a moment), constitute one of the most important notions in philosophy. It is

a fundamental determination which repeatedly occurs throughout the whole of philosophy, the meaning of which is to be clearly grasped and especially distinguished from *nothing*. What is sublated is not thereby reduced to nothing. Nothing is *immediate*; what is sublated, on the other hand, is the result of *mediation*; it is a non-being but as a *result* which had its origin in a being. It still has, therefore, *in itself* the *determinateness from which it originates.*

'*To sublate*' has a twofold meaning in the language: on the one hand it means to preserve, to maintain, and equally it also means to cause to cease, to put an end to. Even 'to preserve' includes a negative element, namely, that something is removed from its immediacy and so from an existence which is open to external influences, in order to preserve it. Thus what is sublated is at the same time preserved; it has only lost its immediacy but is not on that account annihilated. The two definitions of 'to sublate' which we have given can be quoted as two dictionary *meanings* of this word. But it is certainly remarkable to find that a language has come to use one and the same word for two opposite meanings. It is a delight to speculative thought to find in the language words which have in themselves a speculative meaning; the German language has a number of such. The double meaning of the Latin *tollere* (which has become famous through the Ciceronian pun: *tollendum est Octavium*) does not go so far; its affirmative determination signifies only a lifting-up. Something is sublated only in so far as it has entered into unity with its opposite; in this more particular signification as something reflected, it may fittingly be called a *moment.* In the case of the lever, weight and distance from a point are called its mechanical moments on account of the sameness of their effect, in spite of the contrast otherwise between something real, such as a weight, and something ideal, such as a mere spatial determination, a line.[1] We shall often have occasion to notice that the technical language of philosophy employs Latin terms for reflected determinations, either because the mother tongue has no words for them or if it has, as here, because its expression calls to mind more what is immediate, whereas the foreign language suggests more what is reflected.

The more precise meaning and expression which being and nothing receive, now that they are *moments*, is to be ascertained

[1] See *Encycl.* 3rd edition, Section 261, Remark.

from the consideration of determinate being as the unity in which they are preserved. Being is being, and nothing is nothing, only in their contradistinction from each other; but in their truth, in their unity, they have vanished as these determinations and are now something else. Being and nothing are the same; ***but just because they are the same they are no longer being and nothing***, but now have a different significance. In becoming they were coming-to-be and ceasing-to-be; in determinate being, a differently determined unity, they are again differently determined moments. This unity now remains their base from which they do not again emerge in the abstract significance of being and nothing.

CHAPTER 2

DETERMINATE BEING

In considering determinate being the emphasis falls on its determinate character; the determinateness is in the form of *being*, and as such it is *quality*. Through its quality, something is determined as opposed to an other, as *alterable* and *finite*; and as negatively determined not only against an other but also in its own self. This its negation as at first opposed to the finite something is the *infinite*; the abstract opposition in which these determinations appear resolves itself into the *infinity* which is free from the opposition, into *being-for-self*.

The treatment of determinate being falls therefore into three parts:

A. Determinate being as such
B. Something and other, finitude
C. Qualitative infinity.

A. DETERMINATE BEING AS SUCH

In determinate being (*a*) *as such*, its determinateness is first of all (*b*) to be distinguished as *quality*. This, however, is to be taken as well in the one determination of determinate being as in the other—as *reality* and *negation*. But in these determinatenesses determinate being is equally reflected into itself; and posited as such it is (*c*) *something*, *a* determinate being.

(*a*) *Determinate Being in General*

From becoming there issues determinate being, which is the simple oneness of being and nothing. Because of this oneness it has the form of *immediacy*. Its mediation, becoming, lies behind it; it has sublated itself and determinate being appears, therefore, as a first, as a starting-point for the ensuing development. It is first of all in the one-sided determination of *being*; the other determination, *nothing*, will likewise display itself and in contrast to it.

It is not mere being, but determinate being [*Dasein*], etymologically taken, being in a certain *place*; but the idea of space is irrelevant here. Determinate being as the result of its becoming is, in general, being with a non-being such that this non-being is taken up into simple unity with being. *Non-being* thus taken up into being in such a way that the concrete whole is in the form of being, of immediacy, constitutes *determinateness* as such.

The *whole* is likewise in the form, that is, in the *determinateness* of being, for being has likewise shown itself in becoming to be only a moment—a sublated, negatively determined being; but it is such *for us in our reflection*, it is not yet *posited* as such in its own self. But the determinateness as such of determinate being is the determinateness which is posited, and this is implied in the expression *Dasein* [*there*-being or being which is *there*]. The two are always to be clearly distinguished from each other; only that which is *posited* in a Notion belongs in the dialectical development of that Notion to its content; whereas the determinateness that is not yet posited in the Notion itself belongs to our reflection, whether it concerns the nature of the Notion itself or is an external comparison. To draw attention to a determinateness of the latter kind can only serve to elucidate or indicate in advance the course which will be exhibited in the development itself. That the whole, the unity of being and nothing, is in the one-sided determinateness of being is an external reflection; but in the negation, in *something* and *other* and so on, it will come to be *posited.* It was necessary here to draw attention to the distinction referred to; but to take account of all the remarks which may be prompted by reflection would lead to the prolixity of anticipating what must yield itself in the subject matter. Such reflections may facilitate a general view and thereby an understanding of the development, but they also have the disadvantage of appearing as unjustified assertions, grounds and foundations for what is to follow. They should therefore not be taken for more than they are supposed to be and should be distinguished from what is a moment in the development of the subject matter itself.

Determinate being corresponds to *being* in the previous sphere, but being is indeterminate and therefore no determinations issue from it. *Determinate* being, however, is *concrete*; consequently a number of determinations, distinct relations of its moments, make their appearance in it.

(b) Quality

Because of the immediacy of the oneness of being and nothing in determinate being, they do not extend beyond each other; so far as determinate being is in the form of being, so far is it non-being, so far is it determinate. Being is not the *universal*, determinateness not the *particular*. Determinateness has not yet severed itself from being; and indeed it will no more sever itself from being, for the truth which from now on underlies them as ground is the unity of non-being with being; on this as ground all further determinations are developed. But the relation in which determinateness here stands to being is the immediate unity of both, so that as yet no differentiation of this unity is posited.

Determinateness thus isolated by itself in the form of *being* is *quality*—which is wholly simple and immediate. *Determinateness* as such is the more universal term which can equally be further determined as quantity and so on. Because of this simple character of quality as such, there is nothing further to be said about it.

Determinate being, however, in which nothing no less than being is contained, is itself the criterion for the one-sidedness of quality as a determinateness which is only *immediate* or only in the form of *being*. It is equally to be posited in the determination of nothing, when it will be posited as a differentiated, reflected determinateness, no longer as immediate or in the form of being. Nothing, as thus the determinate element of a determinateness, is equally something reflected, a *negation*. Quality, taken in the distinct character of *being*, is *reality*; as burdened with a negative it is *negation* in general, likewise a quality but one which counts as a deficiency, and which further on is determined as limit, limitation.

Both are determinate being, but in *reality* as quality with the accent on *being*, the fact is concealed that it contains determinateness and therefore also negation. Consequently, reality is given the value only of something positive from which negation, limitation and deficiency are excluded. Negation taken as mere deficiency would be equivalent to nothing; but it is a *determinate* being, a quality, only determined with a non-being.

Remark: Quality and Negation

Reality may seem to be a word of various meanings because it is used of different, indeed of opposed determinations. In philosophy

one may perhaps speak of a *merely empirical* reality as of a worthless existence. But when it is said that thoughts, concepts, theories *have no reality*, this means that they do not possess *actuality*; *in itself* or in its notion, the idea of a Platonic Republic, for example, may well be true. Here the worth of the idea is not denied and it is left its place *alongside* the reality. But as against *mere* ideas, *mere* notions, the real alone counts as true. The sense in which, on the one hand, outer existence is made the criterion of the truth of a content is no less one-sided than when the idea, essential being, or even inner feeling is represented as indifferent to outer existence and is even held to be the more excellent the more remote it is from reality.

In connection with the term 'reality', mention must be made of the former metaphysical concept of God which, in particular, formed the basis of the so-called ontological proof of the existence of God. God was defined as the sum-total of all realities, and of this sum-total it was said that no contradiction was contained in it, that none of the realities cancelled any other; for a reality is to be taken only as a perfection, as an *affirmative* being which contains no negation. Hence the realities are not opposed to one another and do not contradict one another.

Reality as thus conceived is assumed to survive when all negation has been thought away; but to do this is to do away with all determinateness. Reality is quality, determinate being; consequently, it contains the moment of the negative and is through this alone the determinate being that it is. Reality, taken as we are supposed to take it, in the so-called *eminent sense* or as *infinite* —in the usual meaning of the word—is expanded into indeterminateness and loses its meaning. God's goodness is not to be goodness in the ordinary, but in the eminent sense; not different from justice but *tempered* by it (a mediatory expression used by Leibniz), just as, conversely, justice is tempered by goodness; and so goodness is no longer goodness, nor justice any more justice. Power is supposed to be tempered by wisdom, but in that case it is not power as such for it would be subject to wisdom; wisdom is supposed to be expanded into power, in which case it vanishes as the wisdom which determines the end and measure of things. The true Notion of the infinite and its *absolute* unity which will present itself later, is not to be understood as a *tempering*, a *reciprocal restricting* or a *mixing*; such a superficial con-

ception of the relationship leaves it indefinite and nebulous and can satisfy only a Notion-less way of thinking. When reality, taken as a determinate quality as it is in the said definition of God, is extended beyond its determinateness it ceases to be reality and becomes abstract being; God as the *pure* reality in all realities, or as the sum total of all realities, is just as devoid of determinateness and content as the empty absolute in which all is one.

If, on the other hand, reality is taken in its determinateness, then, since it essentially contains the moment of the negative, the sum-total of all realities becomes just as much a sum-total of all negations, the sum-total of all contradictions; it becomes then straightway the absolute *power* in which everything determinate is absorbed; but reality itself *is*, only in so far as it is still confronted by a being which it has not sublated; consequently, when it is thought as expanded into realized, limitless power, it becomes the abstract nothing. The said reality in all realities, the being in all determinate being, which is supposed to express the concept of God, is nothing else than abstract being, which is the same as nothing.

Determinateness is negation posited as affirmative and is the proposition of Spinoza: *omnis determinatio est negatio.* This proposition is infinitely important; only, negation as such is formless abstraction. However, speculative philosophy must not be charged with making negation or nothing an ultimate: negation is as little an ultimate for philosophy as reality is for it truth.

Of this proposition that determinateness is negation, the unity of Spinoza's substance—or that there is only one substance—is the necessary consequence. *Thought* and *being* or extension, the two attributes, namely, which Spinoza had before him, he had of necessity to posit as one in this unity; for as determinate realities they are negations whose infinity is their unity. According to Spinoza's definition, of which more subsequently, the infinity of anything is its affirmation. He grasped them therefore as attributes, that is, as not having a separate existence, a self-subsistent being of their own, but only as sublated, as moments; or rather, since substance in its own self lacks any determination whatever, they are for him not even moments, and the attributes like the modes are distinctions made by an external intellect. Similarly, the substantiality of individuals cannot persist in the face of that proposition. The individual is a relation-to-self through its setting limits

to everything else; but these limits are thereby also limits of itself, relations to an other, it does not possess its determinate being within itself. True, the individual is *more* than merely an entity bounded on all sides, but this *more* belongs to another sphere of the Notion; in the metaphysics of being, the individual is simply a determinate something, and in opposition to the independence and self-subsistence of such something, to the finite as such, determinateness effectively brings into play its essentially negative character, dragging what is finite into that same negative movement of the understanding which makes everything vanish in the abstract unity of substance.

Negation stands directly opposed to reality: further on, in the special sphere of reflected determinations, it becomes opposed to the *positive*, which is reality reflecting the negation—the reality in which the negative *has an illusory being* [*scheint*], the negative which in reality as such is still hidden.

Quality is especially a *property* only where, in an *external* relation, it manifests itself as an *immanent* determination. By properties of herbs, for instance, we understand determinations which not only are *proper* to something, but are the means whereby this something in its relations with other somethings *maintains* itself in its own peculiar way, counteracting the alien influences posited in it and making its own determinations *effective* in the other—although it does not keep this at a distance. The more stable determinatenesses, on the other hand, such as figure, shape, are not called properties, nor even qualities perhaps, because they are conceived as alterable, as not identical with the *being* [of the object].

'*Qualierung*' or '*Inqualierung*', an expression of Jacob Boehme's, whose philosophy goes deep, but into a turbid depth, signifies the movement of a quality (of sourness, bitterness, fieriness, etc.) within itself in so far as it posits and establishes itself in its negative nature (in its '*Qual*' or torment) from out of an other —signifies in general the quality's own internal unrest by which it produces and maintains itself only in conflict.

(*c*) *Something*

In determinate being its determinateness has been distinguished as quality; in quality as determinately present, there *is* distinction —of reality and negation. Now although these distinctions are

present in determinate being, they are no less equally void and sublated. Reality itself contains negation, is determinate being, not indeterminate, abstract being. Similarly, negation is determinate being, not the supposedly abstract nothing but posited here as it is in itself, as affirmatively present [*als seiend*], belonging to the sphere of determinate being. Thus quality is completely unseparated from determinate being, which is simply determinate, qualitative being.

This sublating of the distinction is more than a mere taking back and external omission of it again, or than a simple return to the simple beginning, to determinate being as such. The distinction cannot be omitted, for it *is*. What is, therefore, in fact present is determinate being in general, distinction in it, and sublation of this distinction; determinate being, not as devoid of distinction as at first, but as *again* equal to itself through sublation of the distinction, the simple oneness of determinate being *resulting* from this sublation. This sublatedness of the distinction is determinate being's *own* determinateness; it is thus *being-within-self*: determinate being is *a determinate being*, a *something*.

Something is the *first negation of negation*, as simple self-relation in the form of being. Determinate being, life, thought, and so on, essentially determine themselves to become a determinate being, a living creature, a thinker (ego) and so on. This determination is of supreme importance if we are not to remain at the stage of determinate being, life, thought, and so on—also the Godhead (instead of God)—as generalities. In our ordinary way of thinking, *something* is rightly credited with reality. However, something is still a very superficial determination; just as reality and negation, determinate being and its determinateness, although no longer blank being and nothing, are still quite abstract determinations. It is for this reason that they are the most current expressions and the intellect which is philosophically untrained uses them most, casts its distinctions in their mould and fancies that in them it has something really well and truly determined. The negative of the negative is, as *something*, only the beginning of the subject [*Subjekt*]—being-within-self, only as yet quite indeterminate. It determines itself further on, first, as *a being-for-self* and so on, until in the Notion it first attains the concrete intensity of the subject. At the base of all these determinations lies the negative unity with itself. But in all this, care must be

taken to distinguish between the *first* negation as negation *in general*, and the second negation, the negation of the negation: the latter is concrete, *absolute* negativity, just as the former on the contrary is only *abstract* negativity.

Something is the negation of the negation in the form of *being*; for this second negation is the restoring of the simple relation to self; but with this, something is equally *the mediation of itself with itself*. Even in the simple form of *something*, then still more specifically in *being-for-self*, *subject*, and so on, self-mediation is present; it is present even in *becoming*, only the mediation is quite abstract. In *something*, mediation with self is *posited*, in so far as something is determined as a simple identity. Attention can be drawn to the presence of mediation in general, as against the principle of the alleged mere immediacy of knowledge, from which mediation is supposed to be excluded; but there is no further need to draw particular attention to the moment of mediation, for it is to be found everywhere, in every Notion.

This mediation with itself which something is *in itself*, taken only as negation of the negation, has no concrete determinations for its sides; it thus collapses into the simple oneness which is *being*. Something *is*, and *is*, then, also a determinate being; further, it is *in itself* also *becoming*, which, however, no longer has only being and nothing for its moments. One of these, being, is now determinate being, and, further, *a* determinate being. The second is equally a *determinate* being, but determined as a negative of the something—an *other*. Something as a *becoming* is a transition, the moments of which are themselves somethings, so that the transition is *alteration*—a becoming which has already become *concrete*. But to begin with, something alters only in its Notion; it is not yet *posited* as mediating and mediated, but at first only as simply maintaining itself in its self-relation, and its negative is posited as equally qualitative, as only an *other* in general.

B. FINITUDE

(*a*) Something *and* other are at first indifferent to one another; an other is also immediately a determinate being, a something; the negation thus falls outside both. Something is *in itself* as against its *being-for-other*. But the determinateness also belongs to its *in-itself* and is

(*b*) its *determination*; this equally passes over into *constitution* which, being identical with the determination, constitutes the immanent and at the same time negated being-for-other, the *limit* of the something. This limit is

(*c*) the immanent determination of the something itself, which latter is thus the *finite*.

In the first section, in which *determinate being* in general was considered, this had, as at first taken up, the determination of *being*. Consequently, the moments of its development, quality and something, equally have an affirmative determination. In this section, on the other hand, the negative determination contained in determinate being is developed, and whereas in the first section it was at first only negation in general, the *first* negation, it is now determined to the point of the *being-within-self* or the *inwardness* of the something, to the negation of the negation.

(*a*) *Something and an Other*

1. Something and other are, in the first place, both determinate beings or somethings.

Secondly, each is equally an other. It is immaterial which is first named and solely for that reason called *something*; (in Latin, when they both occur in a sentence, both are called *aliud*, or 'the one, the other', *alius alium*; when there is reciprocity the expression *alter alterum* is analogous). If of two things we call one A, and the other B, then in the first instance B is determined as the other. But A is just as much the other of B. Both are, in the same way, *others*. The word 'this' serves to fix the distinction and the something which is to be taken affirmatively. But 'this' clearly expresses that this distinguishing and signalizing of the one something is a subjective designating falling outside the something itself. The entire determinateness falls into this external pointing out; even the expression 'this' contains no distinction; each and every something is just as well a 'this' as it is also an other. By 'this' we *mean* to express something completely determined; it is overlooked that speech, as a work of the understanding, gives expression only to universals, except in the *name* of a single object; but the individual name is meaningless, in the sense that it does not express a universal, and for the same reason appears as something merely posited and arbitrary; just as proper names, too, can be arbitrarily assumed, given or also altered.

Otherness thus appears as a determination alien to the determinate being thus characterized, or as the other *outside* the one determinate being; partly because a determinate being is determined as other only through being *compared* by a Third, and partly because it is only determined as other on account of the other which is outside it, but is not an other on its own account. At the same time, as has been remarked, every determinate being, even for ordinary thinking, determines itself as an other, so that there is no determinate being which is determined only as such, which is not outside a determinate being and therefore is not itself an other.

Both are determined equally as something and as other, and are thus the same, and there is so far no distinction between them. But this self-sameness of the determinations likewise arises only from external reflection, from the *comparing* of them; but the other as at first posited, although an other in relation to the something, is nevertheless also an other on its own account, apart from the something.

Thirdly, therefore, the other is to be taken as isolated, as in relation to itself, *abstractly* as the *other*; the τὸ ἕτερον of Plato, who opposes it as one of the moments of totality to the One, and in this way ascribes to the other a *nature* of its own. Thus the other, taken solely as such, is not the other of something but the other in its own self, that is, the other of itself. Such an other, determined as other, is physical nature; it is the other of spirit. This its determination is thus at first a mere relativity by which is expressed, not a quality of nature itself, but only a relation external to it. However, since spirit is the true something and nature, consequently, in its own self is only what it is as contrasted with spirit, the quality of nature taken as such is just this, to be the *other* in its own self, that which is *external to itself* (in the determinations of space, time and matter).

The other simply by itself is the other in its own self, hence the other of itself and so the other of the other—it is, therefore, that which is absolutely dissimilar within itself, that which negates itself, *alters* itself. But in so doing it remains identical with itself, for that into which it alters is the other, and this is its sole determination; but what is altered is not determined in any different way but in the same way, namely, to be an other; in this latter, therefore, it only unites with its own self. It is thus posited as

reflected into itself with sublation of the otherness, as a self-identical something from which, consequently, the otherness, which is at the same time a moment of it, is distinct from it and does not appertain to the something itself.

2. Something *preserves* itself in the negative of its determinate being [*Nichtdasein*]; it is essentially *one* with it and essentially *not one* with it. It stands, therefore, in a *relation* to its otherness and is not simply its otherness. The otherness is at once contained in it and also still *separate* from it; it is a *being-for-other*.

Determinate being as such is immediate, without relation to an other; or, it is in the determination of *being*; but as including within itself non-being, it is *determinate* being, being negated within itself, and then in the first instance an other—but since at the same time it also preserves itself in its negation, it is only a *being-for-other*.

It preserves itself in the negative of its determinate being and is being, but not being in general, but as self-related in *opposition* to its relation to other, as self-equal in opposition to its inequality. Such a being is *being-in-itself*.

Being-for-other and being-in-itself constitute the two moments of the something. There are here present *two pairs* of determinations: 1. Something and other, 2. Being-for-other and being-in-itself. The former contain the unrelatedness of their determinateness; something and other fall apart. But their truth is their relation; being-for-other and being-in-itself are, therefore, the above determinations posited as *moments* of one and the same something, as determinations which are relations and which remain in their unity, in the unity of determinate being. Each, therefore, at the same time, also contains within itself its other moment which is distinguished from it.

Being and nothing in their unity, which is determinate being, are no longer being and nothing—these they are only outside their unity—thus in their unstable unity, in becoming, they are coming-to-be and ceasing-to-be. The being in something is *being-in-itself*. Being, which is self-relation, equality with self, is now no longer immediate, but is only as the non-being of otherness (as determinate being reflected into itself). Similarly, non-being as a moment of something is, in this unity of being and non-being, not negative determinate being in general, but an other, and more specifically—seeing that being is differentiated from it—at the

same time a *relation* to its negative determinate being, a being-for-other.

Hence being-in-itself is, first, a negative relation to the negative determinate being, it has the otherness outside it and is opposed to it; in so far as something is *in itself* it is withdrawn from otherness and being-for-other. But secondly it has also present in it non-being itself, for it is itself the *non-being* of being-for-other.

But being-for-other is, first, a negation of the simple relation of being to itself which, in the first instance, is supposed to be determinate being and something; in so far as something is in an other or is for an other, it lacks a being of its own. But secondly it is not negative determinate being as pure nothing; it is negative determinate being which points to being-in-itself as to its own being which is reflected into itself, just as, conversely, being-in-itself points to being-for-other.

3. Both moments are determinations of what is one and the same, namely, the something. Something is *in itself* in so far as it has returned into itself out of the being-for-other. But something also has *in itself* (here the accent falls on *in*) or *within it*, a determination or circumstance in so far as this circumstance is outwardly *in it*, is a being-for-other.

This leads to a further determination. Being-in-itself and being-for-other are, in the first instance, distinct; but that something also has *within it* the same character that it is *in itself*, and, conversely, that what it is as being-for-other it also is in itself—this is the identity of being-in-itself and being-for-other, in accordance with the determination that the something itself is one and the same something of both moments, which, therefore, are undividedly present in it. This identity is already formally given in the sphere of determinate being, but more expressly in the consideration of *essence* and of the relation of *inner* and *outer*, and most precisely in the consideration of the Idea as the unity of the Notion and *actuality*. People fancy that they are saying something lofty with the expression 'in itself', as they do in saying 'the inner'; but what something is *only in itself*, is also *only* in *it*; 'in itself' is only an abstract, and so even external determination. The expressions: there is nothing *in it*, or, there is something *in it*, imply, though somewhat obscurely, that what is *in* a thing also belongs to the thing's *being-in-itself*, to its inner, true worth.

It may be observed that the meaning of the *thing-in-itself* is

here revealed; it is a very simple abstraction but for some while it counted as a very important determination, something superior, as it were, just as the proposition that we do not know what things are in themselves ranked as a profound piece of wisdom. Things are called 'in themselves' in so far as abstraction is made from all being-for-other, which means simply, in so far as they are thought devoid of all determination, as nothings. In this sense, it is of course impossible to know *what* the *thing-in-itself* is. For the question: *what*? demands that *determinations* be assigned; but since the things of which they are to be assigned are at the same time supposed to be *things in-themselves*, which means, in effect, to be without any determination, the question is thoughtlessly made impossible to answer, or else only an absurd answer is given. The thing-in-itself is the same as that *absolute* of which we know nothing except that in it all is one. What is *in* these things-in-themselves, therefore, we know quite well; they are as such nothing but truthless, empty abstractions. What, however, the thing-in-itself is in truth, what truly is in itself, of this logic is the exposition, in which however something better than an abstraction is understood by 'in-itself', namely, what something is in its Notion; but the Notion is concrete within itself, is comprehensible simply as Notion, and as determined within itself and the connected whole of its determinations, is cognizable.

Being-in-itself, in the first instance, has being-for-other as its contrasted moment; but *positedness*, too, is contrasted with it. This expression, it is true, includes also being-for-other, but it specifically contains the already accomplished bending back of that which is not *in itself* into that which is its being-in-itself, in which it is *positive*. Being-in-itself is generally to be taken as an abstract way of expressing the Notion; *positing*, properly speaking, first occurs in the sphere of *essence*, of objective reflection; the *ground posits* that which is grounded by it; still more, the *cause produces* or *brings forth* an *effect*, a determinate being whose self-subsistence is *immediately* negated and which carries the meaning of having its matter [*Sache*], its being, in an other. In the sphere of being, determinate being only *proceeds* from *becoming*, or, with the something an other is posited, with the finite, the infinite; but the finite does not bring forth the infinite, does not *posit* it. In the sphere of being, the *self-determining* even of the Notion is at first only *in itself* or *implicit*—as such it is called a transition; and the

reflected determinations of being such as something and other, or finite and infinite, although they essentially refer to each other or are as a being-for-other, they too count as *qualitative*, as existing on their own account; the other *is*, the finite ranks equally with the infinite as an immediate, affirmative being, standing fast on its own account; the meaning of each appears to be complete even without its other. On the other hand positive and negative, cause and effect, however much they may be taken as isolated from each other, are at the same time meaningless one without the other. There is *present in them* their showing or reflection in each other, the showing or reflection in each of its other. In the different spheres of determination and especially in the progress of the exposition, or more precisely in the progress of the Notion towards the exposition of itself, it is of capital importance always clearly to distinguish what is still *in itself* and what is *posited*, the determinations as they are in the Notion, and as they are as posited, or as being-for-other. This is a distinction which belongs only to the dialectical development and which is unknown to metaphysical philosophizing, which also includes the critical philosophy; the definitions of metaphysics, like its presuppositions, distinctions and conclusions, seek to assert and produce only what comes under the category of *being*, and that, too, of *being-in-itself.*

Being-for-other is, in the unity of the something with itself, identical with its *in-itself*; the being-for-other is thus present *in* the something. The determinateness thus reflected into itself is, therefore, again in the simple form of *being*, and hence is again a quality: *determination.*

(*b*) *Determination, Constitution and Limit*

The *in-itself* into which something is reflected into itself out of its being-for-other is no longer an abstract in-itself, but as negation of its being-for-other is mediated by the latter, which is thus its moment. It is not only the immediate identity of the something with itself, but the identity through which there is present *in* the the something that which it is *in itself*; being-for-other is present *in it* because the *in-itself* is the sublation of the being-for-other, has returned *out of* the being-for-other into itself; but equally, too, simply because it is abstract and therefore essentially burdened with negation, with being-for-other. There is present here

not only quality and reality, determinateness in the form of simple being, but determinateness in the form of the *in-itself*; and the development consists in *positing* this determinateness as reflected into itself.

1. The quality which is constituted by the essential unity of the in-itself in the simple something with its other moment, the *presence in it* of its being-for-other, can be called the *determination* of the in-itself, in so far as this word in its exact meaning is distinguished from *determinateness* in general. Determination is affirmative determinateness as the in-itself with which something in its determinate being remains congruous in face of its entanglement with the other by which it might be determined, maintaining itself in its self-equality, and making its determination hold good in its being-for-other. Something *fulfils* its determination in so far as the further determinateness which at once develops in various directions through something's relation to other, is congruous with the in-itself of the something, becomes its filling. Determination implies that what something is *in itself*, is also *present in* it.

The *determination* of man is thinking reason; thought in general, thought as such, in his simple *determinateness*—by it he is distinguished from the brute; *in himself* he is thought, in so far as this is also distinguished from his being-for-other, from his own natural existence and sense-nature through which he is directly connected with his other. But thought is also present *in* him; man himself is thought, he actually exists as thinking, it is his concrete existence and actuality; and, further, since thought is in his determinate being and his determinate being is in thought, it is to be taken as *concrete*, as having content and filling; it is thinking reason and as such the *determination* of man. But even this determination again is only *in itself* as something which *ought* to be, that is it, together with the filling which is incorporated in its in-itself, is in the form of the in-itself in general, *in contrast to* the determinate being not incorporated in it, which at the same time still confronts it externally as immediate sense-nature and nature.

2. The filling of the in-itself with determinateness is also distinct from the determinateness which is only being-for-other and remains outside the determination. For in the sphere of quality, the differences in their sublated form as moments also retain the form of immediate, qualitative being relatively to one another.

That which something has *in it*, thus divides itself and is from this side an external determinate being of the something, which is also *its* determinate being, but does not belong to the something's in-itself. The determinateness is thus a *constitution*.

Constituted in this or that way, something is involved in external influences and relationships. This external connection on which the constitution depends, and the circumstance of being determined by an other, appears as something contingent. But it is the quality of something to be open to external influences and to have a *constitution*.

In so far as something alters, the alteration falls within its constitution; it is that *in* the something which becomes an other. The something itself preserves itself in the alteration which affects only this unstable surface of its otherness, not its determination.

Determination and constitution are thus distinguished from each other; something, in accordance with its determination, is indifferent to its constitution. But that which something has *in it*, is the middle term connecting them in this syllogism. Or, rather, the *being-in-the-something* showed itself as falling apart into these two extremes. The simple middle term is *determinateness* as such; to its identity belongs both determination and constitution. But determination spontaneously passes over into constitution, and the latter into the former. This is implied in what has been said already; the connection is more precisely this: in so far as that which something is *in itself* is also present *in it*, it is burdened with being-for-other; hence the determination is, as such, open to relationship to other. The determinateness is at the same time a moment, but contains at the same time the qualitative distinction of being different from the in-itself, of being the negative of the something, another determinate being. The determinateness which thus holds the other within it, being united with the in-itself, brings the otherness into the latter or into the determination, which, consequently, is reduced to constitution. Conversely, being-for-other isolated as constitution and posited by itself, is in its own self the same as the other as such, the other in its own self, that is, the other of itself; but thus it is *self-related* determinate being, the in-itself with a determinateness, and therefore a *determination*. Consequently, in so far as determination and constitution are also to be held apart, the latter, which appears to be grounded in some-

thing external, in an other in general, also *depends* on the former, and the determining from outside is at the same time determined by the something's own, immanent determination. But further, the constitution belongs to that which the something is in itself; something alters with its constitution.

This alteration of something is no longer the first alteration of something merely in accordance with its being-for-other; the first was only an implicit (*an sich seiende*) alteration belonging to the inner Notion; now alteration is also *posited* in the something. The something itself is further determined and the negation is posited as immanent in it, as its developed *being-within-self*.

In the first place, the transition of determination and constitution into each other is the sublation of their difference, resulting in the positing of determinate being or something in general; and since this latter results from that difference which equally includes within it qualitative otherness, there are *two* somethings which, however, are not opposed to each other only as others in general—for in that case this negation would still be abstract and would arise only from *comparing* them. But the negation is now *immanent* in the somethings. As *determinate beings* they are indifferent to each other, but this their affirmation is no longer immediate, each relates itself to itself only *by means of* the sublation of the otherness which, in the determination, is reflected into the in-itself.

Thus something *through its own nature* relates itself to the other, because otherness is posited in it as its own moment; its being-within-self includes the negation within it, by means of which alone it now has its affirmative determinate being. But the other is also qualitatively distinguished from this and is thus posited outside the something. The negation of its other is now[1] the quality of the something, for it is as this sublating of its other that it is something. It is only in this sublation that the other is really opposed to another determinate being; the other is only externally opposed to the *first* something, or rather, since in fact they are *directly* connected, that is in their Notion, their connection is this, that determinate being has *passed over* into otherness, something into other, and something is just as much an other as the other itself is. Now in so far as the being-within-self is the non-being of the otherness which is contained in it but which at the same time has a distinct being of its own, the some-

[1] Reading *nun* for *nur*.

thing is itself the negation, *the ceasing of an other in it*; it is posited as relating itself negatively to the other and in so doing preserving itself; this other, the being-within-self of the something as negation of the negation, is its *in-itself*, and at the same time this sublation is *present in it* as a simple negation, namely, as its negation of the other something external to it. There is a *single* determinateness of both, which on the one hand is identical with the being-within-self of the somethings as negation of the negation, and on the other hand, since these negations are opposed to one another as other somethings, conjoins and equally disjoins them through their own nature, each negating the other: this determinateness is *limit*.

3. Being-for-other is the indeterminate, affirmative community of something with its other; in the limit the *non-being*-for-other becomes prominent, the qualitative negation of the other, which is thereby kept apart from the something which is reflected into itself. We must observe the development of this Notion, which manifests itself, however, rather as an entanglement and a contradiction. This contradiction is at once to be found in the circumstance that the limit, as something's negation reflected into itself, contains *ideally* in it the moments of something and other, and these, as distinguished moments, are at the same time posited in the sphere of determinate being as *really, qualitatively distinct*.

(α) Something, therefore, is immediate, self-related determinate being, and has a limit, in the first place, relatively to an other; the limit is the non-being of the other, not of the something itself: in the limit, something limits its other. But the other is itself a something in general, therefore the limit which something has relatively to the other is also the limit of the other as a something, its limit whereby it keeps the first something as *its* other apart from it, or is a *non-being of that something*; it is thus not only non-being of the other, but non-being equally of the one and of the other something, consequently of the something as such.

But the limit is essentially equally the non-being of the other, and so something at the same time *is* through its limit. It is true that something, in limiting the other, is subjected to being limited itself; but at the same time its limit is, as the ceasing of the other in it, itself only the being of the something; *through the limit something is what it is, and in the limit it has its quality*. This relationship is the outward manifestation of the fact that the limit

is simple negation or the *first* negation, whereas the other is, at the same time, the negation of the negation, the being-within-self of the something.

Something, as an immediate determinate being, is, therefore, the limit relatively to another something, but the limit is present in the something itself, which is a something through the mediation of the limit which is just as much the non-being of the something. Limit is the mediation through which something and other each as well *is*, as *is not*.

(β) Now in so far as something in its limit both *is* and *is not*, and these moments are an immediate, qualitative difference, the negative determinate being and the determinate being of the something fall outside each other. Something has its determinate being *outside* (or, as it is also put, on the *inside*) of its limit; similarly, the other, too, because it is a something, is outside it. Limit is the *middle between* the two of them in which they cease. They have their determinate being *beyond* each other and *beyond* their limit; the limit as the non-being of each is the other of both.

It is in accordance with this difference of something from its limit that the line appears as line only outside its limit, the point; the plane as plane outside the line; the solid as solid only outside its limiting surface. It is primarily this aspect of limit which is seized by pictorial thought—the self-externality of the Notion—and especially, too, in reference to spatial objects.

(γ) But further, something as it is outside the limit, the unlimited something, is only a determinate being in general. As such, it is not distinguished from its other; it is only determinate being and therefore has the same determination as its other; each is only a something in general, or each is an other; thus both are the *same*. But this their primarily immediate determinate being is now posited with the determinateness as limit, in which both are what they are, distinguished from each other. Limit is, however, equally their *common* distinguishedness, their unity and distinguishedness, like determinate being. This double identity of both, of determinate being and limit, contains this: that something has its determinate being only in the limit, and that since the limit and the determinate being are each at the same time the negative of each other, the something, which *is* only in its limit, just as much separates itself from itself and points beyond itself to its non-being, declaring this is to be its being and thus passing

over into it. To apply this to the preceding example and taking first the determination that something is what it is only in its limit: as thus determined, the point is therefore the limit of the line, not merely in the sense that the line only ceases in the point, and as a determinate being is outside it; neither is the line the limit of the plane merely in the sense that the plane only ceases in it—and similarly with surface as limit of the solid; on the contrary, in the point the line also *begins*; the point is its absolute beginning. Even when the line is represented as unlimited on either side, or, as it is put, is produced to infinity, the point still constitutes its *element*, just as the line is the element of the plane, and the surface that of the solid. These limits are the *principle* of that which they limit; just as one, for example as hundredth, is the limit, but also the element, of the whole hundred.

The other determination is the unrest of the something in its limit in which it is immanent, an unrest which is the *contradiction* which impels the something out beyond itself. Thus the point is this dialectic of its own self to become a line, the line the dialectic to become a plane, and the plane the dialectic to become total space. A second definition is given of line, plane and total space: namely, that the line originates through the *movement* of the point, the plane through the movement of the line, and so on. But this *movement* of the point, line and so on, is regarded as something contingent or as only thus imagined. This point of view is, however, really retracted in so far as the determinations from which the line and so on are supposed to originate are their elements and principles which, at the same time, are nothing else but their limits; and so the origin is not considered as contingent or as only thus imagined. That point, line and plane by themselves are self-contradictory, are *beginnings* which spontaneously repel themselves from themselves, so that the point, through its Notion, passes out of itself into the line, *moves in itself* and gives rise to the line, and so on, lies in the Notion of limit which is immanent in the something. The application itself, however, belongs to the consideration of space; to give an indication of it here, the point is the wholly abstract limit, *but in a determinate being*; this is taken as still wholly abstract, it is so-called absolute, that is, abstract *space*, a purely continuous asunderness. But the limit is not abstract negation, but is *in this determinate being*, is a *spatial* determinateness; the point is, therefore, spatial, the contradiction

of abstract negation and continuity, and is, therefore, the transition, the accomplished transition into the line, and so on; just as also, for the same reason, *there is* no such thing as a point, line or plane.

Something with its immanent limit, posited as the contradiction of itself, through which it is directed and forced out of and beyond itself, is the *finite*.

(*c*) *Finitude*

The being of something is determinate; something has a quality and in it is not only determined but limited; its quality is its limit and, burdened with this, it remains in the first place an affirmative, stable being. But the development of this negation, so that the opposition between its determinate being and the negation as its immanent limit, is itself the being-within-self of the something, which is thus in its own self only a becoming, constitutes the finitude of something.

When we say of things that *they are finite*, we understand thereby that they not only have a determinateness, that their quality is not only a reality and an intrinsic determination, that finite things are not merely limited—as such they still have determinate being outside their limit—but that, on the contrary, non-being constitutes their nature and being. Finite things *are*, but their relation to themselves is that they are *negatively* self-related and in this very self-relation send themselves away beyond themselves, beyond their being. They *are*, but the truth of this being is their *end*. The finite not only alters, like something in general, but it *ceases to be*; and its ceasing to be is not merely a possibility, so that it could be without ceasing to be, but the being as such of finite things is to have the germ of decease as their being-within-self: the hour of their birth is the hour of their death.

(α) *The Immediacy of Finitude*

The thought of the finitude of things brings this sadness with it because it is qualitative negation pushed to its extreme, and in the singleness of such determination there is no longer left to things an affirmative being *distinct* from their destiny to perish. Because of this qualitative singleness of the negation, which has gone back to the abstract opposition of nothing and ceasing-to-be as opposed to being, finitude is the most stubborn category of the understanding; negation in general, constitution and limit,

reconcile themselves with their other, with determinate being; and even nothing, taken abstractly as such, is given up as an abstraction; but finitude is the negation as *fixed in itself*, and it therefore stands in abrupt contrast to its affirmative. The finite, it is true, lest itself be brought into flux, it is itself this, to be determined or destined to its end, but *only* to its end—or rather, it is the refusal to let itself be brought affirmatively to its affirmative, to the infinite, and to let itself be united with it; it is therefore posited as inseparable from its nothing, and is thereby cut off from all reconciliation with its other, the affirmative. The determination or destiny of finite things takes them no further than their *end.* The understanding persists in this sadness of finitude by making non-being the determination of things and at the same time making it *imperishable* and *absolute.* Their transitoriness could only pass away or perish in their other, in the affirmative; their finitude would then be parted from them; but it is their unalterable quality, that is, their quality which does not pass over into its other, that is, into its affirmative; *it is thus eternal.*

This is a very important consideration; but certainly no philosophy or opinion, or understanding, will let itself be tied to the standpoint that the finite is absolute; the very opposite is expressly present in the assertion of the finite; the finite is limited, transitory, it is *only* finite, not imperishable; this is directly implied in its determination and expression. But the point is, whether in thinking of the finite one holds fast to the *being* of finitude and lets the *transitoriness* continue to be, or whether the *transitoriness* and the *ceasing-to-be cease to be.* But it is precisely in that view of the finite which makes *ceasing-to-be* the *final* determination of the finite, that this does not happen. It is the express assertion that the finite is irreconcilable with the infinite and cannot be united with it, that the finite is utterly opposed to the infinite. Being, absolute being, is ascribed to the infinite; confronting it, the finite thus remains held fast as its negative; incapable of union with the infinite, it remains absolute on its own side; from the affirmative, from the infinite, it would receive affirmation, and would thus cease to be; but a union with the infinite is just what is declared to be impossible. If it is not to remain fixed in its opposition to the infinite but is to cease to be, then, as we have already said, just this ceasing-to-be is its final determination, not the affirmative which would be only the ceasing to be of the

ceasing-to-be. If, however, the finite is not to pass way in the affirmative, but its end is to be grasped as the *nothing*, then we should be back again at that first, abstract nothing which itself has long since passed away.

With this nothing, however, which is supposed to be *only* nothing, and which at the same time is granted an existence in thought, imagination or speech, there occurs the same contradiction as has just been indicated in connection with the finite, but with this difference, that in the case of that first nothing it only *occurs*, whereas in finitude it is *explicitly stated*. There it appears as subjective; here it is asserted that the finite *stands perpetually* opposed to the infinite, that what is in itself null *is*, and is *as* in itself null. We have to become conscious of this; and the development of the finite shows that, having this contradiction present within it, it collapses within itself, yet in doing so actually resolves the contradiction, that not only is the finite transitory and ceases to be, but that the ceasing-to-be, the nothing, is not the final determination, but itself ceases to be.

(β) *Limitation and the Ought*

This contradiction is, indeed, abstractly present simply in the circumstance that the *something* is finite, or that the finite is. But *something* or being is no longer abstractly posited but reflected into itself and developed as being-within-self which possesses a determination and a constitution, and, still more specifically, a limit which, as immanent in the something and constituting the quality of its being-within-self, is finitude. It is to be seen what moment are contained in this Notion of the finite something.

Determination and constitution showed themselves as *sides* for external reflection; but the former already contained otherness as belonging to the something's *in-itself*; the externality of the otherness is on the one hand in the something's own inwardness, on the other hand it remains, as externality, distinguished from it, it is still externality as such, but present *in* the something. But further, since the otherness is determined as *limit*, as itself negation of the negation, the otherness immanent in the something is posited as the connection of the two sides, and the unity with itself of the something which possesses both determination and constitution, is its relation turned towards its own self, the relation of its *implicit* determination to the limit immanent in the some-

thing, a relation in which this immanent limit is negated. The self-identical being-within-self thus relates itself to itself as its own non-being, but as negation of the negation, as negating the non-being which at the same time retains in it determinate being, for determinate being is the quality of its being-within-self. Something's own limit thus posited by it as a negative which is at the same time essential, is not merely limit as such, but *limitation.* But what is posited as negated is not limitation alone; the negation is two-edged, since what is posited by it as negated is the *limit,* and this is in general what is common to both something and other, and is also a determinateness of the *in-itself* of the determination as such. This *in-itself,* therefore, as the negative relation to its limit (which is also distinguished from it), to itself as limitation, is the *ought.*

In order that the limit which is in something as such should be a limitation, something must at the same time in its own self transcend the limit, it must in its own self *be related to the limit as to something which is not.* The determinate being of something lies inertly indifferent, as it were, *alongside* its limit. But something only transcends its limit in so far as it is the accomplished sublation of the limit, is the *in-itself* as negatively related to it. And since the limit is in the *determination* itself as a limitation, something transcends *its own self.*

The ought therefore contains the determination in double form: once as the *implicit* determination counter to the negation, and again as a non-being which, as a limitation, is distinguished from the determination, but is at the same time itself an implicit determination.

The finite has thus determined itself as the relation of its determination to its limit; in this relation, the determination is an ought and the limit is a *limitation.* Both are thus moments of the finite and hence are themselves finite, both the ought and the limitation. But only the limitation is *posited* as finite; the ought is limited only in itself, that is, for us. It is limited through its relation to the limit which is already immanent in the ought itself, but this its restriction is enveloped in the in-itself, for, in accordance with its determinate being, that is, its determinateness relatively to the limitation, it is posited as the in-itself.

What ought to be *is,* and at the same time *is not.* If it *were,* we could not say that it *ought* merely *to be.* The ought has, therefore,

essentially a limitation. This limitation is not alien to it; that which *only* ought to be is the *determination*, which is now posited as it is in fact, namely, as at the same time only a determinateness.

The being-in-itself of the something in its determination reduces itself therefore to an *ought-to-be* through the fact that the same thing which constitutes its in-itself is in one and the same respect a *non-being*; and that, too, in this way, that in the being-within-self, in the negation of the negation, this in-itself as one of the negations (the one that negates) is a unity with the other, which at the same time is a qualitatively distinct limit, through which this unity is a *relation* to it. The limitation of the finite is not something external to it; on the contrary, its own determination is also its limitation; and this latter is both itself and also the ought-to-be; it is that which is common to both, or rather that in which both are identical.

But now further, the finite as the ought *transcends* its limitation; the same determinateness which is its negation is also sublated, and is thus its in-itself; its limit is also not its limit.

Hence as the *ought*, something is *raised above its limitation*, but conversely, it is only as the *ought* that it has its *limitation*. The two are inseparable. Something has a limitation in so far as it has negation in its determination, and the determination is also the accomplished sublation of the limitation.

Remark: The Ought

The ought has recently played a great part in philosophy, especially in connection with morality and also in metaphysics generally, as the ultimate and absolute concept of the identity of the in-itself or *self*-relation, and of the *determinateness* or limit.

'You can, because you ought'—this expression, which is supposed to mean a great deal, is implied in the notion of ought. For the ought implies that one is superior to the limitation; in it the limit is sublated and the in-itself of the ought is thus an identical self-relation, and hence the abstraction of 'can'. But conversely, it is equally correct that: 'you cannot, just because you ought.' For in the ought, the limitation as limitation is equally implied; the said formalism of possibility has, in the limitation, a reality, a qualitative otherness opposed to it and the relation of each to the other is a contradiction, and thus a 'cannot', or rather an impossibility.

In the ought the transcendence of finitude, that is, infinity, begins. The ought is that which, in the further development, exhibits itself in accordance with the said impossibility as the progress to infinity.

With respect to the form of the *limitation* and the *ought*, two prejudices can be criticized in more detail. First of all, great stress is laid on the limitations of thought, of reason, and so on, and it is asserted that the limitation *cannot* be transcended. To make such an assertion is to be unaware that the very fact that something is determined as a limitation implies that the limitation is already transcended. For a determinateness, a limit, is determined as a limitation only in opposition to its other in general, that is, in opposition to that which is *free from the limitation*; the other of a limitation is precisely the *being beyond* it. Stone and metal do not transcend their limitation because this is not a limitation *for them*. If, however, in the case of such general propositions framed by the understanding, such as that limitation cannot be transcended, thought will not apply itself to finding out what is implied in the Notion, then it can be directed to the world of actuality where such proportions show themselves to be completely unreal. Just because thought is *supposed* to be superior to actuality, to dwell apart from it in higher regions and therefore to be itself determined as an *ought-to-be*, on the one hand, it does not advance to the Notion, and, on the other hand, it stands in just as untrue a relation to actuality as it does to the Notion. Because the stone does not think, does not even feel, its limitedness is not a limitation *for it*, that is, is not a negation in it for sensation, imagination, thought, etc., which it does not possess. But even the stone, as a something, contains the distinction of its determination or in-itself and its determinate being, and to that extent it, too, transcends its limitation; the Notion which is implicit in it contains the identity of the stone with its other. If it is a base capable of being acted on by an acid, then it can be oxidized, and neutralized, and so on. In oxidation, neutralization and so on, it overcomes its limitation of existing only as a base; it transcends it, and similarly the acid overcomes its limitation of being an acid. This ought, the obligation to transcend limitations, is present in both acid and caustic base in such a degree that it is only by force that they can be kept fixed as (waterless, that is, purely non-neutral) acid and caustic base.

If, however, an existence contains the Notion not merely as an abstract in-itself, but as an explicit, self-determined totality, as instinct, life, ideation, etc., then in its own strength it overcomes the limitation and attains a being beyond it. The plant transcends the limitation of being a seed, similarly, of being blossom, fruit, leaf; the seed becomes the developed plant, the blossom fades away, and so on. The sentient creature, in the limitation of hunger, thirst, etc., is the urge to overcome this limitation and it does overcome it. It feels *pain*, and it is the privilege of the sentient nature to feel pain; it is a negation in its *self*, and the negation is determined as a *limitation* in its feeling, just because the sentient creature has the feeling of its *self*, which is the totality that transcends this determinateness. If it were not above and beyond the determinateness, it would not feel it as its negation and would feel no pain. But it is reason, thought, which is supposed to be unable to transcend limitation—reason, which is the *universal* explicitly beyond particularity *as such* (that is, *all* particularity), which is nothing but the overcoming of limitation! Granted, not every instance of transcending and being beyond limitation is a genuine liberation from it, a veritable affirmation; even the ought itself, and abstraction in general, is an imperfect transcending. However, the reference to the wholly abstract universal is a sufficient reply to the equally abstract assertion that limitation cannot be transcended, or, again, even the reference to the infinite in general is a sufficient refutation of the assertion that the finite cannot be transcended.

In this connection we may mention a seemingly ingenious fancy of Leibniz: that if a magnet possessed consciousness it would regard its pointing to the north as a determination of its will, as a law of its freedom. On the contrary, if it possessed consciousness and consequently will and freedom, it would be a thinking being. Consequently, space for it would be *universal*, embracing *every* direction, so that the single direction to the north would be rather a limitation on its freedom, just as much as being fixed to one spot would be a limitation for a man although not for a plant.

On the other hand, the ought is the transcending, but still only *finite transcending*, of the limitation. Therefore, it has its place and its validity in the sphere of finitude where it holds fast to being-in-itself in opposition to limitedness, declaring the former

to be the regulative and essential factor relatively to what is null. Duty is an ought directed against the particular will, against self-seeking desire and capricious interest and it is held up as an ought to the will in so far as this has the capacity to isolate itself from the true. Those who attach such importance to the ought of morality and fancy that morality is destroyed if the ought is not recognized as ultimate truth, and those too who, reasoning from the level of the understanding, derive a perpetual satisfaction from being able to confront everything there is with an ought, that is, with a 'knowing better'—and for that very reason are just as loth to be robbed of the ought—do not see that as regards the finitude of their sphere the ought receives full recognition. But in the world of actuality itself, Reason and Law are not in such a bad way that they only *ought* to be—it is only the abstraction of the in-itself that stops at this—any more than the ought is in its own self perennial and, what is the same thing, that finitude is absolute. The philosophy of Kant and Fichte sets up the ought as the highest point of the resolution of the contradictions of Reason; but the truth is that the ought is only the standpoint which clings to finitude and thus to contradiction.

(γ) *Transition of the Finite into the Infinite*

The ought as such contains limitation, and limitation contains the ought. Their relation to each other is the finite itself which contains them both in its being-within-self. These moments of its determination are qualitatively opposed; limitation is determined as the negative of the ought and the ought likewise as the negative of limitation. The finite is thus inwardly self-contradictory; it sublates itself, ceases to be. But this its result, the negative as such, is (α) its very *determination*; for it is the negative of the negative. Thus, in ceasing to be, the finite has not ceased to be; it has become in the first instance only *another* finite which, however, is equally a ceasing-to-be as transition into another finite, and so on to *infinity*. But (β) closer consideration of this result shows that the finite in its ceasing-to-be, in this negation of itself has attained its being-in-itself, is *united with itself*. Each of its moments contains precisely this result; the ought transcends the limitation, that is, transcends itself; but beyond itself or its other, is only the limitation itself. The limitation, however, points directly beyond itself to its other, which is the ought; but this

latter is the same duality of *being-in-itself* and *determinate being* as the limitation; it is the same thing; in going beyond itself, therefore, it equally only unites with itself. This *identity with itself*, the negation of negation, is affirmative being and thus the other of the finite, of the finite which is supposed to have the first negation for its determinateness; this other is the *infinite*.

C. INFINITY

The infinite in its simple Notion can, in the first place, be regarded as a fresh definition of the absolute; as indeterminate self-relation it is posited as *being* and *becoming*. The forms of *determinate being* find no place in the series of those determinations which can be regarded as definitions of the absolute, for the individual forms of that sphere are immediately posited only as determinatenesses, as finite in general. The infinite, however, is held to be absolute without qualification for it is determined expressly as negation of the finite, and reference is thus expressly made to limitedness in the infinite—limitedness of which being and becoming could perhaps be capable, even if not possessing or showing it—and the presence in the infinite of such limitedness is denied.

But even so, the infinite is not yet really free from limitation and finitude; the main point is to distinguish the genuine Notion of infinity from spurious infinity, the infinite of reason from the infinite of the understanding; yet the latter is the *finitized* infinite, and it will be found that in the very act of keeping the infinite pure and aloof from the finite, the infinite is only made finite.

The infinite is:

(*a*) in its *simple determination*, affirmative as negation of the finite

(*b*) but thus it is in *alternating determination* with the *finite*, and is the abstract, *one-sided* infinite

(*c*) the self-sublation of this infinite and of the finite, as a *single* process—this is the *true* or *genuine infinite*.

(*a*) *The Infinite in General*

The infinite is the negation of the negation, affirmation, *being* which has restored itself out of limitedness. The infinite *is*, and more intensely so than the first immediate being; it is the true being, the elevation above limitation. At the name of the infinite,

the heart and the mind light up, for in the infinite the spirit is not merely abstractly present to itself, but rises to its own self, to the light of its thinking, of its universality, of its freedom.

The Notion of the infinite as it first presents itself is this, that determinate being in its being-in-itself determines itself as finite and transcends the limitation. It is the very nature of the finite to transcend itself, to negate its negation and to become infinite. Thus the infinite does not stand as something finished and complete above or superior to the finite, as if the finite had an enduring being *apart from* or *subordinate to* the infinite. Neither do *we* only, as subjective reason, pass beyond the finite into the infinite; as when we say that the infinite is the Notion of reason and that through reason we rise superior to temporal things, though we let this happen without prejudice to the finite which is in no way affected by this exaltation, an exaltation which remains external to it. But the finite itself in being raised into the infinite is in no sense acted on by an alien force; on the contrary, it is its nature to be related to itself as limitation,—both limitation as such and as an ought—and to transcend the same, or rather, as self-relation to have negated the limitation and to be beyond it. It is not in the sublating of finitude in general that infinity in general comes to be; the truth is rather that the finite is only this, through its own nature to become itself the infinite. The infinite is its *affirmative determination*, that which it truly is in itself.

Thus the finite has vanished in the infinite and what *is*, is only the *infinite*.

(*b*) *Alternating Determination of the Finite and the Infinite*

The infinite *is*; in this immediacy it is at the same time the *negation* of an other, of the finite. As thus in the form of simple being and at the same time as the *non-being* of an *other*, it has fallen back into the category of *something* as a determinate being in general—more precisely, into the category of something with a limit, because the infinite is determinate being reflected into itself, resulting from the sublating of determinateness in general, and hence is determinate being *posited* as distinguished from its determinateness. In keeping with this determinateness, the finite stands opposed to the infinite as a *real determinate being*; they stand thus in a qualitative relation, each *remaining* external to the other; the *immediate being* of the infinite resuscitates the *being* of

its negation, of the finite again which at first seemed to have vanished in the infinite.

But the infinite and the finite are not in these categories of relation only; the two sides are determined beyond the stage of being merely *others* to each other. Finitude, namely, is limitation posited as limitation; determinate being is posited with the *determination* to pass over into its *in itself*, to *become* infinite. Infinity is the nothing of the finite, it is what the latter is *in itself*, what it *ought to be*, but this ought-to-be is at the same time reflected into itself, is *realized*; it is a purely self-related, wholly affirmative being. In infinity we have the satisfaction that all determinateness, alteration, all limitation and with it the ought itself, are posited as vanished, as sublated, that the nothing of the finite is posited. As this negation of the finite the in-itself is determinate and thus, as negation of the negation, is affirmative within itself. But this affirmation as qualitative, is *immediate* self-relation, is *being*; and thus the infinite is reduced to the category of a being which has the finite confronting it as an other; its negative nature is posited as the simply *affirmative*, hence as the first and immediate negation. The infinite is in this way burdened with the opposition to the finite which, as an other, remains at the same time a determinate reality although in its in-itself, in the infinite, it is at the same time posited as sublated; this infinite is the non-finite—a being in the determinateness of negation. Contrasted with the finite, with the sphere of affirmative determinatenesses, of realities, the infinite is the indeterminate void, the beyond of the finite, whose being-in-itself is not present in its *determinate* reality.

The infinite as thus posited over against the finite, in a relation wherein they are as qualitatively distinct others, is to be called the *spurious infinite*, the infinite of the understanding, for which it has the value of the highest, the absolute Truth. The understanding is satisfied that it has truly reconciled these two, but the truth is that it is entangled in unreconciled, unresolved, absolute contradiction; it can only be brought to a consciousness of this fact by the contradictions into which it falls on every side when it ventures to apply and to explicate these its categories.

This contradiction occurs as a direct result of the circumstance that the finite remains as a determinate being opposed to the infinite, so that there are *two* determinatenesses; *there are* two

worlds, one infinite and one finite, and in their relationship the infinite is only the *limit* of the finite and is thus only a determinate infinite, an *infinite which is itself finite*.

This contradiction develops its content into more explicit forms. The finite is real determinate being which persists as such even when transition is made to its non-being, to the infinite; this, as has been shown, has only the first, immediate negation for its determinateness relatively to the finite, just as the finite as opposed to that negation has, as negated, only the significance of an other and is, therefore, still [only] *something*. When, therefore, the understanding, raising itself above this finite world, ascends to its highest, to the infinite, this finite world remains for it on *this* side, so that the infinite is only set *above* or *beyond* the finite, is *separated* from it, with the consequence that the finite is separated from the infinite; each is *assigned* a *distinct* place—the finite as determinate being here, on *this* side, and the infinite, although the *in-itself* of the finite, nevertheless as a beyond in the dim, inaccessible distance, *outside* of which the finite is and remains.

As thus separated they are just as much essentially *connected* by the very negation which separates them. This negation which connects them—the *somethings* reflected into themselves—is the limit of the one relatively to the other, and that, too, in such a manner that each of them does not have the limit *in it* merely relatively to the other, but the negation is their *being-in-itself*; the limit is thus present in each on its own account, in separation from the other. But the limit is in the form of the first negation and thus both are limited, finite in themselves. However, each as affirmatively self-related is also the negation of its limit; each thus immediately repels the limit, as its non-being, from itself and, as qualitatively separated from it, posits it as *another being* outside it, the finite positing its non-being as this infinite and the infinite, similarly, the finite. It is readily conceded that there is a necessary transition from the finite to the infinite—necessary through the determination of the finite—and that the finite is raised to the form of being-in-itself, since the finite, although persisting as a determinate being, is at the same time *also* determined as *in itself* nothing and therefore as destined to bring about its own dissolution; whereas the infinite, although determined as burdened with negation and limit, is at the same time also determined as possessing *being-in-itself*, so that this abstraction of self-related affir-

mation constitutes its determination, and hence finite determinate being is not present in it. But it has been shown that the infinite itself attains affirmative being only *by means of* negation, as the negation of negation, and that when this its affirmation is taken as merely simple, qualitative being, the negation contained in it is reduced to a simple immediate negation and thus to a determinateness and limit, which then, as in contradiction with the being-in-itself of the infinite is posited as excluded from it, as not belonging to it, as, on the contrary, opposed to its being-in-itself, as the finite. As therefore each is in its own self and through its own determination the positing of its other, they are *inseparable.* But this their unity is *concealed* in their *qualitative* otherness, it is the *inner* unity which only lies at their base.

This determines the manner in which this unity is manifested: posited in *determinate being*, the unity is a changing or transition of the finite into the infinite, and vice versa; so that the infinite only *emerges* in the finite and the finite in the infinite, the other in the other; that is to say, each arises *immediately* and independently in the other, their connection being only an external one.

The process of their transition has the following detailed shape. We pass from the finite to the infinite. This transcending of the finite appears as an external act. In this void beyond the finite, what arises? What is the positive element in it? Owing to the inseparability of the infinite and the finite—or because this infinite remaining aloof on its own side is itself limited—there arises a limit; the infinite has vanished and its other, the finite, has entered. But this entrance of the finite appears as a happening external to the infinite, and the new limit as something that does not arise from the infinite itself but is likewise found as given. And so we are faced with a relapse into the previous determination which has been sublated in vain. But this new limit is itself only something which has to be sublated or transcended. And so again there arises the void, the nothing, in which similarly the said determinateness, a new limit, is encountered—*and so on to infinity.*

We have before us the alternating determination of the *finite* and the *infinite*; the finite is finite only in its relation to the ought or to the infinite, and the latter is only infinite in its relation to the finite. They are inseparable and at the same time mutually related as sheer others; each has in its own self the other of itself. Each

is thus the unity of itself and its other and is in its determinateness *not* that which it itself is, and which its other is.

It is this alternating determination negating both its own self and its negation, which appears as the *progress to infinity*, a progress which in so many forms and applications is accepted as something ultimate beyond which thought does not go but, having got as far as this 'and so on to infinity', has usually reached its goal. This progress makes its appearance wherever *relative* determinations are pressed to the point of opposition, with the result that although they are in an inseparable unity, each is credited with a self-subsistent determinate being over against the other. The progress is, consequently, a *contradiction* which is not resolved but is always only enunciated as *present*.

What we have here is an abstract transcending of a limit, a transcending which remains incomplete because *it is not itself transcended*. Before us is the infinite; it is of course transcended, for a new limit is posited, but the result is rather only a return to the finite. This spurious infinity is in itself the same thing as the perennial ought; it is the negation of the finite it is true, but it cannot in truth free itself therefrom. The finite reappears *in the infinite itself* as its other, because it is only in its *connection* with its other, the finite, that the infinite is. The progress to infinity is, consequently, only the perpetual repetition of one and the same content, one and the same tedious *alternation* of this finite and infinite.

The infinity of the infinite progress remains burdened with the finite as such, is thereby limited and is itself *finite*. But this being so, the infinite progress would in fact be posited as the unity of the finite and the infinite; but this unity is not reflected on. Yet it is this unity alone which evokes the infinite in the finite and the finite in the infinite; it is, so to speak, the mainspring of the infinite progress. This progress is the *external* aspect of this unity at which ordinary thinking halts, at this perpetual repetition of one and the same alternation, of the vain unrest of advancing beyond the limit to infinity, only to *find* in this infinite a new limit in which, however, it is as little able to rest as in the infinite. This infinite has the fixed determination of a *beyond*, which cannot be reached, for the very reason that *it is not meant* to be reached, because the determinateness of the beyond, of the *affirmative* negation, is not let go. In accordance with this determination the

infinite has the finite opposed to it as a being *on this side*, which is equally unable to raise itself into the infinite just because it has this determination of an *other*, of a *determinate being* which perpetually generates itself in its beyond, a beyond from which it is again distinct.

(*c*) *Affirmative Infinity*

In this alternating determination of the finite and the infinite from one to the other and back again, their truth is already implicitly *present*, and all that is required is to take up what is before us. This transition from one to the other and back again constitutes the external realization of the Notion. In this realization is *posited* the content of the Notion, but it is posited as *external*, as falling *asunder*; all that is required is to compare these different moments which yield the *unity* which gives the Notion itself; the *unity* of the infinite and the finite is—as has often been remarked already but here especially is to be borne in mind—the one-sided expression for the unity as it is in truth; but the elimination, too, of this one-sided determination must lie in the externalization of the Notion now before us.

Taken according to their first, only immediate determination, the infinite is only the *beyond* of the *finite*; according to its determination it is the negation of the finite; thus the finite is only that which must be transcended, the negation of itself in its own self, which is infinity. In *each*, therefore, there lies the *determinateness of the other*, although according to the standpoint of the infinite progress these two are supposed to be shut out from each other and only to follow each other alternately; neither can be posited and grasped without the other, the infinite not without the finite, nor the latter without the infinite. In *saying* what the infinite is, namely the negation of the *finite*, the latter is itself included in what is *said*; it cannot be dispensed with for the definition or determination of the infinite. One only needs to *be aware of what one is saying* in order to find the determination of the finite in the infinite. As regards the finite, it is readily conceded that it is the null; but its very nullity is the infinity from which it is thus inseparable. In this way of conceiving them, each may seem to be taken in its *connection* with its other. But if they are taken as *devoid of connection* with each other so that they are only joined by 'and', then each confronts the other as self-subsistent,

as in its own self only affirmatively present. Let us see how they are constituted when so taken. The infinite, in that case, is *one of the two*; but as *only* one of the two it is itself finite, it is not the whole but only *one* side; it has its limit in what stands over against it; it is thus the *finite infinite*. There are present only *two finites*. It is precisely this holding of the infinite *apart* from the finite, thus giving it a *one-sided* character, that constitutes its finitude and, therefore, its unity with the finite. The finite, on the other hand, characterized as independent of and apart from the infinite, is that *self-relation* in which its relativity, its dependence and transitoriness is removed; it is the same self-subsistence and affirmation which the infinite is supposed to be.

The two modes of consideration at first seem to have a different determinateness for their point of departure, inasmuch as the former is supposed to be only the *connection* of the infinite and the finite, of each with its other, and the latter is supposed to hold them apart in complete separation from each other; but both modes yield one and the same result: the infinite and the finite viewed as *connected* with each other—the connection being only external to them but also essential to them, without which neither is what it is—each contains its own other in its own determination, just as much as each, taken *on its own account*, considered *in its own self*, has its other present within it as its own moment.

This yields the decried unity of the finite and the infinite—the unity which is itself the infinite which embraces both itself and finitude—and is therefore the infinite in a different sense from that in which the finite is regarded as separated and set apart from the infinite. Since now they must also be distinguished, each is, as has just been shown, in its own self the unity of both; thus we have two such unities. The common element, the unity of the two determinatenesses, as unity, posits them in the first place as negated, since each is supposed to be what it is in its distinction from the other; in their unity, therefore, they lose their qualitative nature—an important reflection for rebutting that idea of the unity which insists on holding fast to the infinite and finite in the quality they are supposed to have when taken in their separation from each other, a view which therefore sees in that unity *only* contradiction, but not also resolution of the contradiction through the negation of the qualitative determinate-

ness of both; thus the unity of the infinite and finite, simple and general in the first instance, is falsified.

But further, since now they are also to be taken as distinct, the *unity* of the infinite which each of these moments is, is differently determined in each of them. The infinite determined as such, has present in it the finitude which is distinct from it; the former is the *in-itself* in this unity, and the latter is only determinateness, limit in it; but it is a limit which is the sheer other of the in-itself, is its opposite; the infinite's determination, which is the in-itself as such, is ruined by the addition of such a quality; it is thus a *finitized infinite*. Similarly, since the finite as such is only the negation of the in-itself, but by reason of this unity also has its opposite present in it, it is exalted and, so to say, infinitely exalted above its worth; the finite is posited as the *infinitized* finite.

Just as before, the simple unity of the infinite and finite was falsified by the understanding, so too is the double unity. Here too this results from taking the infinite in one of the two unities not as negated, but rather as the in-itself, in which, therefore, determinateness and limitation are not to be explicitly present, for these would debase and ruin it. Conversely, the finite is likewise held fast as not negated, although in itself it is null; so that in its union with the infinite it is exalted to what it is not and is thereby infinitized in opposition to its determination as finite, which instead of vanishing is perpetuated.

The falsification of the finite and infinite by the understanding which holds fast to a qualitatively distinct relation between them and asserts that each in its own nature is separate, in fact absolutely separate from the other, comes from forgetting what the Notion of these moments is for the understanding itself. According to this, the unity of the finite and infinite is not an external bringing together of them, nor an incongruous combination alien to their own nature in which there would be joined together determinations inherently separate and opposed, each having a simple affirmative being independent of the other and incompatible with it; but each is in its own self this unity, and this only as a *sublating* of its own self in which neither would have the advantage over the other of having an in-itself and an affirmative determinate being. As has already been shown, finitude *is* only as a transcending of itself; it therefore contains infinity, the other of itself. Similarly, infinity *is* only as a transcending of

the finite; it therefore essentially contains its other and is, consequently, in its own self the other of itself. The finite is not sublated by the infinite as by a power existing outside it; on the contrary, its infinity consists in sublating its own self.

This sublating is, therefore, not alteration or otherness as such, not the sublating of a *something*. That in which the finite sublates itself is the infinite as the negating of finitude; but finitude itself has long since been determined as only the *non-being* of determinate being. It is therefore only *negation* which *sublates* itself in the *negation*. Thus infinity on *its* side is determined as the negative of finitude, and hence of determinateness in general, as the empty beyond; the sublating of itself in the finite is a return from an empty flight, a *negation* of the beyond which is in its own self a *negative*.

What is therefore present is the same negation of negation in each. But this is *in itself* self-relation, affirmation, but as return to itself, that is through the *mediation* which the negation of negation is. These are the determinations which it is essential to keep in view; but secondly it is to be noted that they are also *posited* in the infinite progress, and how they are posited in it, namely, as not yet in their ultimate truth.

In the first place, both the infinite and the finite are negated in the infinite progress; both are transcended in the same manner. Secondly, they are posited one after the other as distinct, each as positive on its own account. We thus compare these two determinations in their separation, just as in our comparison—an external comparing—we have separated the two modes of considering the finite and the infinite: on the one hand in their connexion, and on the other hand each on its own account. But the infinite progress expresses more than this; in it there is also posited the *connexion* of terms which are also distinct from each other, although at first the connexion is still only a transition and alternation; only a simple reflection on our part is needed to see what is in fact present.

In the first place, the negation of the finite and infinite which is posited in the infinite progress can be taken as simple, hence as separate and merely successive. Starting from the finite, the limit is transcended, the finite negated. We now have its beyond, the infinite, but in this the limit *arises* again; and so we have the transcending of the infinite. This double sublation, however, is

partly only an external affair, an alternation of the moments, and partly it is not yet posited as a *single unity*; the transcending of each moment starts independently, is a fresh act, so that the two processes fall apart. But in addition there is also present in the infinite progress their *connexion*. First there is the finite, then this is transcended and this negative or beyond of the finite is the infinite, and then this negation is again transcended, so that there arises a new limit, a *finite* again. This is the complete, self-closing movement which has arrived at that which constituted the beginning; what arises is the *same* as that from which the movement *began*, that is, the finite is restored; it has therefore united *with itself*, has in its beyond only found *itself* again.

The same is the case with the infinite. In the infinite, the beyond of the limit, there arises only another limit which has the same fate, namely, that as finite it must be negated. Thus what is present again is the *same* infinite which had previously disappeared in the new limit; the infinite, therefore, through its sublating, through its transcending of the new limit, is not removed any further either from the finite—for the finite is only this, to pass over into the infinite—or from itself, for it has arrived *at its own self*.

Thus, both finite and infinite are this *movement* in which each returns to itself through its negation; they *are* only as *mediation* within themselves, and the affirmative of each contains the negative of each and is the negation of the negation. They are thus a *result*, and consequently not what they are in the determination of their *beginning*; the finite is not a *determinate being* on *its* side, and the infinite a *determinate being* or *being-in-itself* beyond the determinate being, that is, beyond the being determined as finite. The reason why understanding is so antagonistic to the unity of the finite and infinite is simply that it presupposes the limitation and the finite, as well as the in-itself, as *perpetuated*; in doing so it *overlooks* the negation of both which is actually present in the infinite progress, as also the fact that they occur therein only as moments of a whole and that they come on the scene only by means of their opposite, but essentially also by means of the sublation of their opposite.

If, at first, the return into self was considered to be just as much a return of the finite to itself as return of the infinite to itself, this very result reveals an error which is connected with the one-

sidedness just criticized: first the finite and then the infinite is taken as the *starting point* and it is only this that gives rise to *two* results. It is, however, a matter of complete indifference which is taken as the beginning; and thus the difference which occasioned the *double* result disappears of itself. This is likewise explicit in the line—unending in both directions—of the infinite progress in which each of the moments presents itself in equal alternation, and it is quite immaterial what point is fixed on or which of the two is taken as the beginning. They are distinguished in it but each is equally only the moment of the other. Since both the finite and the infinite itself are moments of the progress they are *jointly or in common the finite*, and since they are equally together negated in it and in the result, this result as negation of the finitude of both is called with truth the infinite. Their difference is thus the *double* meaning which both have. The finite has the double meaning of being first, only the finite *over against* the infinite which stands opposed to it, and secondly, of being the finite and *at the same time* the infinite opposed to it. The infinite, too, has the double meaning of being *one* of these two moments—as such it is the spurious infinite—and also the infinite in which both, the infinite and its other, are only moments. The infinite, therefore, as now before us is, in fact, the process in which it is deposed to being only *one* of its determinations, the opposite of the finite, and so to being itself only one of the finites, and then raising this its difference from itself into the affirmation of itself and through this mediation becoming the *true* infinite.

This determination of the true infinite cannot be expressed in the *formula*, already criticized, of a *unity* of the finite and infinite; *unity* is abstract, inert self-sameness, and the moments are similarly only in the form of inert, simply affirmative being. The infinite, however, like its two moments, is essentially only as a *becoming*, but a becoming now *further determined* in its moments. Becoming, in the first instance, has abstract being and nothing for its determinations; as alteration, its moments possess determinate being, something and other; now, as the infinite, they are the finite and the infinite, which are themselves in process of becoming.

This infinite, as the consummated return into self, the relation of itself to itself, is *being*—but not indeterminate, abstract being, for it is posited as negating the negation; it is, therefore, also *determinate* being for it contains negation in general and hence

determinateness. It *is* and *is there*, present before us. It is only the spurious infinite which is the *beyond*, because it is *only* the negation of the finite posited as *real*—as such it is the abstract, first negation; determined *only* as negative, the affirmation of *determinate* being is lacking in it; the spurious infinite, held fast as only negative, is even *supposed to be not there*, is supposed to be unattainable. However, to be thus unattainable is not its grandeur but its defect, which is at bottom the result of holding fast to the *finite* as such as a *merely affirmative being*. It is what is untrue that is unattainable, and such an infinite must be seen as a falsity. The image of the progress to infinity is the *straight line*, at the two limits of which alone the infinite is, and always only is where the line—which is determinate being—is not, and which goes *out beyond* to this negation of its determinate being, that is, to the indeterminate; the image of true infinity, bent back into itself, becomes the *circle*, the line which has reached itself, which is closed and wholly present, without *beginning* and *end*.

True infinity taken thus generally as *determinate* being which is posited as *affirmative* in contrast to the abstract negation, is *reality* in a higher sense than the former reality which was *simply* determinate; for here it has acquired a concrete content. It is not the finite which is the real, but the infinite. Thus reality is further determined as essence, Notion, Idea, and so on. It is, however, superfluous to repeat an earlier, more abstract category such as reality, in connexion with the more concrete categories and to employ it for determinations which are more concrete than it is in its own self. Such repetition as to say that essence, or the Idea, is the real, has its origin in the fact that for untrained thinking, the most abstract categories such as being, determinate being, reality, finitude, are the most familiar.

The more precise reason for recalling the category of reality here is that the negation to which it is opposed as the affirmative is here negation of the negation; as such it is itself opposed to that reality which finite determinate being is. The negation is thus determined as ideality; ideal being [*das Ideelle*][1] is the finite

[1] '*Das Ideale*' has a more precise meaning (of the beautiful and its associations) than '*das Ideelle*'; the former is not yet appropriate here and for this reason we have used the expression '*ideell*'. We do not make this distinction though when speaking of reality; the expressions '*reell*' and '*real*' are used practically synonymously and no interest is served by giving the words different shades of meaning. [Author's note.]

as it is in the true infinite—as a determination, a content, which is distinct but is not an *independent, self-subsistent* being, but only a *moment*. Ideality has this more concrete signification which is not fully expressed by the negation of finite determinate being. With reference to reality and ideality, however, the opposition of finite and infinite is grasped in such a manner that the finite ranks as the real but the infinite as the 'ideal' [*das Ideelle*]; in the same way that further on the Notion, too, is regarded as an 'ideal', that is, as a *mere* 'ideal', in contrast to determinate being as such which is regarded as the real. When they are contrasted in this way, it is pointless to reserve the term 'ideal' for the concrete determination of negation in question; in that opposition we return once more to the one-sidedness of the abstract negative which is characteristic of the spurious infinite, and perpetuate the affirmative determinate being of the finite.

TRANSITION

Ideality can be called the *quality* of infinity; but it is essentially the process of *becoming*, and hence a transition—like that of becoming in determinate being—which is now to be indicated. As a sublating of finitude, that is, of finitude as such, and equally of the infinity which is merely its opposite, merely negative, this return into self is *self-relation, being*. As this being contains negation it is *determinate*, but as this negation further is essentially negation of the negation, the self-related negation, it is that determinate being which is called *being-for-self*.

Remark 1: *The Infinite Progress*

The infinite—in the usual meaning of the spurious infinity—and the progress to infinity are, like the ought, the expression of a contradiction which is itself put forward as the final solution. This infinite is a first elevation of sensuous conception above the finite into thought, the content of which, however, is only nothing, that is, it is *expressly* in the form of not-being—a flight beyond limited being which does not inwardly collect itself and does not know how to bring the negative back to the positive. This *incomplete reflection* has completely before it both determinations of the genuine infinite: the *opposition* of the finite and infinite, and their *unity*, but it does not bring these two thoughts together;

the one inevitably evokes the other, but this reflection lets them only *alternate*. This alternation, the infinite progress, is exhibited whenever one remains fixed in the contradiction of the *unity* of two determinations and of their *opposition*. The finite is the sublating of itself, it includes within itself its negation, infinity—the *unity* of both; there is a movement away from the finite to the infinite, to the beyond of the finite—the *separation* of both; but beyond the infinite is another finite—the beyond; the infinite contains finitude—the *unity* of both; but this finite, too, is a negative of the infinite—the *separation* of both; and so on. Thus, in the causal relation, cause and effect are inseparable; a cause which had no effect would not be a cause, just as an effect which had no cause would no longer be an effect. This relation yields, therefore, the infinite progress of causes and effects; something is determined as cause, but as finite (and it is finite for the very reason that it is separated from its effect) it, too, has a cause, that is, it is also an effect; hence the *same thing* that was determined as cause is also determined as effect—the *unity* of cause and effect; now that which is determined as effect again has a cause, that is, the cause has to be separated from its effect and posited as a different something; but this fresh cause is itself only an effect—the *unity* of cause and effect; it has an other for its cause—the separation of both determinations, and so on to *infinity*.

Thus the progress can be put in the following more characteristic form. The assertion is made: the finite and infinite are a single unity; this false assertion must be corrected by the opposite: they are absolutely different and opposed to each other; this must be corrected again by declaring that they are inseparable, that the determination of each lies in the other, by the assertion of their unity, and so on to infinity. What is required in order to see into the nature of the infinite is nothing difficult: it is to be aware that the infinite progress, the developed infinite of the understanding, is so constituted as to be the *alternation* of the two determinations, of the *unity* and the *separation* of both moments and also to be aware that this unity and this separation are themselves inseparable.

The resolution of this contradiction is not the recognition of the equal correctness and equal incorrectness of the two assertions—this is only another form of the abiding contradiction—but the *ideality* of both, in which as distinct, reciprocal negations, they are only *moments*. The said monotonous alternation is actually

the negation of their unity and also of their separation. In it, also actually, there is present what was pointed out above: that the beyond of the finite is the infinite, but equally beyond the infinite again the finite finds itself reborn; consequently, in this process the finite is united only with itself, and the same is true of the infinite—so that the same negation of the negation results in an affirmation, this result thus proving itself to be their truth and primary determination. In this being which is thus the *ideality* of the distinct moments, the contradiction has not vanished abstractly, but is resolved and reconciled, and the thoughts are not only complete, but they are also *brought together*. In this detailed example, there is revealed the specific nature of speculative thought, which consists solely in grasping the opposed moments in their unity. Each moment actually shows that it contains its opposite within itself and that in this opposite it is united with itself; thus the affirmative truth is this immanently active unity, the taking together of both thoughts, their infinity—the relation to self which is not immediate but infinite.

Thinkers have often placed the essence of philosophy in the answering of the question: how does the infinite go forth from itself and become finite? This, it is supposed, cannot be made comprehensible. In the course of this exposition the infinite, the Notion of which we have reached, will *further determine* itself and will show in all its varied forms what is demanded, that is, *how* (if we want so to express it) *the infinite becomes finite*. Here we are considering this question only in its immediacy and with respect to the meaning considered above which is usually attached to the infinite.

It is supposed that it depends altogether on the answering of this question *whether there is a philosophy*; and while people pretend that they are willing to let the decision rest on this, they also believe themselves to possess in the question itself a sort of puzzle, an invincible talisman, by which they are firmly secured against the answering of the question and consequently against philosophy and the entering of its portals. Even in other subjects, to understand how to put questions presupposes a certain education, and this holds good even more in the case of philosophical topics if one is to obtain a better answer than that the question is an idle one.

With such questions it is usually claimed as a reasonable assumption that the matter does not depend on the words, but that the

point at issue can be understood from one or other of the ways in which the question is expressed. Expressions of sensuous conception like *going forth* and suchlike, which are used in the question, arouse the suspicion that they spring from the level of ordinary conception and that for the answer, too, conceptions which are current in everyday life are expected and in the form of a sensuous simile.

If instead of the infinite being as such is taken, then the *determining* of being, a negation or finitude in it, seems easier to understand. Being, it is true, is itself the indeterminate; but that it is the opposite of determinate being, this is not directly expressed in it. In the infinite, on the other hand, this is expressed; it is the *not*-finite. The unity of the finite and infinite thus seems to be directly excluded, and that is why incomplete reflection is most stubbornly opposed to this unity.

But it has been shown that it is at once evident without going into further detail about the determination of the finite and infinite, that the infinite as understood by said reflection, namely, as opposed to the finite, has in it its other, just because it is opposed to the finite, and therefore is already limited and itself finite—the spurious infinite. The answer, therefore, to the question: how does the infinite become finite? is this: that *there is not* an infinite which is first of all infinite and only subsequently has need to become finite, to go forth into finitude; on the contrary, it is on its own account just as much finite as infinite. The question assumes that the infinite, on the one side, exists by itself, and that the finite which has gone forth from it into a separate existence—or from whatever source it might have come—is in its separation from the infinite truly real; but it should rather be said that this separation is *incomprehensible*. Neither such a finite nor such an infinite has truth; and what is untrue is incomprehensible. But equally it must be said that they are comprehensible, to grasp them even as they are in ordinary conception, to see that in the one there lies the determination of the other, the simple insight into their inseparability, means to comprehend them; *this inseparability is their Notion*. But the separate self-subsistence of the said infinite and finite assumed in the question is an untrue content, and the question already implies an untrue connection between them. Instead, therefore, of answering the question, we must deny the false presuppositions contained in

it, that is, the question itself. The question as to the truth of the said infinite and finite involves a change of standpoint and this change will cause a recoil upon the first question of the embarrassment it was intended to produce. This question of ours is something *new* for the reflection which is the source of the first question, since such reflection does not contain the speculative interest which, for its own sake and before it connects determinations, sets out to ascertain whether these, as presupposed, are something true. But in so far as the falsity of that abstract infinite, and of the finite which equally is supposed to remain standing on *its* side, is recognized, there is this to be said about the coming or going forth of the finite from the infinite: the infinite goes forth *out* of itself into finitude because, being grasped as an abstract unity, it has no truth, no enduring being within it; and conversely the finite goes *into* the infinite for the same reason, namely that it is a nullity. Or rather it should be said that the infinite has eternally gone forth into finitude, that, solely by itself and without having its other present *within it*, the infinite no more *is* than pure *being* is.

The question as to how the infinite goes forth to the finite can contain still another presupposition, namely that the infinite *in itself* includes the finite, hence is in itself the unity of itself and its other, so that the difficulty is connected chiefly with the *separating* of them, which is in opposition to the presupposed unity of both. In this presupposition, the opposition which is insisted on merely has another form; the *unity* and the *differentiation* are separated and isolated from each other. But if the former is taken, not as the abstract indeterminate unity but, as it already is in the presupposition, as the determinate unity of the finite and the infinite, then the differentiation of both is already present in it too—a differentiation which is thus at the same time not a releasing of them into a separate self-subsistence, but which leaves them as *ideal* moments in the unity. This *unity* of the finite and infinite and the *distinction* between them are just as inseparable as are finitude and infinity.

Remark 2: *Idealism*

The proposition that the finite is ideal [*ideell*] constitutes idealism. The idealism of philosophy consists in nothing else than in recognizing that the finite has no veritable being. Every philosophy

is essentially an idealism or at least has idealism for its principle, and the question then is only how far this principle is actually carried out. This is as true of philosophy as of religion; for religion equally does not recognize finitude as a veritable being, as something ultimate and absolute or as something underived, uncreated, eternal. Consequently the opposition of idealistic and realistic philosophy has no significance. A philosophy which ascribed veritable, ultimate, absolute being to finite existence as such, would not deserve the name of philosophy; the principles of ancient or modern philosophies, water, or matter, or atoms are *thoughts*, universals, ideal entities, not things as they immediately present themselves to us, that is, in their sensuous individuality—not even the water of Thales. For although this is also empirical water, it is at the same time also the *in-itself* or *essence* of all other things, too, and these other things are not self-subsistent or grounded in themselves, but are *posited* by, are *derived* from, an *other*, from water, that is they are ideal entities. Now above we have named the principle or the universal the *ideal* (and still more must the Notion, the Idea, spirit be so named); and then again we have described individual, sensuous things as *ideal* in principle, or in their Notion, still more in spirit, that is, as sublated; here we must note, in passing, this twofold aspect which showed itself in connection with the infinite, namely that on the one hand the ideal is concrete, veritable being, and on the other hand the moments of this concrete being are no less ideal—are sublated in it; but in fact what is, is only the one concrete whole from which the moments are inseparable.

By the ideal [*dem Ideellen*] is meant chiefly the form of figurate conception and imagination, and what is simply *in* my conception, or *in* the Notion, or *in* the idea, *in* imagination, and so on, is called *ideal*, so that even fancies are counted as ideals—conceptions which are not only distinct from the real world, but are supposed to be essentially *not* real. In point of fact, the spirit is the *idealist* proper; in spirit, even as feeling, imagination and still more as thinking and comprehending, the content is not present as a so-called *real existence;* in the simplicity of the ego such external being is present only as sublated, it is *for me*, it is *ideally* in me. This subjective idealism, either in the form of the unconscious idealism of consciousness generally, or consciously enunciated and set up as a principle, concerns only the *form* of

conception according to which a content is mine; in the systematic idealism of subjectivity this form is declared to be the only true exclusive form in opposition to the form of objectivity or reality, of the *external existence* of that content. Such idealism is [merely] formal because it disregards the *content* of imagination or thought, which content in being imagined or thought can remain wholly in its finitude. In such an idealism nothing is lost, just as much because the reality of such a finite content, the existence filled with finitude, is preserved, as because, in so far as abstraction is made from such finite reality, the content is supposed to be of no consequence *in itself*; and in it nothing is gained for the same reason that nothing is lost, because the ego, conception, spirit, remains filled with the same content of finitude. The opposition of the form of subjectivity and objectivity is of course one of the finitudes; but the *content*, as taken up in sensation, intuition or even in the more abstract element of conception, of thought, contains finitudes in abundance and with the exclusion of only one of the modes of finitude, namely, of the said form of subjective and objective, these finitudes are certainly not eliminated, still less have they spontaneously fallen away.

CHAPTER 3

BEING-FOR-SELF

In being-for-self, *qualitative* being finds its *consummation*; it is infinite being. The being of the beginning lacks all determination. Determinate being is sublated but only immediately sublated being. It thus contains, to begin with, only the first negation, which is itself immediate; it is true that being, too, is preserved in it and both are united in determinate being in a simple unity, but for that very reason they are in themselves still unequal to each other and their unity is not yet *posited*. Determinate being is therefore the sphere of difference, of dualism, the field of finitude. Determinateness is determinateness as such, in which being is only relatively, not absolutely determined. In being-for-self, the difference between being and determinateness or negation is posited and equalized; quality, otherness, limit—like reality, being-in-itself, the ought, and so on—are the imperfect embodiments of the negation in being in which the difference of both still lies at the base. Since, however, in finitude the negation has passed into infinity, into the *posited* negation of negation, it is simple self-relation and consequently in its own self the equalization with being, *absolutely determined being*.

Being-for-self is first, immediately *a* being-for-self—the One.

Secondly, the One passes into a *plurality of ones—repulsion*—and this otherness of the ones is sublated in their ideality—*attraction*.

Thirdly, we have the alternating determination of repulsion and attraction in which they collapse into equilibrium, and quality, which in being-for-self reached its climax, passes over into *quantity*.

A. BEING-FOR-SELF AS SUCH

We have arrived at the general Notion of being-for-self. All that is now necessary to justify our use of the term for this Notion is to demonstrate that the said Notion corresponds to the general idea associated with the expression, being-for-self. And so indeed

it seems; we say that something is for itself in so far as it transcends otherness, its connexion and community with other, has repelled them and made abstraction from them. For it, the other has being only as sublated, as *its moment;* being-for-self consists in having so transcended limitation, its otherness, that it is, as this negation, the infinite *return* into itself. Consciousness, even as such, contains in principle the determination of being-for-self in that it *represents* to itself an object which it senses, or intuits, and so forth; that is, it has *within it* the content of the object, which in this manner has an 'ideal' being; in its very intuiting and, in general, in its entanglement with the negative of itself, with its other, consciousness is still only in the presence of its own self. Being-for-self is the polemical, negative attitude towards the limiting other, and through this negation of the latter is a reflected-ness-into-self, although *along* with this return of consciousness into itself and the ideality of the object, the *reality* of the object is *also* still preserved, in that it is *at the same time* known as an external existence. Consciousness thus belongs to the sphere of *Appearance*, or is the dualism, on the one hand of knowing an alien object external to it, and on the other hand of being for its own self, having the object ideally [*ideell*] present in it; of being not only in the presence of the other, but therein being in the presence of its own self. *Self-consciousness*, on the other hand, is being-for-self as *consummated* and *posited;* the side of connexion with an other, with an external object, is removed. Self-consciousness is thus the nearest example of the presence of infinity; granted, of an infinity which is still abstract, yet which, at the same time, is a very different concrete determination from being-for-self in general, the infinity of which has a determinateness which is still quite qualitative.

(*a*) *Determinate Being and Being-for-self*

As already mentioned, being-for-self is infinity which has collapsed into simple being; it is *determinate* being in so far as the negative nature of infinity, which is the negation of negation, is from now on in the explicit form of the *immediacy* of being, as only negation in general, as simple qualitative determinateness. But being, which in such determinateness is *determinate* being, is also at once distinct from being-for-self, which is only being-for-self in so far as its determinateness is the infinite one above-mentioned;

nevertheless, determinate being is at the same time also a moment of being-for-self; for this latter, of course, also contains being charged with negation. Thus the determinateness which in determinate being as such is an other, and a being-for-other, is bent back into the infinite unity of being-for-self, and the moment of determinate being is present in being-for-self as a ***being-for-one***.

(*b*) *Being-for-one*

This moment expresses the manner in which the finite is present in its unity with the infinite, or is an ideal being [*Ideelles*]. In being-for-self, negation is not present as a determinateness or limit, or consequently as a relation to a determinate being which is for it an other. Now though this moment has been designated as *being-for-one*, there is as yet nothing present for which it would be—no *one*, of which it would be the moment. There is, in fact, nothing of the kind as yet fixed in being-for-self; that for which something (and here there is no something) would be, whatever the other side as such might be, is likewise a moment, is itself only a being-for-one, not yet a one. Consequently, what we have before us is still an undistinguishedness of the two sides which may be suggested by being-for-one; there is only *one* being-for-other, and because there is only *one*, this too is only a being-for-one; there is only the *one* ideality of that, for which or in which there is supposed to be a determination as moment, and of that which is supposed to be a moment in it. Being-for-one and being-for-self are, therefore, not genuinely opposed determinatenesses. If the difference is assumed for a moment and we speak of *a* being-for-self, then it is this itself which, as the sublatedness of otherness, relates itself to itself as the sublated other, and is therefore '*for one*'; it is related in its other only to its own self. Ideal being [*Ideelles*] is necessarily 'for one', but it is not for an other; the one for which it is, is only itself. The ego, therefore, spirit as such, or God, are 'ideal' because they are infinite; but as being for themselves they are not 'ideally' different from that which is 'for one'. For if they were, they would be only immediate existences, or, more precisely, determinate being and a being-for-other, because that which would be for them would be, not themselves but an other, if they were supposed to lack the moment of being 'for one'. God is, therefore, *for **himself*** in so far as he himself is that which is *for **him***.

To be 'for self' and to be 'for one' are therefore not different meanings of ideality, but are essential, inseparable moments of it.

Remark: The German Expression, 'What For a Thing' (Meaning 'What Kind of a Thing')

The German expression for enquiring after the quality of anything —an expression which appears strange at first sight—'What for a thing [*was für ein Ding*]' something is, brings into prominence, in its reflection-into-self, the moment here considered. This expression is in its origin idealistic, since it does not ask what this thing A is for *another* thing B, not what this man is for another man; on the contrary, it asks what this thing, this man, is *for a thing*, *for a man*, so that this being-for-one has at the same time returned into this thing, into this man himself; in other words that *which is*, and that *for which* it is, are one and the same—an identity, such as ideality also must be considered to be.

Ideality attaches in the first place to the sublated determinations as distinguished from that *in which* they are sublated, which by contrast can be taken as the real. But thus the ideal is again one of the moments, and the real the other; but the significance of ideality is that both determinations are equally only *for one* and count only for *one*, which *one* ideality is, without distinction, reality. In this sense self-consciousness, spirit, God, is ideal as an infinite relation purely to self. Ego is for ego, both are the same, the ego is twice named, but so that each of the two is only a 'for-one', is ideal; spirit is only for spirit, God only for God, and this unity alone is God, God as spirit. Self-consciousness, however, as consciousness, enters into the difference *of itself* and of an *other*—or of its *ideality*, in which it produces conceptions, and of its *reality*, inasmuch as its conception has a determinate content which still has the side of being known as the unsublated negative, as a real, determinate being. However, to call thought, spirit, God, *only* an ideal being, presupposes the standpoint from which finite being counts as the real, and the ideal being or being-for-one has only a one-sided meaning.

In a previous Remark the principle of idealism was indicated and it was said that in any philosophy the precise question was, how far has the principle been carried through.[1] As to the manner in which it is carried through, a further observation may be made

[1] P. 155.

in connection with the category we have reached. This carrying through of the principle depends primarily on whether the finite reality still retains an independent self-subsistence alongside the being-for-self, but also on whether in the infinite itself the moment of *being-for-one*, a relationship of the ideal to itself as ideal, is posited. Thus the Eleatic Being or Spinoza's substance is only the abstract negation of all determinateness, without ideality being posited in substance itself. With Spinoza, as will be mentioned later, infinity is only the absolute *affirmation* of a thing, hence only the unmoved unity; consequently, substance does not even reach the determination of being-for-self, much less that of subject and spirit. The idealism of the noble Malebranche is in itself more explicit. It contains the following fundamental thoughts: because God includes within himself all eternal truths, the ideas and perfections of all things, so that they are *his* and his alone, we see them only in him; God awakens in us our sensations of objects by an action in which there is nothing sensuous, whereby we imagine to ourselves that we obtain not only the idea of the object which represents its essential nature, but also the sensation of its existence.[1] As then the eternal truths and Ideas (essentialities) of things are in God, are ideal, so also is their existence in God ideal, not an actual existence; though they are *our* objects, they are only *for one*. This moment of explicit and concrete idealism which is lacking in Spinozism is present here, in that absolute ideality is characterized as a *knowing*. Pure and profound as this idealism is, the above relations on the one hand still contain much that is indeterminate for thought and, on the other hand, their content is directly quite concrete (sin and salvation, etc., enter directly into them); the logical determination of infinity on which they would have to be based is not explicitly realized, and thus this lofty and rich idealism, though it is the product of a pure, speculative spirit, is still not the product of a pure, speculative thinking which alone can truly establish it.

The Leibnizian idealism lies more within the bounds of the abstract Notion. The ideating being, the monad, of Leibniz is essentially ideal [*Ideelles*]. Ideation is a being-for-self in which the determinatenesses are not limits, and consequently not a determinate being, but only moments. Ideation is also, it is true, a more concrete determination, but here it has no further signification

[1] *De la Recherche de la Vérité, Éclairc.* 'Sur la nature des idées' etc.

than that of ideality; for with Leibniz, even that which lacks consciousness is an ideating, percipient being. In this system, then, otherness is sublated; spirit and body, or the monads generally, are not others for one another, they do not limit one another and do not affect one another; all relationships generally which are based on a determinate being fall away. The diversity is only ideal and inner and in it the monad remains related only to itself; the alterations develop within the monad and are not relations of it to others. What is taken to be, in accordance with the real determination, a determinately existent relation of the monads to one another, is an independent, only *simultaneous becoming*, enclosed within the being-for-self of each of them. That there is a *plurality of monads*, that therefore they are also determined as others, does not concern the monads themselves; this is the reflection, external to them, of a third. They are not *in themselves others to one another;* the being-for-self is kept pure, and is free from the accompaniment of any real being. But herein lies, too, the inadequacy of this system. The monads are such ideating, percipient beings only *in themselves*, or *in God* as the monad of monads, or *even in the system.* Otherness is equally present, whether in the ideation itself or in whatever shape the third assumes which considers them as others, as a plurality. The plurality of their determinate being is only excluded, and that only momentarily, the monads being posited as not-others only by abstraction. If it is a third which posits their otherness, it is also a third which sublates it; but this *entire movement which gives them their ideality* falls outside them. Should it be pointed out that this movement of thought itself falls, nevertheless, only within an ideating monad, the reply must be that the very *content* of such thinking is *within itself external to itself.* The transition from the oneness of absolute ideality (the monad of monads) to the category of the abstract (connectionless) *plurality* of determinate being, is immediate and uncomprehended (it is effected through the image of creation); and the transition from this plurality back to the oneness is equally abstract. Ideality, ideation generally, remains something formal, as also does ideation raised to the form of consciousness. Just as consciousness is conceived as a one-sided form which is indifferent to its determination and content, in the above-mentioned fancy of Leibniz—namely, that the magnetic needle, if it possessed consciousness, would regard

its direction to the north as freely determined by itself—so, too, ideality in the monads is a form remaining external to the plurality. Ideality is supposed to be immanent in them, their nature is supposed to be ideation; but their relationship is on the one hand their harmony, which does not fall within the sphere of their determinate being and is, consequently, pre-established; and on the other hand, this determinate being of theirs is not grasped as a being-for-other, or, further, as ideality, but is determined only as an abstract plurality; the ideality of the plurality and the further determination of it to harmony are not immanent in and proper to this plurality itself.

Other idealisms, as for example those of Kant and Fichte, do not go beyond the *ought* or the *infinite progress*, and remain in the dualism of determinate being and being-for-self. True, in these systems, the thing-in-itself or the infinite shock or resistance principle [*Anstoss*] enters directly into the ego and becomes only something *for it;* but it proceeds from a free otherness which is perpetuated as a negative being-in-itself. The ego is therefore undoubtedly determined as ideal [*das Ideelle*], as being for itself, as infinite self-relation; but the moment of *being-for-one* is not completed to the point where the beyond, or the direction to the beyond, vanishes.

(*c*) *The One*

Being-for-self is the simple unity of itself and its moment, being-for-one. There is before us only a single determination, the self-relation of the sublating. The *moments* of being-for-self have collapsed into the *undifferentiatedness* which is immediacy or being, but an *immediacy* based on the negating which is posited as its determination. Being-for-self is thus *a* being-for-self, and since in this immediacy its inner meaning vanishes, it is the wholly abstract limit of itself—the *one*.

Attention may be drawn in advance to the difficulty involved in the following exposition of the *development* of the one and to its cause. The *moments* which constitute the Notion of the one as a being-for-self fall *asunder* in the development. They are: (1) negation in general, (2) *two* negations, (3) two that are therefore the *same*, (4) sheer opposites, (5) self-relation, identity as such, (6) relation which is *negative* and yet to *its own self*. The reason for the separation of these moments here is that the form of *immediacy*,

of *being* enters into being-for-self as *a* being-for-self; through this immediacy *each moment is posited* as a *distinct, affirmative determination*, and yet they are no less *inseparable*. Hence of each determination the opposite must equally be asserted; it is this contradiction, together with the abstract nature of the moments, which constitutes the difficulty.

B. THE ONE AND THE MANY

The one is the simple self-relation of being-for-self in which its moments have collapsed in themselves and in which, consequently, being-for-self has the form of *immediacy*, and its moments therefore now have a *determinate* being.

As self-relation of the *negative* the one is a process of determining—and as *self*-relation it is an infinite *self*-determining. But because being-for-self is now in the form of immediacy, these *differences* are no longer posited only as moments of one and the same self-determination, but as at the same time *affirmatively* present. The *ideality* of being-for-self as a totality thus reverts, in the first place, to *reality* and that too in its most fixed, abstract form, as the *one*. In the one, being-for-self is the *posited* unity of simple being and determinate being, as the absolute union of the relation to other and self-relation; but, further, the determinateness of being also stands *opposed* to the determination of the *infinite negation*, to the self-determination, so that what the one is *in itself* is now only *ideally present in it*, and the negative consequently is an other distinct from it. What shows itself to be *present* as distinct from the one is its own self-determining; the unity of the one with itself as thus distinguished from itself is reduced to a *relation*, and as a *negative* unity it is a negation of its own self as *other*, *exclusion* of the one as other from itself, from the one.

(*a*) *The One in its own self*

In its own self the one simply *is;* this its being is neither a determinate being, nor a determinateness as a relation to an other, nor is it a constitution; what it is, in fact, is the accomplished negation of this circle of categories. Consequently, the one is not capable of becoming an other: it is *unalterable*.

It is indeterminate but not, however, like being; its indeter-

minateness is the determinateness which is a relation to its own self, an absolute determinedness—*posited* being-within-self. As the one is in accordance with its Notion a self-related negation, it has difference in it—a turning away from itself to an other; but this movement is immediately turned back on itself, because it follows from this moment of self-determining that there is no other to which the one can go, and the movement has thus returned into itself.

In this simple immediacy the mediation of determinate being and of ideality itself, and with it all difference and manifoldness, has vanished. There is *nothing* in it; this *nothing*, the abstraction of self-relation, is here distinguished from the being-within-self itself; it is a *posited* nothing because this being-within-self no longer has the simple character of something but, as a mediation, has a concrete determination; but as abstract, though it is identical with the one, it is distinct from its determination. This nothing, then, posited as *in the one*, is the nothing as the *void*. The void is thus the *quality* of the *one* in its immediacy.

(*b*) *The One and the Void*

The one is the void as the abstract relation of the negation to itself. However, the void as the nothing is absolutely distinct from the simple immediacy, the also *affirmative* being of the one, and since they stand in one and the same relation, namely, that of the one, their difference is *posited;* but as distinct from the affirmative being of the one, the nothing as the void is *outside* it.

Being-for-self determined in this manner as the one *and* the void has again acquired a *determinate* being. The one and the void have negative relation to self for their common, simple base. The moments of being-for-self emerge from this unity, become external to themselves; through the *simple* unity of the moments there enters the determination of *being* and the unity thus reduces itself to being only *one* side, and so to a determinate being; and in this it is confronted by its other determination, the negation as such, likewise as a determinate being of the nothing, as the void.

Remark: Atomism

The one in this form of determinate being is the stage of the category which made its appearance with the ancients as the

atomistic principle, according to which the essence of things is the atom and the void (*το ἄτομον* or *τὰ ἄτομα καὶ τὸ κενόν*). The abstraction which has developed into this form has acquired a greater determinateness than the ***being*** of Parmenides and the ***becoming*** of Heraclitus. Lofty as is this abstraction, in that it makes this simple determinateness of the one and the void the principle of all things, deriving the infinite variety of the world from this simple antithesis and boldly presuming to know the former from the latter, it is equally easy for figurate conception to picture ***here*** atoms and ***alongside*** them the void. It is, therefore, no wonder that the atomistic principle has at all times been upheld; the equally trivial and external relation of *composition* which must be added to achieve a semblance of concreteness and variety is no less popular than the atoms themselves and the void. The one and the void is being-for-self, the highest qualitative being-within-self, sunk back into complete *externality*; the immediacy or being of the one, because it is the negation of all otherness, is posited as being no longer determinable and alterable; such therefore is its absolute, unyielding rigidity that all determination, variety, conjunction remains for it an utterly external relation.

However, with the first thinkers the atomistic principle did not remain in this externality but besides its abstraction had also a speculative determination in the fact that the void was recognized as the source of movement, which is an entirely different relation of the atom and the void from the mere juxtaposition and mutual indifference of these two determinations. That the void is the source of movement has not the trivial meaning that something can only move into an empty space and not into an already occupied space, for in such a space it would not find any more open room—understood in this sense, the void would be only the presupposition or condition of movement, not its *ground*, just as the movement itself, too, would be presupposed as already existing, the essential point, its ground, being forgotten. The view that the void constitutes the ground of movement contains the profounder thought that in the negative as such there lies the ground of becoming, of the unrest of self-movement—in which sense, however, the negative is to be taken as the veritable negativity of the infinite. The void is the *ground* of movement only as the ***negative*** relation of the one to its ***negative***, to the one,

that is to itself, which however is posited as having determinate being.

But in other respects, the further determinations of the ancients concerning the shape and position of the atoms and the direction of their movement, are arbitrary and external enough and, in addition, stand in direct contradiction to the basic determination of the atom. Physics with its molecules and particles suffers from the atom, this principle of extreme externality, which is thus utterly devoid of the Notion, just as much as does that theory of the State which starts from the particular will of individuals.

(c) Many Ones: Repulsion

The one and the void constitute the first stage of the determinate being of being-for-self. Each of these moments has negation for its determination and is at the same time posited as a determinate being; according to the former determination the one and the void are the *relation* of negation to negation as of an other to its other: the one is negation in the determination of being, and the void is negation in the determination of non-being. But the one is essentially self-relation only as related *negation*, that is, it is itself that which the void outside it is supposed to be. Each, however, is also *posited* as an affirmative *determinate being*, one as a being-for-self as such, the other an an unspecified determinate being in general, and each is related to the other as to *another determinate being*. The being-for-self of the one, is, however, essentially the ideality of determinate being and of other: it relates itself not to an other but only *to itself*. But since being-for-self is fixed as a one, as *affirmatively* for itself, as *immediately* present, its *negative* relation *to itself* is at the same time a relation to an *affirmative* being; and since the relation is just as much negative, that to which it relates itself remains determined as a *determinate being* and an *other;* as essentially *self*-relation, the other is not indeterminate negation as the void, but is likewise a *one*. The one is consequently a *becoming of many ones*.

Strictly, however, this is not really a *becoming*, for becoming is a transition of *being* into *nothing:* the one, on the other hand, becomes only one. The one, as related, contains the negative as a relation, has it therefore *within* it. Instead of a becoming, then, there is present first the immanent relation of the one itself; and secondly, since the relation is negative and the one is at the same

time affirmatively present, the one repels itself *from itself*. The negative relation of the one to itself is *repulsion*.

This repulsion as thus the positing of *many ones* but through the one itself, is the one's own coming-forth-from-itself but to such outside it as are themselves only ones. This is repulsion according to its Notion, repulsion *in itself*. The second repulsion is different from it, it is what is immediately suggested to external reflection: repulsion not as the *generation* of ones, but only as the mutual repelling of ones presupposed as already *present*. We have now to see how the first repulsion, repulsion *in itself*, determines itself to the second, to external repulsion.

First of all we must establish what determinations are possessed by the many ones as such. As an explication of what the one is in itself, the becoming of the many, or the generation of the many, vanishes immediately; the products of the process are ones, and these are not for an other, but relate themselves infinitely to themselves. The one repels only *itself* from itself, therefore does not *become* but *already is*; and what is represented as repelled is likewise a *one*, a one that *is*. To repel and to be repelled applies equally to both, and makes no difference.

The ones are thus *presupposed* relatively to one another—*supposed* or *posited* by the repulsion of the one from itself; *pre*-supposed as *not* posited. Their positedness is sublated, and as related only to *themselves* they are *affirmative* beings relatively to one another.

Thus plurality appears not as an *otherness*, but as a determination completely external to the one. The one, in repelling itself, remains self-related, like that which to begin with is taken as repelled. That the ones are related to one another as *others*, are brought together into the determinateness of plurality, does not therefore concern the ones. If plurality were a relation of the ones themselves to one another then they would limit one another and there would be affirmatively present in them a being-for-other. Their relation—and this they have through their *implicit* unity—as here *posited* is determined as none: it is again the previously posited *void*. The void is their limit but a limit which is external to them, in which they are not to be *for one another*. The limit is that in which what are limited both *are* and *are not*: but the void is determined as pure non-being, and this alone constitutes their limit.

The repulsion of the one from itself is the explication of that which the one is in itself; but infinity as *explicated* is here the infinity which has *come forth from itself;* it has come forth from itself by virtue of the immediacy of the infinite, the one. It is a simple relating of the one to the one, and no less also the absolute absence of relation in the one; it is the former according to the simple, affirmative self-relation of the one, and the latter according to the self-same relation as negative. In other words, the plurality of the one is its own positing; the one is nothing but the *negative* relation of the one to itself, and this relation—and therefore the one itself—is the plural one. But equally, plurality is absolutely external to the one; for the one is, precisely, the sublating of otherness; repulsion is its self-relation and simple equality with itself. The plurality of ones is infinity as a contradiction which unconstrainedly produces itself.

Remark: The Monad of Leibniz.

We have previously referred to the Leibnizian idealism. We may add here that this idealism which started from the *ideating monad,* which is determined as being for itself, advanced only as far as the repulsion just considered, and indeed only to *plurality* as such, in which each of the ones is only for its own self and is indifferent to the determinate being and being-for-self of the others; or, in general, for the one, there are no others at all. The monad is, by itself, the entire closed universe; it requires none of the others. But this inner manifoldness which it possesses in its ideational activity in no way affects its character as a being-for-self. The Leibnizian idealism takes up the *plurality* immediately as something *given* and does not grasp it as a *repulsion* of the monads. Consequently, it possesses plurality only on the side of its abstract externality. The atomistic philosophy does not possess the Notion of ideality; it does not grasp the one as an ideal being, that is, as containing *within itself* the two moments of being-for-self and being-for-it, but only as a simple, dry, real being-for-self. It does, however, go beyond mere indifferent plurality; the atoms become further determined in regard to one another even though, strictly speaking, this involves an inconsistency; whereas, on the contrary, in that indifferent independence of the monads, plurality remains as a fixed *fundamental determination,* so that the connexion

between them falls only in the monad of monads, or in the philosopher who contemplates them.

C. REPULSION AND ATTRACTION

(*a*) *Exclusion of the One*

The many ones have affirmative being; their determinate being or relation to one another is a non-relation, is external to them—the abstract void. But they themselves are now this negative relation to themselves as to *affirmatively present* others—the demonstrated contradiction, infinity posited in the immediacy of being. Thus repulsion now simply *finds immediately before* it that which is repelled by it. In this determination repulsion is an *exclusion*; the one repels from itself only the many ones which are neither generated nor posited by it. This mutual or all-round repelling is relative, is limited by the being of the ones.

The plurality is, in the first place, non-posited otherness, the limit is only the void, only that in which the ones *are not*. But in the limit they also *are*; they are in the void, or their repulsion is their *common relation*.

This mutual repulsion is the posited *determinate being* of the many ones; it is not their being-for-self, for according to this they would be differentiated as many only in a third, but it is their own differentiating which preserves them. They negate one another reciprocally, posit one another as being only *for-one*. But at the same time they equally *negate* this *being only for-one*; they *repel* this their *ideality* and *are*. Thus the moments which in ideality are absolutely united are separated. The one is, in its being-for-self, also *for-one*, but this one for which it is is its own self; its differentiation of itself is immediately sublated. But in plurality the differentiated one has a being; the being-for-one as determined in exclusion is, consequently, a being-for-other. Each is thus repelled by an other, is sublated and made into that which is not *for itself* but *for-one*, and that another one.

The being-for-self of the many ones shows itself, therefore, as their self-preservation through the mediation of their mutual repulsion in which they mutually sublate themselves and each posits the others as a mere being-for-other; but the self-preservation at the same time consists in repelling this ideality and in positing the ones as not being for-an-other. This self-preservation

of the ones through their negative relation to one another is, however, rather their dissolution.

The ones not only *are*, but they maintain themselves through their reciprocal exclusion. Now in the first place that which should enable the ones to maintain their diversity in opposition to their being negated is their *being*, in fact, their being-*in-itself* as opposed to their relation-to-other; this being-in-itself is that they are *ones*. But *this is what they all are*; they are in their being-in-itself the *same* instead of this latter being the fixed point of their diversity. Secondly, their determinate being and their relation to one another, that is, their *positing of themselves as ones*, is the reciprocal negating of themselves; but this likewise is *one and the same* determination of them all, through which then they rather posit themselves as identical; similarly, because they are in themselves the same, their ideality, instead of being posited through others, is *their own*, and they therefore repel it just as little. Consequently, as regards both their being and their positing, they are only *one* affirmative unity.

This reflection that the ones, determined both as simply being and as inter-related, show themselves to be one and the same and indistinguishable, is a comparison made by us. But we have also to see what is *posited* in them in their *inter-relatedness*. They *are*—this is presupposed in this inter-relatedness—and they *are* only in so far as they reciprocally negate one another and at the same time hold themselves aloof from this their ideality, their negatedness, that is, negate this reciprocal negating. But they *are* only in so far as they negate; consequently, since this their negating is negated, their being is negated. It is true that since they *are*, they would not be negated by this negating, which for them is only something external; this negating by the other rebounds off them and touches only their surface. And yet it is only through this negating of the others that the ones return into themselves: they *are* only as this mediation, and this their return is their self-preservation and their being-for-self. Since their negating is ineffectual because of the resistance offered by the ones either as simply affirmative or as negating, they do not return into themselves, do not preserve themselves, and so are not.

The observation was made above that the ones are the same, each of them is a *one* like the other. This is not only a relating of them by us, an external bringing of them together; on the

contrary, repulsion is itself a relating; the one which excludes the ones relates itself to them, to the ones, that is, to its own self. Hence the negative relationship of the ones to one another is only a *going-together-with-self*. This identity into which their repelling passes over is the sublating of their diversity and externality which they, as excluding, ought rather to maintain relatively to one another.

This positing of themselves by the many ones into a single one is *attraction*.

Remark: The unity of the One and the Many

Self-subsistence pushed to the point of the one as a being-for-self is abstract, formal, and destroys itself. It is the supreme, most stubborn error, which takes itself for the highest truth, manifesting in more concrete forms as abstract freedom, pure ego and, further, as Evil. It is that freedom which so misapprehends itself as to place its essence in this abstraction, and flatters itself that in thus being with itself it possesses itself in its purity. More specifically, this self-subsistence is the error of regarding as negative that which is its own essence, and of adopting a negative attitude towards it. Thus it is the negative attitude towards itself which, in seeking to possess its own being destroys it, and this its act is only the manifestation of the futility of this act. The reconciliation is the recognition that the object of this negative attitude is rather its own essence, and is only the *letting go* of the negativity of *its* being-for-self instead of holding fast to it.

It is an ancient proposition that the one is many and especially that the many are one. We may repeat here the observation that the truth of the one and the many expressed in propositions appears in an inappropriate form, that this truth is to be grasped and expressed only as a becoming, as a process, a repulsion and attraction—not as being, which in a proposition has the character of a stable unity. We have already mentioned and recalled the dialectic of Plato in the Parmenides concerning the derivation of the many from the one, namely, from the proposition: the one is. The inner dialectic of the Notion has been stated; it is easiest to grasp the dialectic of the proposition, that the many are one, as an external reflection; and it may properly be grasped externally here inasmuch as the object too, the many, are mutually external. It directly follows from this comparison of the many with one

another that any one is determined simply like any other one; each is a one, each is one of the many, *is* by excluding the others—so that they are absolutely the same, there is present one and only one determination. This is the *fact*, and all that has to be done is to grasp this simple fact. The only reason why the understanding stubbornly refuses to do so is that it has *also* in mind, and indeed rightly so, the difference; but the existence of this difference is just as little excluded because of the said fact, as is the certain existence of the said fact in spite of the difference. One could, as it were, comfort understanding for the naïve manner in which it grasps the fact of the difference, by assuring it that the difference will also come in again.

(*b*) *The one One of Attraction*

Repulsion is the self-differentiating of the one, at first into many, whose negative relationship is without effect because they presuppose one another as affirmatively present; it is only the *ought-to-be* of ideality. In attraction, however, ideality is realized. Repulsion passes over into attraction, the many ones into *one* one. Both repulsion and attraction are in the first place distinct from each other, the former as the reality of the ones, the latter as their posited ideality. The relation of attraction to repulsion is such that the former has the latter for *presupposition*. Repulsion provides the material for attraction. If there were no ones there would be nothing to attract; the conception of a perpetual attraction, of an absorption of the ones, presupposes an equally perpetual production of them. Spatial attraction as pictorially conceived makes the flow of attracted ones proceed uninterruptedly; in place of the atoms which vanish in the centre of attraction, another multitude comes forth from the void, and so on to infinity if one wishes. If attraction were conceived as accomplished, the many being brought to the point of the one one, then there would be present only an inert one and no longer any attraction. The ideality present in attraction still also bears within itself the determination of the negation of itself, the many ones of which it is the relatedness: attraction is inseparable from repulsion.

In the first place, attraction belongs equally to each of the many ones as *immediately* present; none has any precedence over another; this would result in an equilibrium in attraction, or, strictly speaking, an equilibrium of attraction and repulsion

itself, and an inert state of rest in which ideality would have no determinate being. But there can be no question here of a precedence of such a one over another, for this would presuppose a specific difference between them; rather is attraction the positing of the immediately present undifferentiatedness of the ones. It is only attraction itself that is a *positing* of a one distinct from other ones; these are only immediate ones which should maintain themselves through repulsion; but through their posited negation arises the one of attraction, which is consequently determined as mediated, the *one posited as one*. The first ones, as immediate, do not in their ideality return into themselves but have this ideality in another one.

The one one, however, is the realized ideality, posited in the one; it is attraction through the mediation of repulsion, and it contains this mediation within itself as *its determination*. Thus it does not absorb the attracted ones into itself as into a centre, that is, it does not sublate them abstractly. Since it contains repulsion in its determination, this latter at the same time preserves the ones as many in it; through its attracting, so to speak, it acquires something for itself, obtains an extension or filling. There is thus in it the unity of repulsion and attraction in general.

(*c*) *The Relation of Repulsion and Attraction*

The difference of the one and the many is now determined as the difference of their *relation* to one another, with each other, a relation which splits into two, repulsion and attraction, each of which is at first independent of the other and stands apart from it, the two nevertheless being essentially connected with each other. Their as yet indeterminate unity is to be more precisely ascertained.

Repulsion, as the basic determination of the one, appears first and as *immediate*, like its ones which although generated by repulsion are yet also posited as immediate. As such, repulsion is indifferent to attraction which is externally added to it as thus presupposed. Attraction on the other hand is not presupposed by repulsion in such a manner that the former is supposed to have no part in the positing and being of the latter, that is, as if repulsion were not already in its own self the negation of itself and the ones were not already in themselves negated. In this way we have repulsion abstractly on its own account and, similarly, attraction

relatively to the ones as *affirmatively* present, has the side of an immediate determinate being and comes to them as an other.

If repulsion is thus taken merely by itself, then it is the dispersion of the many ones into somewhere undetermined, outside the sphere of repulsion itself; for repulsion is this, to negate the inter-relatedness of the many: the absence of any relation between them is the determination of the many taken abstractly. But repulsion is not merely the void; the ones, as unrelated, do not repel or exclude one another, this constitutes their determination. Repulsion is, although negative, still essentially *relation;* the mutual repulsion and flight is not a liberation from what is repelled and fled from, the one as excluding still remains *related* to what it excludes. But this moment of relation is attraction and thus is in repulsion itself; it is the negating of that abstract repulsion according to which the ones would be only self-related affirmative beings not excluding one another.

In starting, however, with the repulsion of the determinately present ones and so, too, with attraction posited as externally connected with it, the two determinations although inseparable are held apart as distinct; but it has been found that not merely is repulsion presupposed by attraction, but equally, too, there is a reverse relation of repulsion to attraction, and the former equally has its presupposition in the latter.

As thus determined they are inseparable and at the same time each is determined as an ought and a limitation relatively to the other. Their ought is their abstract determinateness in the form of the *in-itself;* but with this determinateness each is simply directed away from itself and relates itself to the *other*, and thus each *is* through the mediation of the *other* as *other*; their self-subsistence consists in the fact that in this mediation each is posited for the other as a *different* determining: repulsion as the positing of the many, attraction as the positing of the one, the latter as at the same time a negation of the many, and the former as a negation of their ideality in the one, so that attraction, too, is attraction only *through the mediation* of repulsion, just as repulsion is repulsion through the mediation of attraction. But the fact that in this interdependence the mediation of each through the other is rather negated, each of these determinations being a self-mediation, becomes evident after a closer consideration of them and brings them back to the unity of their Notion.

In the first place, that each presupposes *itself*, is related only to itself in its presupposition, this is already implied in the relationship between repulsion and attraction in their initially still relative character.

Relative repulsion is the mutual repelling of the *present* many ones which are supposed to be immediately *given*. But that *there are* many ones, this is repulsion itself; any presupposition which it might have is only its own positing. Further, the determination of *being* which might belong to the ones apart from the circumstance that they are posited—whereby they would be *already there*—belongs likewise to repulsion. The repelling is that whereby the ones manifest and maintain themselves as ones, whereby they *are* as such. Their being is repulsion itself, which is thus not a relative determinate being over against another such, but relates itself simply and solely to its own self.

Attraction is the positing of the one as such, of the real one, in contrast to which the many in their determinate being are determined as only ideal [*ideell*] and as vanishing. Attraction thus directly presupposes itself—in the determination, namely, of the other ones as ideal, which ones are otherwise supposed to be *for themselves*, repelling *others*, and therefore also any attracting one. Ideality, as opposed to this determination of repulsion, does not belong to the ones only through the relation to attraction; on the contrary, it is presupposed, it is the ideality *inherent* in the ones in that, as ones—including the one conceived as attracting —they are not distinguished from one another, are one and the same.

Further, this self-presupposing of the two determinations each for itself, means that each contains the other as a moment within it. The *self-presupposing* as such is the one's positing of itself in a one as the *negative* of itself—repulsion; and what is therein presupposed is *the same* as that which presupposes—attraction. That each is *in itself* only a moment, is the transition of each out of itself into the other, the self-negating of each in itself and the self-positing of each as its own other. The one as such, then, is a coming-out-of-itself, is only the positing of itself as its own other, as many; and the many, similarly, is only this, to collapse within itself and to posit itself as *its* other, as one, and in this very act to be related only to its own self, each continuing itself in its other. Thus there is already present in principle (*an sich*) the

undividedness of the coming-out-of-itself (repulsion) and the self-positing as one (attraction). But in the relative repulsion and attraction, which presuppose immediate, *determinately* existent ones, it is *posited* that each is in its own self this negation of itself and is thus also the continuity of itself in its other. The repulsion of the determinately existent ones is the self-preservation of the one through the mutual repulsion of the others, so that (1) the other ones are negated *in it*—this is the side of its determinate being or of its being-for-other; but this is thus attraction as the ideality of the ones; and (2) the one is *in itself*, without relation to the others; but not only has being-in-itself as such long since passed over into being-for-self, but the one *in itself*, by its determination, is the aforesaid becoming of many ones. The *attraction* of the determinately existent ones is their ideality and the positing of the one, in which, accordingly, attraction as a negating and a generating of the one sublates itself, and as a positing of the one is in its own self the negative of itself, repulsion.

With this, the development of being-for-self is completed and has reached its conclusion. The one as *infinitely self*-related—infinitely, as the posited negation of negation—is the mediation in which it repels from itself its own self as its absolute (that is, abstract) *otherness*, (the *many*), and in relating itself negatively to this its non-being, that is, in sublating it, it is only self-relation; and one is only this *becoming* in which it is no longer determined as having a *beginning*, that is, is no longer posited as an immediate, affirmative being, neither is it as result, as having restored itself as the one, that is, the one as equally *immediate* and excluding; the process which it is posits and contains it throughout only as sublated. The sublating, at first determined as only a relative sublating of the *relation* to another determinately existent one—a relation which is thus itself not an indifferent repulsion and attraction—equally displays itself as passing over into the infinite relation of mediation through negation of the external relations of the immediate, determinately existent ones, and as having for result that very process of becoming which, in the instability of its moments, is the collapse, or rather going-together-with-itself, into simple immediacy. This being, in the determination it has now acquired, is *quantity*.

A brief survey of the moments of this *transition of quality into quantity* shows us that the fundamental determination of quality

is being and immediacy, in which limit and determinateness are so identical with the being of something, that with its alteration the something itself vanishes; as thus *posited* it is determined as finite. Because of the immediacy of this unity, in which the *difference* has vanished but is *implicitly* present in the unity of *being* and *nothing*, the difference as *otherness* in general falls *outside* this unity. This relation to other contradicts the immediacy in which qualitative determinateness is self-relation. This otherness sublates itself in the infinity of being-for-self which makes explicit the difference (which in the negation of the negation is present in it) in the form of the one and the many and their relations, and has raised the qualitative moment to a genuine unity, that is, a unity which is no longer immediate but is posited as accordant with itself.

This unity is, therefore, (α) *being*, only as *affirmative*, that is *immediacy*, which is self-mediated through negation of the negation; being is posited as the unity which *pervades* its determinatenesses, limit, etc., which are posited in it as sublated; (β) *determinate being:* in such determination it is the negation or determinateness as a moment of affirmative being, yet determinateness no longer as immediate, but as reflected into itself, as related not to an other but to itself; a being determined simply *in itself*—the one; the otherness as such is itself a being-for-self; (γ) *being-for-self*, as that being which continues itself right through the determinateness and in which the one and the intrinsic determinedness is itself posited as sublated. The one is determined simultaneously as having gone beyond itself, and as *unity*; hence the one, the absolutely determined limit, is posited as the limit which is no limit, which is present in being but is indifferent to it.

Remark: The Kantian Construction of Matter from the Forces of Attraction and Repulsion

Attraction and repulsion, as we know, are usually regarded as *forces*. This determination of them and the relationships connected with it have to be compared with the Notions which have resulted from our consideration of them. Conceived as forces, they are regarded as self-subsistent and therefore as not connected with each other through their own nature; that is, they are considered not as moments, each of which is supposed to pass into the other,

but rather as fixed in their opposition to each other. Further, they are imagined as meeting in a third, in *matter*, but not in such a manner that this unification is counted as their truth; on the contrary, each is regarded as a first, as being in and for itself, and matter, or its determinations, are supposed to be realized and produced by them. When it is said that matter has the forces *within itself*, they are understood to be so conjoined in this unity that they are at the same time presupposed as intrinsically free and independent of each other.

Kant, as we know, *constructed* matter from the forces of attraction and repulsion, or at least he has, to use his own words, set up the metaphysical elements of this construction. It will not be without interest to examine this construction more closely. This *metaphysical* exposition of a subject matter which not only itself but also in its determinations seemed to belong only to *experience* is noteworthy, partly because as an experiment with the Notion it at least gave the impulse to the more recent philosophy of nature, to a philosophy which does not make nature as given in sense-perception the basis of science, but which goes to the absolute Notion for its determinations; and partly because in many cases no advance is made beyond the Kantian construction which is held to be a philosophical beginning and foundation for physics.

Now it is true that matter as it exists for sense perception is no more a subject matter of logic than are space and its determinations. But the forces of attraction and repulsion, in so far as they are regarded as forces of empirical matter, are also based on the pure determinations here considered of the one and the many and their inter-relationships, which, because these names are most obvious, I have called repulsion and attraction.

Kant's method in the deduction of matter from these forces, which he calls a *construction*, when looked at more closely does not deserve this name, unless any exercise of reflection, even analytical reflection, is to be called a construction; and later philosophers of nature have in fact given the name of construction to the shallowest reasoning and the most baseless concoction of unbridled imagination and thoughtless reflection—and it is especially for the so-called factors of attraction and repulsion that such philosophers have shown a predilection.

For Kant's method is basically analytical, not constructive. He

presupposes the idea of matter and then asks what forces are required to maintain the determinations he has presupposed. Thus, on the one hand, he demands the force of attraction because, properly speaking, through repulsion alone and without attraction matter could not exist;[1] and on the other hand he derives repulsion, too, from matter and gives as the reason that we think of matter as impenetrable, since it presents itself under this category to the sense of touch by which it manifests itself to us. Consequently, he proceeds, repulsion is at once thought in the *concept* of matter because it is immediately *given* therein, whereas attraction is added to the concept *syllogistically*. But these syllogisms, too, are based on what has just been said, namely, that matter which possessed repulsive force alone, would not exhaust our conception of matter. It is evident that this is the method of a cognition which reflects on experience, which first *perceives* the determinations in a phenomenon, then makes these the foundation, and for their so-called *explanation* assumes corresponding *basic elements* or *forces* which are supposed to produce those determinations of the phenomenon.

With respect to this difference as to the way in which cognition finds the forces of repulsion and attraction in matter, Kant further remarks that the force of attraction certainly just as much belongs to the concept of matter '*although it is not contained in it*'; this last expression is italicized by Kant. However, it is hard to perceive what this difference is supposed to be; for a determination which belongs to the concept of anything *must* be truly *contained in it*.

What causes the difficulty and gives rise to this vain subterfuge, is that Kant from the start one-sidedly attributes to the concept of matter only the determination of impenetrability, which we are supposed to perceive by the sense of touch, for which reason the force of repulsion as the holding off of an other from itself is immediately *given*. But if, further, the *existence* of matter is supposed to be impossible without attraction, then this assertion is based on a conception of matter taken from sense perception; consequently, the determination of attraction, too, must come within the range of sense perception. It is indeed easy to perceive that matter, besides its being-for-self, which sublates the being-for-other (offers resistance), has also a *relation* between its *self-*

[1] *Anfangsgr. der Naturwissensch.*, p. 53 *et seq.*

determined parts, a spatial *extension* and *cohesion*, and in rigidity and solidity the cohesion is very firm. Physics explains that the tearing apart, etc., of a body requires a force which shall be stronger than the mutual *attraction* of the parts of the body. From this observation reflection can just as directly derive the force of attraction or assume it as *given*, as it did with the force of repulsion. In point of fact, if we consider Kant's arguments from which the force of attraction is supposed to be deduced (the proof of the proposition that the possibility of matter requires a force of attraction as a second fundamental force, *loc. cit.*), it is apparent that their sole content is this, that through repulsion alone matter would not be *spatial*. Matter being presupposed as filling space, it is credited with continuity, the ground of which is assumed to be the force of attraction.

Now if the merit of such a construction of matter were at most that of an analysis (though a merit diminished by the faulty exposition), still the fundamental thought, namely, the derivation of matter from these two opposite determinations as its fundamental forces, must always be highly esteemed. Kant is chiefly concerned to banish the vulgar mechanistic way of thinking which stops short at the one determination of impenetrability, of self-determined and self-subsistent punctuality, and converts into something *external* the opposite determination, the *relation* of matter within itself or the relation of a plurality of matters, which in turn are regarded as particular ones—a way of thinking which, as Kant says, will admit no motive forces except pressure and thrust, that is, only action from without. This *external* manner of thinking always presupposes motion as already externally *present* in matter, and it does not occur to it to regard motion as something immanent and to comprehend motion itself in matter, which latter is thus assumed as, on its own account, motionless and inert. This stand-point has before it only ordinary mechanics, not immanent and free motion. It is true that Kant sublates this externality in so far as he makes attraction (the *relation* of matters to one another in so far as these are assumed as separated from one another, or matter generally in its self-externality) a *force of matter itself*; still, on the other hand, his two fundamental forces within matter remain external to and completely independent of each other.

The fixed difference of these two forces attributed to them

from that external standpoint is no less null than any other distinction must show itself to be which, in respect of its specific content, is made into something supposedly fixed; because these forces are only moments which pass over into each other, as we saw above when they were considered in their truth. I go on to consider these other distinctions as they are stated by Kant.

He defines the force of attraction as a penetrative force by which one bit of matter can act *directly* on the parts of another even beyond the area of contact; the force of repulsion, on the other hand, he defines as a surface force through which bits of matter can act on each other only in the common area of contact. The reason adduced that the latter can be only a surface force is as follows: 'The parts in *contact* each limit the sphere of action of the other, and the force of repulsion cannot move any more distant part except through the agency of the intervening parts; an immediate action of one part of matter on another passing right across these intervening parts by forces of expansion (which means here, forces of repulsion) is impossible.'[1]

But here we must remember that in assuming 'nearer' or 'more distant' parts of matter, the same distinction would likewise arise with respect to attraction, namely, that though one atom acted on another, yet a third, more distant atom (between which and the first atom, the second atom would be), would first enter into the sphere of attraction of the intervening atom nearer to it; therefore the first atom would not have an *immediate*, simple action on the third, from which it would follow that the action of the force of attraction, like that of repulsion, is equally mediated. Further, the *genuine penetration* of the force of attraction could of necessity consist only in this, that every part of matter was in and for itself attractive, not that a certain number of atoms behaved passively and only one atom actively. But we must at once remark with respect to the force of repulsion itself that in the passage quoted, 'parts in contact' are mentioned which implies *solidity* and *continuity* of a matter already *finished and complete* which would not permit the passage through it of a repelling force. But this solidity of matter in which parts are in *contact* and are no longer separated by the void already presupposes that the force of repulsion is sublated; according to the sensuous conception of repulsion which prevails here, parts

[1] See *Anfangsgr. der Naturwissensch. Erklärung und Zusätze*, p. 67.

in contact are to be taken as those which do not repel each other. It therefore follows, quite tautologically, that where repulsion is assumed to be *not*, there no repulsion can take place. But from this nothing else follows which could serve to determine the force of repulsion. However, reflection on the statement that parts in contact are in contact only in so far as they hold themselves *apart*, leads directly to the conclusion that the force of repulsion is not merely *on the surface* of matter but within the sphere which was supposed to be only a sphere of attraction.

Kant assumes further that 'through the force of attraction, matter only occupies space but does not fill it';[1] and 'because matter through the force of attraction does not fill space, this force can act across empty space since there is no intervening matter to limit it'. This distinction is much the same as the one mentioned above where a determination was supposed to belong to the concept of a thing but not to be contained in it; here, then, matter is supposed only to *occupy* a space but not to *fill* it. There it is repulsion, if we stop at the first determination of matter, through which the ones repel one another and so are only negatively related to one another, here that means, by empty space. Here, however, it is the force of attraction which keeps space empty; it does not fill space by its connection of the atoms, in other words, it keeps the atoms in a negative relation to one another. We see that Kant here unconsciously realizes what is implicit in the nature of the subject matter, when he attributes to the force of attraction precisely what, in accordance with the first determination, he attributed to the opposite force. While he was busy with establishing the difference between the two forces, it happened that one had passed over into the other. Thus through repulsion, on the other hand, matter is supposed to fill a space, and consequently through repulsion the empty space left by the force of attraction vanishes. In point of fact repulsion, in doing away with empty space, also destroys the negative relation of the atoms or ones, that is, their repulsion of one another; in other words, repulsion is determined as the opposite of itself.

To this effacing of the differences there is added the confusion arising from the fact that, as we observed at the beginning, Kant's exposition of the opposed forces is analytic; and whereas matter is supposed to be derived from its elements, it is presented through-

[1] *Ibid.*

out the entire discourse as already formed and constituted. In the definition of surface and penetrative force both are assumed as motive forces by means of which matter is supposed to be able to act in one or other of these ways. Here, therefore, they are represented as forces, not through which matter first comes into being but through which matter, as an already finished product, is only set in motion. But in so far as we are speaking of the forces through which different bodies act on one another and are set in motion, this is something quite different from the determination and relation which these forces were supposed to have as [constitutive] moments of matter.

The same opposition of attractive and repulsive forces is made by their more developed form of centripetal and centrifugal forces. These appear to offer an essential distinction, since in their sphere there is a fixed single one, a centre, in relation to which the other ones behave as *not for themselves*, so that the difference between the forces can be linked to this presupposed difference between a single central one and the others which are not independent relatively to it. But if they are to be used for explanation—for which purpose they are assumed to be (like the forces of repulsion and attraction) in an inverse quantitative ratio so that the one increases as the other decreases—then the phenomenon of the motion and its inequality ought to be the result of these forces which were assumed for the purpose of explanation. However, one need only examine the accounts (any of them will do) of a phenomenon like the unequal velocity of a planet in its orbit round the sun, based on the opposition of these forces, to become aware of the confusion which prevails in such explanations, and the impossibility of disentangling the magnitudes of the forces, so that the one which in the explanation is assumed to be decreasing can just as well be assumed to be increasing, and *vice versa*. To make this evident would require a lengthier exposition than could be given here; but what is necessary for this purpose is adduced later on in connection with the inverted relation.

Section Two: Magnitude (Quantity)

The difference between quantity and quality has been stated. Quality is the first, immediate determinateness, quantity is the determinateness which has become indifferent to being, a limit which is just as much no limit, being-for-self which is absolutely identical with being-for-other—a repulsion of the many ones which is directly the non-repulsion, the continuity of them.

Because that which is for itself is now posited as not excluding its other, but rather as affirmatively continuing itself into it, it is otherness in so far as *determinate being* again appears in this continuity and its determinateness is *at the same time* no longer in a simple self-relation, no longer an immediate determinateness of the determinately existent something, but is posited as self-repelling, as in fact having the relation-to-self as a determinateness in another something (which is *for itself*); and since they are *at the same time* indifferent, relationless limits reflected into themselves, the determinateness in general is outside itself, an absolutely *self-external* determinateness and an equally external something; such a limit, the indifference of the limit within itself and of the something to the limit, constitutes the *quantitative* determinateness of the something.

In the first place, *pure quantity* is to be distinguished from itself as a *determinate* quantity, from *quantum*. As the former, it is in the first place real being-for-self which has returned into itself and which as yet contains no determinateness: a compact, infinite unity which continues itself into itself.

Secondly, this develops a determinateness which is posited in it as one which is at the same time no determinateness, as only an *external* one. It becomes *quantum*. Quantum is indifferent determinateness, that is, a self-transcending, self-negating determinateness; as this otherness of otherness it relapses into the infinite progress. But the infinite quantum is the indifferent determinateness *sublated*, it is the restoration of quality.

Thirdly, quantum in a qualitative form is quantitative *ratio*. Quantum transcends itself only generally: in ratio, however, its transition into its otherness is such that this otherness in which it has its determination is at the same time posited, is another

quantum. Thus quantum has returned into itself and in its otherness is related to itself.

At the base of this ratio there is still the externality of quantum; the quanta which are related to each other are *indifferent*, that is, they have their self-relation in such self-externality. The ratio is thus only a formal unity of quality and quantity. Its dialectic is its transition into their absolute unity, into *Measure*.

Remark: In something, its limit as quality is essentially its determinateness. If, however, by limit we mean quantitative limit, then when, for example, a field alters its limit it still remains what it was before, a field. If on the other hand its qualitative limit is altered, then since this is the determinateness which makes it a field, it becomes a meadow, wood, and so on. A red, whether brighter or paler, is still red; but if it altered its quality it would cease to be red, would become blue or some other colour. The determination of *magnitude* as quantum reached above, namely that it has a permanent substratum of being *which is indifferent to its determinateness*, can be found in any other example.

By magnitude *quantum* is meant, as in the examples cited, not quantity; and it is chiefly for this reason that this foreign term must be used.

The definition of magnitude given in mathematics likewise concerns quantum. A magnitude is usually defined as that which can be increased or diminished. But to increase means to make the magnitude more, to decrease, to make the magnitude less. In this there lies a *difference* of magnitude as such from itself and magnitude would thus be that of which the magnitude can be altered. The definition thus proves itself to be inept in so far as the same term is used in it which was to have been defined. Since the same term must not be used in the definition, the *more* and *less* can be resolved into an affirmative addition which, in accordance with the nature of quantum, is likewise external, and a subtraction, as an equally external negation. It is this *external* form both of reality and of negation which in general characterizes the nature of *alteration* in quantum. In that imperfect expression, therefore, one cannot fail to recognize the main point involved, namely the indifference of the alteration, so that the alteration's own *more* and *less*, its indifference to itself, lies in its very Notion.

CHAPTER I

QUANTITY

A. PURE QUANTITY

Quantity is sublated being-for-self; the repelling one which related itself only negatively to the excluded one, having passed over into relation to it, treats the other as identical with itself, and in doing so has lost its determination: being-for-self has passed over into attraction. The absolute brittleness of the repelling *one* has melted away into this *unity* which, however, as containing this one, is at the same time determined by the immanent repulsion, and as unity of the self-externality is unity with itself. Attraction is in this way the moment of *continuity* in quantity.

Continuity is, therefore, simple, self-same self-relation, which is not interrupted by any limit or exclusion; it is not, however, an *immediate* unity, but a unity of ones which possess being-for-self. The *asunderness of the plurality* is still contained in this unity, but at the same time as not differentiating or *interrupting* it. In continuity, the plurality is posited as it is in itself; the many are all alike, each is the same as the other and the plurality is, consequently, a simple, undifferentiated sameness. Continuity is this moment of self-sameness of the asunderness, the self-continuation of the different ones into those from which they are distinguished.

In continuity, therefore, magnitude immediately possesses the moment of *discreteness*—repulsion, as now a moment in quantity. Continuity is self-sameness, but of the Many which, however, do not become exclusive; it is repulsion which expands the self-sameness to continuity. Hence discreteness, on its side, is a coalescent discreteness, where the ones are not connected by the void, by the negative, but by their own continuity and do not interrupt this self-sameness in the many.

Quantity is the unity of these moments of continuity and discreteness, but at first it is so in the *form* of one of them, *continuity*, as a result of the dialectic of being-for-self, which has collapsed into the form of self-identical immediacy. Quantity is, as such, this simple result in so far as being-for-self has not yet developed its moments and posited them within itself. It *contains*

them to begin with as being-for-self posited as it is in truth. The determination of being-for-self was to be a self-sublating relation-to-self, a perpetual coming-out-of-itself. But what is repelled is itself; repulsion is, therefore, the creative flowing away of itself. On account of the self-sameness of what is repelled, this distinguishing or differentiation is an uninterrupted continuity; and because of the coming-out-of-itself this continuity, without being interrupted, is at the same time a plurality, which no less immediately remains in its self-identicalness.

Remark 1: *The Conception of Pure Quantity*

Pure quantity has not as yet any limit or is not as yet quantum; and even in so far as it becomes quantum it is not bounded by limit but, on the contrary, consists precisely in not being bounded by limit, in having being-for-self within it as a sublated moment. The presence in it of discreteness as a moment can be expressed by saying that quantity is simply the omnipresent *real possibility* within itself of the one, but conversely that the one is no less absolutely continuous.

In thinking that is not based on the Notion, continuity easily becomes mere *composition*, that is, an *external* relation of the ones to one another, in which the one is maintained in its absolute brittleness and exclusiveness. But it has been shown that the one essentially and spontaneously (*an und für sich selbst*) passes over into attraction, into its ideality, and that consequently continuity is not external to it but belongs to it and is grounded in its very nature. It is just this *externality* of continuity for the ones to which atomism clings and which ordinary thinking finds it difficult to forsake. Mathematics, on the other hand, rejects a metaphysics which would make time *consist* of points of time; space in general—or in the first place the line—*consist* of points of space; the plane, of lines; and total space of planes. It allows no validity to such discontinuous ones. Even though, for instance, in determining the magnitude of a plane, it represents it as the *sum* of infinitely many lines, this discreteness counts only as a momentary representation, and the sublation of the discreteness is already implied in the *infinite* plurality of the lines, since the space which they are supposed to constitute is after all bounded.

It is the notion of pure quantity as opposed to the mere image

of it that Spinoza, for whom it had especial importance, has in mind when he speaks of quantity as follows:

'Quantitas duobus modis a nobis concipitur, abstracte scilicet sive superficialiter, prout nempe ipsam imaginamur; vel ut substantia, quod a solo intellectu fit. Si itaque ad quantitatem attendimus, prout in imaginatione est, quod saepe et facilius a nobis fit, reperietur finita, divisibilis et ex partibus conflata, si autem ad ipsam, prout in intellectu est, attendimus, et eam, quatenus substantia est, concipimus, quod difficillime fit,—infinita, unica et indivisibilis reperietur. Quod omnibus, qui inter imaginationem et intellectum distinguere sciverint, satis manifestum erit.'[1]

More specific examples of pure quantity, if they are wanted, are space and time, also matter as such, light, and so forth, and the ego itself: only by quantity, as already remarked, is not to be understood quantum. Space, time and the rest, are expansions, pluralities which are a coming-out-of-self, a flowing which, however, does not pass over into its opposite, into quality or the one, but as a coming-out-of-self they are a perennial *self-production* of their unity.

Space is this absolute *self-externality* which equally is absolutely uninterrupted, a perpetual becoming-other which is self-identical; time is an absolute coming-out-of-itself, a generating of the one, (a point of time, the now) and immediately the annihilation of it, and again the continuous annihilation of this passing away; so that this spontaneous generating of non-being is equally a simple self-sameness and self-identity. As regards *matter* as quantity, among the seven surviving propositions of the first dissertation of Leibniz there is one, the second, which runs: *Non omnino improbabile est, materiam et quantitatem esse realiter idem.*[2] In fact, the distinction between these two concepts is simply this, that quantity is a determination of pure thought, whereas matter is the

[1] Eth. P. I. Prop, XV, Schol.: 'Quantity is conceived by us in two manners, to wit, abstractly and superficially, as an offspring of imagination or as a substance, which is done by the intellect alone. If, then, we look at quantity as it is in the imagination, which we often and very easily do, it will be found to be finite, divisible, and composed of parts; but if we look at it as it is in the intellect and conceive it, in so far as it is a substance, which is done with great difficulty, then as we have already sufficiently shown, it will be found to be infinite, without like, and indivisible. This, to all who know how to distinguish between the imagination and the intellect, will be quite clear.'

[2] Left-hand page of Part I of his works.

same determination in outer existence. The determination of pure quantity belongs also to the ego which is an absolute becoming-other, an infinite removal or all-round repulsion to the negative freedom of being-for-self which, however, remains utterly simple continuity—the continuity of universality or being-with-self uninterrupted by the infinitely manifold limits, by the content of sensations, intutions, and so forth. Those who reject the idea of plurality as a simple unity and besides the *Notion* of it, to wit, that each of the many is the same as every other, namely, a one of the many—since here we are not speaking of the many as further determined, as green, red, and so on, but of the many considered in and for itself—demand also a *representation* of this unity, will find plenty of instances in those continua which exhibit the deduced Notion of quantity as present in simple intutition.

Remark 2: *The Kantian Antinomy of the Indivisibility and the Infinite Divisibility of Time, Space and Matter*

It is the nature of quantity, this simple unity of discreteness and continuity, that gives rise to the conflict or antinomy of the infinite divisibility of space, time, matter, etc.

This antinomy consists solely in the fact that discreteness must be asserted just as much as continuity. The one-sided assertion of discreteness gives infinite or absolute dividedness, hence an indivisible, for principle: the one-sided assertion of continuity, on the other hand, gives infinite divisibility.

It is well known that the Kantian *Critique of Pure Reason* sets up four (cosmological) antinomies, the second of which deals with the antithesis constituted by the moments of quantity.

These Kantian antinomies will always remain an important part of the critical philosophy; they, more than anything else, brought about the downfall of previous metaphysics and can be regarded as a main transition into more recent philosophy since they, in particular, helped to produce the conviction of the nullity of the categories of finitude in regard to their *content*—which is a more correct method than the formal one of a subjective idealism, according to which their defect is supposed to be, not what they are in themselves, but only that they are subjective. But this exposition with all its merits is imperfect; its course is impeded and tangled, and also it is false in regard to its result,

which presupposes that cognition has no other forms of thought than finite categories. In both respects these antinomies deserve a more exact critical appraisal which will not only throw more light on their standpoint and method but will also free the main point at issue from the useless form into which it has been forced.

In the first place, I remark that Kant wanted to give his four cosmological antinomies a show of competeness by the principle of classification which he took from his schema of the categories. But profounder insight into the antinomial, or more truly into the dialectical nature of reason demonstrates *any* Notion whatever to be a unity of opposed moments to which, therefore, the form of antinomial assertions could be given. Becoming, determinate being, etc., and any other Notion, could thus provide its particular antinomy, and thus as many antinomies could be constructed as there are Notions. Ancient scepticism did not spare itself the pains of demonstrating this contradiction or antinomy in every notion which confronted it in the sciences.

Further, Kant did not take up the antinomy in the Notions themselves, but in the already *concrete* form of cosmological determinations. In order to possess the antinomy in its purity and to deal with it in its simple Notion, the determinations of thought must not be taken in their application to and entanglement in the general idea of the world, of space, time, matter, etc; this concrete material must be omitted from consideration of these determinations which it is powerless to influence and which must be considered purely on their own account since they alone constitute the essence and the ground of the antinomies.

Kant's conception of the antinomies is that they are 'not sophisms but contradictions which reason must necessarily *come up against*' (a Kantian expression); and this is an important view. 'Reason, when it sees into the ground of the natural illusion of the antinomies is, it is true, no longer imposed on by them but yet continues to be deceived.' The Kantian solution, namely, through the so-called transcendental ideality of the world of perception, has no other result than to make the so-called conflict into something *subjective*, in which of course it remains still the same illusion, that is, is as unresolved, as before. Its genuine solution can only be this: two opposed determinations which belong necessarily to one and the same Notion cannot be valid each on its

own in its one-sidedness; on the contrary, they are true only as sublated, only in the unity of their Notion.

The Kantian antinomies on closer inspection contain nothing more than the quite simple categorical assertion of *each* of the two opposed moments of a determination, each being taken on its own in isolation from the other. But at the same time this simple categorical, or strictly speaking assertoric statement is wrapped up in a false, twisted scaffolding of reasoning which is intended to produce a semblance of proof and to conceal and disguise the merely assertoric character of the statement, as closer consideration will show.

The relevant antinomy here concerns the so-called infinite divisibility of matter and rests on the antithesis of the moments of continuity and discreteness which are contained in the Notion of quantity.

The thesis of the same as expounded by Kant runs thus:

'Every composite substance in the world consists of simple parts, and nowhere does there exist anything but the simple or what is compounded from it.'

To the simple, the atom, there is here opposed the composite, which is a very inferior determination compared to the continuous. The substrate given to these abstractions, namely, substances in the world, here means nothing more than things as sensuously perceived and it has no influence on the antinomy itself; space or time could equally well be taken. Now since the thesis speaks only of composition instead of continuity it is really as it stands an analytical or *tautological* proposition. That the composite is not in its own self a one, but only something externally put together and *consisting of what is other than itself*, this is its immediate determination. But the other of the composite is the simple. It is therefore tautological to say that the composite consists of the simple. To ask of what something consists is to ask for an indication of something else, the compounding of which constitutes the said something. If ink is said to consist simply of ink, the meaning of the inquiry after the something else of which it consists has been missed and the question is not answered but only repeated. A further question then is whether that which is under discussion is supposed to *consist of something* or not. But the composite is simply that which is supposed to be a combination of something else. If, however, the simple which is the other of the composite is

taken only as *relatively* simple and is itself composite, too, then the question still remains unanswered. What ordinary thinking has in mind is, perhaps, only some composite or other of which something or other, too, would be assigned as *its* simple, such particular something being composite on its own account. But what is under discussion here is the *composite as such.*

Now as regards the Kantian *proof* of the thesis this, like all the Kantian proofs of the rest of the antinomial propositions, makes the detour of being apagogic, a detour which will prove to be quite superfluous.

'Assume', he begins, 'that composite substances do not consist of simple parts; then if *all* composition were thought away no composite part and (since we have just assumed that there are no simple parts) no simple part—hence nothing at all—would remain; consequently, no substance would have been given.'

This conclusion is quite correct: if nothing but composite substances exist and all that is composite is thought away, then nothing whatever remains; this will be conceded, but this tautological redundance could be omitted and the proof straightway begin with what follows, namely:

'Either it is impossible to think away all composition, or else there remains after such removal in thought something which is not composite, that is, the simple.

'In the first case, however, the composite again would not consist of substances (because with these, composition is only a contingent relation of substances[1] which, apart from such relation, must still persist on their own account). Now since this case contradicts what was assumed, only the second case is left: namely, that all composite substances consist of simple parts.'

The very reason which is the main point, and in face of which all that precedes is completely superfluous, is mentioned by the way, in a parenthesis. The dilemma is this: either the composite persists, or else the simple. If the former, that is, the composite, persists, then what persists would not be substances, for composition is for these only a contingent relation; but substances do persist, therefore, what persists is the simple.

It is clear that the apagogical detour could be omitted and the

[1] In addition to the redundance of the proof itself there is here also a redundance of language—'because *with these*' (namely the substances) 'composition is only a contingent relation of *substances*'.

thesis: 'composite substance consists of simple parts', could be directly followed by the reason: *because* composition is merely a *contingent* relation of substances, and is therefore external to them and does not concern the substances themselves. If the composition is in fact contingent then, of course, substances are essentially simple. But this contingency which is the sole point at issue is not proved but straightway assumed, and casually, too, in a parenthesis—as something self-evident or of secondary importance. True, it goes without saying that composition is a contingent and external determination; but if the point at issue were only a contingent togetherness instead of continuity, it would not be worth while constructing an antinomy about it—or rather it would not be possible to formulate one. Therefore, the assertion that the parts are simple is, as remarked, only a tautology.

The apagogical detour thus contains the very assertion which should result from it. The proof therefore can be put more concisely thus:

Let us assume that substances do not consist of simple parts but are only composite. But now all composition can be thought away (for it is only a contingent relation); after its removal, therefore, there are no substances left unless we assume that they consist of simple parts. But substances we must have for we have assumed them; *everything* is not meant to vanish, *something* must be left over, for we have presupposed something persistent which we called substance. Therefore this something must be simple.

To complete the whole, we have still to consider the conclusion which runs as follows:

'From this it follows, as a direct consequence, that all things in the world without exception are simple entities, that composition is only an external state of them, and that reason must think the elementary substances as simple entities.'

Here we see the externality, that is contingency, of composition put forward as a *consequence* after it had already been introduced parenthetically and used in the proof.

Kant strongly protests that he is not looking for sophisms in the conflicting statements of the antinomy for the purpose, as it were, of special pleading. But the defect of the proof in question is not so much that it is a sophism, as that its laboured, tortuous complexity serves no other purpose than to produce the merely outward semblance of a proof and partially to obscure the quite

transparent fact that what was supposed to emerge as a consequence is, parenthetically, that on which the proof hinges; that there is no proof at all, but only an assumption.

The antithesis runs:

'No composite thing in the world consists of simple parts and nowhere in the world does there exist anything simple.'

The proof has equally an apagogical turn and, in a different way, is just as faulty as the previous one.

'Suppose', it runs, 'that a composite thing as a substance consists of simple parts. Because all *external* relation, and consequently all composition of substances, is possible only in *space*, therefore, the space occupied by the composite substance must consist of as many parts as those of which the composite substance consists. Now space does not consist of simple parts but of spaces. Therefore each part of the composite substance must occupy a space.'

'But the absolutely primary parts of everything composite are simple.'

'Therefore the simple occupies a space.'

'Now since every real thing that occupies a space comprises a manifold of mutually external parts and is consequently composite, consisting of substances, it would follow that the simple is a composite substance—which is self-contradictory.'

This proof can be called a whole *nest* (to use an expression elsewhere employed by Kant) of faulty procedure.

In the first place, its apagogical form is a groundless illusion. For the assumption that whatever is substantial is spatial, but that space does not consist of simple parts is a direct assertion which is made the immediate ground of what is to be proved, and with this there is an end to the proving of the antithesis.

Next, this apagogical proof begins with the proposition: 'that all composition from substances is an *external* relation', but oddly enough immediately forgets it. For it then goes on to conclude that composition is possible only in *space*, that space does not consist of simple parts, and that therefore the real thing occupying a space is composite. But once composition is assumed as an external relation, then spatiality itself (in which alone composition is supposed to be possible) is for that very reason an external relation for the substances, which does not concern them or affect their nature any more than anything else does

that can be inferred from the determination of spatiality. For this very reason, the substances ought not to have been put into space.

Further, it is assumed that the space in which the substances here are placed does not consist of simple parts, because it is an intuition, that is, according to the Kantian definition, a representation which can only be given through a single object, and is not a so-called discursive concept. This Kantian distinction between intuition and concept has, as everyone knows, given rise to a deal of nonsense about the former, and to avoid the labour of comprehension the value and sphere of intuition have been extended to the whole field of cognition. What is pertinent here is just this, that space, and also intuition itself, must be grasped in terms of their *Notions* if, that is, we want really to *comprehend.* And thus the question would arise whether space, even though a simple continuity for intuition, ought not to be grasped, in accordance with its Notion, as consisting of simple parts, or whether it would be involved in the same antinomy which applied only to substance. As a matter of fact, if the antinomy is grasped abstractly, it concerns, as we remarked, quantity *as such*, and hence equally space and time.

But it is assumed in the proof that space does not consist of simple parts; this therefore ought to have been the reason for not placing the simple in this element which is incompatible with the nature of the simple. There is also involved here a clash between the continuity of space and composition; the two are confused with each other, the former being substituted for the latter (which results in a *quaternio terminorum* in the conclusion). With Kant, space has the express determination of being 'sole and single, its parts resting only on limitations, so that they do not precede the one, all-embracing space as, so to speak, its component parts from which it could be compounded'.[1] Here continuity is quite correctly and definitely predicated of space in denial of its composition from parts. On the other hand, in the argument the placing of substances in space is supposed to involve 'a manifold of mutually external parts' and, more particularly, 'consequently a composite'. Yet, as we have quoted, the way in which manifoldness is present in space is expressly intended to exclude composition and component parts antecedent to the unity of space.

[1] *Critique of Pure Reason*, 2nd edition, p. 39.

In the remark to the proof of the antithesis we are also expressly reminded of the other fundamental conception of the critical philosophy, namely, that we have a *notion* of bodies only as appearances or phenomena; as such, however, they necessarily presuppose space as the condition of the possibility of all outer appearance. If by substances we are meant here to understand only bodies as we see, touch, taste them, and so on, then we are not really discussing them as they are in their Notion but only as sensuously perceived. The proof of the antithesis, then, amounted in short to this: all our visual, tactile, and other experience shows us only what is composite; even the best microscopes and the keenest knives have not enabled us to *come across* anything simple. Then neither should reason expect to come across anything simple.

When we look more closely into the opposition of this thesis and antithesis, freeing their proofs from all pointless redundancy and tortuousness, we see that the proof of the antithesis dogmatically assumes *continuity* (by placing substances in space) and also that the proof of the thesis, by assuming that composition is the mode of relation of substances, dogmatically assumes the *contingency of this relation*, and hence assumes that substances are *absolute ones*. Thus the whole antinomy reduces to the separation of the two moments of quantity and the direct assertion of them as absolutely separate. When substance, matter, space, time, etc., are taken only as *discrete*, they are absolutely divided; their principle is the one. When they are taken as *continuous*, this one is only a sublated one; division remains a divisibility, it remains the *possibility* of division, as a possibility, without actually reaching the atom. Now even if we stop at the determination given in what has been said about these antitheses, then the moment of the atom is contained in continuity itself, for this is simply the possibility of division; just as said dividedness, discreteness, sublates all distinction of the ones—for each of the simple ones is what the other is—consequently, also contains their sameness and hence their continuity. Since each of the two opposed sides contains its other within itself and neither can be thought without the other, it follows that neither of these determinations, taken alone, has truth; this belongs only to their unity. This is the true dialectical consideration of them and also the true result.

Infinitely more ingenious and profound than this Kantian

antinomy are the dialectical examples of the ancient Eleatic school, especially those concerning motion, which likewise are based on the Notion of quantity and in it find their solution. To consider them here, too, would be too lengthy a business; they concern the Notions of space and time and can be dealt with at the same time as these subjects and in the history of philosophy. They reflect the greatest credit on the intelligence of their inventors; they have for *result* the pure being of Parmenides, in that in them is demonstrated the dissolution of all determinate being; they are thus in themselves the *flux* of Heraclitus. For this reason they deserve a more thorough consideration than the usual explanation that they are just sophisms; which assertion sticks to empirical perception, following the procedure of Diogenes (a procedure which is so illuminating to ordinary common sense) who, when a dialectician pointed out the contradiction contained in motion, made no effort to reason it out but, by silently walking up and down, is supposed to have referred to the evidence of sight for an answer. Such assertion and refutation is certainly easier to make than to engage in thinking and to hold fast and resolve by thought alone the complexities originating in thought, and not in abstruse thought either, but in the thoughts spontaneously arising in ordinary consciousness.

The solutions propounded by Aristotle of these dialectical forms merit high praise, and are contained in his genuinely speculative Notions of space, time and motion. To infinite divisibility (which, being imagined as actually carried out, is the same as infinite dividedness, as the atoms) on which is based the most famous of those proofs, he opposes continuity, which applies equally well to time as to space, so that the infinite, that is, *abstract* plurality is contained only *in principle* [*an sich*], as a *possibility*, in continuity. What is actual in contrast to abstract plurality as also to abstract continuity, is their concrete forms, space and time themselves, just as these latter are abstract relatively to matter and motion. What is *abstract* has only an implicit or potential being; it only *is* as a moment of something real. Bayle, who finds Aristotle's solution of the Zenonic dialectic '*pitoyable*',[1] does not understand the meaning of the statement that matter is only *potentially* infinitely divisible; he rejoins that if matter is infinitely divisible, then it *actually* contains an infinite

[1] P. Bayle: *Dictionnaire*, Article 'Zenon'.

number of parts, that, therefore, this infinite is not an infinite *en puissance* but an infinite that really and actually exists. On the contrary, divisibility itself even is only a possibility, not an *existing* of the parts, and the plurality as such is posited in the continuity only as a moment, as sublated. Acute understanding, in which Aristotle, too, is certainly unsurpassed, is not competent to grasp and to decide on speculative Notions, any more than the crudity of sensuous conception instanced above is adequate to refute the reasoning of Zeno. Such intellect commits the error of holding such mental fictions, such abstractions, as an infinite number of parts, to be something true and actual; but this sensuous consciousness does not let itself be brought beyond the empirical element to thought.

The Kantian solution of the antinomy likewise consists solely in the supposition that reason should not soar beyond sensuous perception and should take the world of appearance, the phenomenal world, as it is. This solution leaves the content of the antinomy itself on one side; it does not attain to the nature of the *Notion* of its determinations, each of which, isolated on its own, is null and is in its own self only the transition into its other, the unity of both being *quantity* in which they have their truth.

B. CONTINUOUS AND DISCRETE MAGNITUDE

1. Quantity contains the two moments of continuity and discreteness. It is to be posited in both of them as determinations of itself. It is already their *immediate* unity, that is, quantity is posited at first only in one of its determinations, continuity, and as such is continuous magnitude.

Or we may say that continuity is indeed one of the moments of quantity which requires the other moment, discreteness, to complete it. But quantity is a concrete unity only in so far as it is the unity of *distinct* moments. These, are, therefore, also to be taken as distinct, but are not to be resolved again into attraction and repulsion, but are to be taken as they are in their truth, each remaining in its unity with the other, that is, remaining the *whole*. Continuity is only coherent, compact unity as unity of the discrete; *posited* as such it is no longer only a moment but the whole of quantity, *continuous magnitude*.

2. *Immediate* quantity is continuous magnitude. But quantity is not an immediate at all; immediacy is a determinateness the sublatedness of which is quantity itself. It is, therefore, to be posited in the determinateness immanent in it, and this is the one. Quantity is *discrete magnitude.*

Discreteness is, like continuity, a moment of quantity but it is itself also the whole of quantity just because it is a moment in it, in the whole, and therefore as a distinct moment it does not stand outside the whole, outside its unity with the other moment. Quantity is in itself asunderness, and continuous magnitude is this asunderness continuing itself without negation as an internally self-same connectedness. But discrete magnitude is this asunderness as discontinuous, as interrupted. With this plurality of ones, however, we are not again in the presence of the plurality of atoms and the void, repulsion in general. Because discrete magnitude is quantity, its discreteness is itself continuous. This continuity in the discrete consists in the ones being the same as one another, or in having the same *unity*. Discrete magnitude is, therefore, the asunderness of the manifold one as self-same, not the manifold one in general but posited as the *many of a unity*.

Remark: The Usual Separation of These Magnitudes

In the usual ideas of continuous and discrete magnitude, it is overlooked that *each* of these magnitudes contains both moments, continuity and discreteness, and that the distinction between them consists only in this, that in one of the moments the determinateness is *posited* and in the other it is only *implicit*. Space, time, matter, and so forth are continuous magnitudes in that they are repulsions from themselves, a streaming forth out of themselves which at the same time is not their transition or relating of themselves to a qualitative other. They possess the absolute possibility that the one may be posited in them at any point—not the empty possibility of a mere otherness (as when it is said, it is possible that a tree might stand in the place of this stone), but they contain the principle of the one within themselves; it is one of the determinations which constitute them.

Conversely, in discrete magnitude continuity is not to be overlooked; this moment is, as has been shown, the one as unity.

Continuous and discrete magnitude can be regarded as *species* of quantity, provided that magnitude is posited, not under any

external determinateness, but under the determinatenesses of its own moments; the ordinary transition from genus to species allows *external* characteristics to be attributed to the former according to some *external* basis of classification. And besides, continuous and discrete magnitude are not yet quanta; they are only quantity itself in each of its two forms. They are, perhaps, called magnitudes in so far as they have in common with quantum simply this—to be a determinateness in quantity.

C. LIMITATION OF QUANTITY

Discrete magnitude has first the one for its principle; secondly, it is a plurality of ones; and thirdly, it is essentially continuous; it is the one as at the same time sublated, as *unity*, the continuation of itself as such in the discreteness of the ones. Consequently, it is posited as *one* magnitude, the determinateness of which is the one which, in this posited and determinate being is the *excluding* one, a limit in the unity. Discrete magnitude as such is immediately not limited; but as distinguished from continuous magnitude it is a determinate being, a something, with the one as its determinateness and also as its first negation and limit.

This limit, which is related to the unity and is the negation *in it*, is also, as the one, self-related; it is thus the enclosing, encompassing limit. Limit here is not at first distinguished from its determinate being as something, but, as the one, is immediately this negative point itself. But the being which here is limited is essentially a continuity, by virtue of which it passes beyond the limit, beyond this one, to which it is indifferent. Real discrete quantity is thus *a* quantity, or quantum—quantity as a determinate being and a something.

Since the one which is a limit includes within itself the many ones of discrete quantity, it equally posits them as sublated within it; and because it is a limit of continuity simply as such, the distinction between continuous and discrete magnitude is here of no significance; or, more correctly, it is a limit to the continuity of the one as much as of the other; *both* undergo transition into quanta.

CHAPTER 2

QUANTUM

Quantum, which to begin with is quantity with a determinateness or limit in general is, in its complete determinateness, number. Quantum differentiates itself *secondly*, into (*a*) *extensive* quantum, in which the limit is a limitation of the determinately existent plurality; and (*b*) intensive quantum or *degree*, the determinate being having made the transition into being-for-self. Intensive quantum as both *for itself* and at the same time immediately *outside itself*—since it is an indifferent limit—has its determinateness in an other. As this manifest contradiction of being determined simply within itself yet having its determinateness outside it, pointing outside itself for it, quantum posited as being in its own self external to itself, passes over *thirdly*, into *quantitative infinity*.

A. NUMBER

Quantity is quantum, or has a limit, both as continuous and as discrete magnitude. The difference between these two kinds has here, in the first instance, no immediate significance.

The very nature of quantity as sublated being-for-self is *ipso facto* to be indifferent to its limit. But equally, too, quantity is not unaffected by the limit or by being a quantum; for it contains within itself as its own moment the one, which is absolutely determined and which, therefore, as posited in the continuity or unity of quantity, is its limit, but a limit which remains what it has become, simply a one.

This one is thus the principle of quantum, but as the one of *quantity*. Hence, first, it is continuous, it is a *unity*; secondly, it is discrete, a plurality of ones, which is implicit in continuous, or explicit in discrete magnitude, the ones having equality with one another, possessing the said continuity, the same unity. Thirdly, this one is also a negation of the many ones as a simple limit, an excluding of its otherness from itself, a determination of itself in opposition to *other* quanta. Thus the one is (α) *self-relating*, (β) *enclosing* and (γ) *other-excluding* limit.

Quantum completely posited in these determinations is ***number***. The complete positedness lies in the existence of the limit as a *plurality* and so in its distinction from the unity. Consequently, number appears as a discrete magnitude, but in the unity it equally possesses continuity. It is, therefore, also quantum in its complete *determinateness*, for in it the limit is present as a specific *plurality* which has for its principle the one, the absolutely determinate. Continuity, in which the one is present only *in principle*, as a sublated moment—posited as a unity—is the form of indeterminateness.

Quantum, merely as such, is limited generally; its limit is an abstract, simple determinateness of it. But in quantum as number, this limit is posited as *manifold within itself*. It contains the many ones which constitute its determinate being, but does not contain them in an indeterminate manner, for the determinateness of the limit falls in them; the limit excludes other determinate being, that is, other pluralities and the ones it encloses are a specific aggregate, the *amount*—which is the form taken by discreteness in number—the other to which is the *unit*, the continuity of the amount. *Amount* and *unit* constitute the *moments* of number.

As regards amount, we must see more closely how the many ones of which it *consists* are present in the limit; it is correct to say of amount that it *consists* of the many, for the ones are in it not as sublated but as *affirmatively present*, only posited with the excluding limit to which they are indifferent. This, however, is not indifferent to them. In the sphere of determinate being, the relation of the limit to it was primarily such that the determinate being persisted as the affirmative on this side of its limit, while the limit, the negation, was found outside on the border of the determinate being; similarly, the breaking-off [in the counting] of the many ones and the exclusion of other ones appears as a determination falling outside the enclosed ones. But in the qualitative sphere it was found that the limit pervades the determinate being, is co-extensive with it, and consequently that it lies in the nature of something to be limited, that is, finite. In the quantitative sphere a number, say a hundred, is conceived in such a manner that the hundredth one alone limits the many to make them a hundred. In one sense this is correct; but on the other hand none of the hundred ones has precedence over any other for they are only equal—each is equally the hundredth;

thus they all belong to the limit which makes the number a hundred and the number cannot dispense with any of them for its determinateness. Hence, relatively to the hundredth one, the others do not constitute a determinate being that is in any way different from the limit, whether they are outside or inside it. Consequently, the number is not a plurality over against the enclosing, limiting one, but itself constitutes this limitation which is a specific quantum; the many constitute a number, a two, a ten, a hundred, and so on.

Now the limiting one is the number as determined relatively to other numbers, as distinguished from them. But this distinguishing does not become a qualitative determinateness but remains quantitative, falling only within the comparing *external* reflection; the number, as a one, remains returned into itself and indifferent to others. This *indifference* of a number to others is an essential determination of it and constitutes the implicit determinedness of the number, but also the number's own externality. Number is thus a *numerical* one as the absolutely determinate one, which at the same time has the form of simple immediacy and for which, therefore, the relation to other is completely external. Further, one as a *number* possesses *determinateness* (in so far as this is a relation to other) as the moments of itself contained within it, in its difference of unit and amount; and amount is itself a plurality of ones, that is, this absolute externality is in the one itself. This contradiction of number or of quantum as such within itself is the quality of quantum, in the further determinations of which this contradiction is developed.

Remark 1: *The Species of Calculation in Arithmetic; Kant's Synthetic Propositions* a priori *of Intuition*

Spatial magnitude and numerical magnitude are usually regarded as two species, the former being on its own account a determinate magnitude just as much as the latter; their difference is held to consist only in the different determinations of continuity and discreteness, but as quantum they stand on the same level. In spatial magnitude, geometry has, in general, continuous magnitude for its subject matter while the subject matter of arithmetic is the discrete magnitude of number. But with this dissimilarity of their subject matter, the manner and completeness of their limitation or determinedness is also different. Spatial magnitude

possesses only limitation generally; if it is to be considered as a thoroughly determinate quantum then *number* is required. Geometry as such does not *measure* spatial figures (it is not mensuration), but only *compares* them. In its definitions, too, the determinations are in part derived from the *equality* of the sides and angles, or from *equidistance*. Thus the circle, because it is based solely on the *equidistance* of all possible points in it from a centre, does not require number for its determination. These determinations based on equality or inequality are genuinely geometrical. But they are not sufficient, and for other figures, for example, the triangle or rectangle, number is requisite; this in its principle, the one, contains a *self*-determinedness, it is determined without the aid of an other and therefore not through comparison. It is true that in the point, spatial magnitude has a determinateness corresponding to the one; but the point, in becoming external to itself, becomes an other, becomes the line; because it is essentially only a *spatial* one, it becomes in the *relation* a continuity in which the nature of the point, the self-determinedness, the one, is sublated. In so far as the self-determinedness is supposed to be preserved in the self-externality, the line must be represented as an aggregate of ones, and to the *limit* must be imparted the determination of the *many* ones, that is, the magnitude of the line—and similarly of other spatial determinations—must be taken as a number.

Arithmetic considers number and its figures; or rather does not consider them but operates with them. For number is the determinateness which is indifferent, inert; it must be actuated from without and so brought into a relation. The modes of relation are the species of calculation. In arithmetic they are presented *seriatim* and it is clear that one depends on the other; but the thread which links the progressive stages is not made prominent in arithmetic. However, the systematic arrangement justly claimed for the presentation of these elements in the text-books is readily provided by the determinations of number itself stemming from its Notion. These cardinal determinations will be briefly noted here.

Number has for its principle the one and is, therefore, simply an aggregate externally put together, a purely analytic figure devoid of any inner connectedness; and because it is produced in this merely external manner all calculation is the production of

numbers, a *counting* or, more specifically, a *counting up*. Any diversity in this external production, which always proceeds to the same end, can lie only in a difference between the numbers which are to be counted up; this difference must itself come from elsewhere and from an external determination.

The qualitative difference which constitutes the determinateness of number is, we have seen, that of unit and amount; consequently, every determinateness of the notion of number which can occur in the species of calculation can be reduced to this difference. But the difference which belongs to numbers as quanta, is external identity and external difference, *equality* and *inequality*, which are moments of *reflection* and fall to be considered under the determinations of essence when we come to deal with *difference*.

Further, we must premise that numbers can, in general, be produced in two ways, either by aggregation, or by separation of an aggregate already given; in both cases the same specific kind of counting is employed, so that to an aggregating of numbers there corresponds what may be called a *positive* species of calculation, and to a separating of them, a *negative* species; the determination of the species of calculation itself is independent of this antithesis.

1. After these remarks we proceed to indicate the species of calculation. The *first* production of number is the aggregating of the many as such, each of which then is posited as only a one—numbering or counting. Since the ones are mutually external their representation is illustrated sensuously, and the operation by which number is generated is a process of counting on the fingers, dots, and so on. What four, five, etc., *is*, can only be *pointed out*. Seeing that the limit is an external one, the breaking off of the counting, the amount to be aggregated, is contingent, arbitrary. The difference of amount and unit which appears in the progress from one species of calculation to another, establishes a *system* of numbers, dyadic, decadic, and so on; such a system rests on the whole on the arbitrary choice of an amount which is consistently to be taken as unit.

The number produced by counting are in turn themselves counted; and as thus immediately posited they are determined as still lacking any connection with one another, as indifferent to equality or inequality, and their magnitude relatively to one another is contingent—they are, therefore, simply *unequal*; this

is addition. We learn that $7 + 5 = 12$ by adding (on our fingers or in some other way) five more ones on to the seven; the result is then memorized, learnt by rote, for the procedure has no inner meaning. Similarly, we learn that $7 \times 5 = 35$ by counting on the fingers, etc., to one seven, adding on another, and repeating this five times, the result being likewise memorized. The trouble of this counting, of finding the sums or products, is eliminated by the completed addition and multiplication tables, which have only to be learnt by heart.

Kant considers the proposition: $7 + 5 = 12$ to be a synthetic proposition. 'We might indeed', he says, 'at first think (of course!) that it is a merely analytic proposition obtained by the law of contradiction from the concept of a sum of seven and five.' The concept of the sum means nothing more than the abstract determination that these two numbers are *meant* to be aggregated and, as numbers, in an external, that is, mechanical (*begrifflose*) fashion —that we are to count on from seven until the ones to be added on (their amount is fixed at five) have been exhausted; the result bears the otherwise familiar name of twelve. 'But', continues Kant, 'if we look more closely we find that the concept of the sum of 7 and 5 contains nothing save the *union* of the two numbers into one, and in all this there is no *thinking* at all about *which* is the single number which combines both' . . . 'I may analyse my concept of such a possible sum as much as I please, still, I shall not find the twelve in it.'[1] With the *thinking* of the sum, with the analysis of the concept, the transition from the problem to the result has, of course, nothing whatever to do; and he adds, 'we must go outside these *concepts* and have recourse to intuition, to our five fingers and so on, thus adding on to the *concept* of seven the five units given in *intuition*. Five is, of course, given in intuition, that is, it is a wholly *external* aggregation of the arbitrarily repeated thought of one; but seven equally is not a concept; there are no concepts to go outside of or beyond. The sum of 5 and 7 means the mechanical [*begrifflose*] conjunction of the two numbers, and the counting from seven onwards thus mechanically continued until the five units are exhausted can be called a putting together, a synthesis, just like counting from one onwards; but it is a synthesis wholly analytical in nature, for the connection is quite artificial, there is nothing in it or put into it which is not

[1] Introduction to the *Critique of Pure Reason*, V.

quite externally given. The postulate that 5 be added to 7, bears the same relation to the postulate of simply counting, as does the postulate that a straight line be produced to the postulate that a straight line be drawn.

Just as meaningless as the expression 'synthesis', is its characterization as occurring *a priori*. Counting is not, of course, determined by sensation which, according to Kant's definition of intuition is all that remains over for the *a posteriori*, and counting is certainly an activity based on abstract intuiting, that is, an intuiting determined by the category of the one, and in which abstraction is made from all other determinations of sensation, no less than from concepts, too. The *a priori* is altogether too vague a characterization; feeling, determined as impulse, sense, and so on, has in it the *a priori* moment, just as much as space and time, in the shape of spatial and temporal existence, is determined *a posteriori*.

We may add in this connection that Kant's assertion of the synthetic nature of the foundations of pure geometry is equally without any solid basis. He admits that several are really analytic but only adduces one in support of his assertion that they are synthetic, namely, the axiom that the straight line is the shortest line between two points. 'For my notion of straightness contains nothing pertaining to magnitude, but only a quality; the notion of the shortest is, therefore, wholly an addition and cannot be inferred from any analysis of the notion of a straight line; we must therefore have recourse to intuition here which alone makes the synthesis possible.' But here again the question is not of a notion of straightness as such but of a straight line, and this is already something spatial and intuited. The determination (or, if you like, the concept) of the straight line is, after all, none other than that the line is absolutely simple, that is, in coming out of itself (the so-called movement of the point) the line is purely self-related, and its extension does not involve any alteration in its determination, or reference to another point or line outside itself; it is a simple direction purely internal to the line. This simplicity is indeed its quality, and should it seem difficult to define the straight line analytically this would be due solely to the simplicity and self-relation of the determination, and merely because reflection thinks of determining primarily in terms of a plurality, a determining through something else. But there is no

inherent difficulty whatever in grasping this determination of the simplicity of extension within the line, of the absence of determination by anything else; Euclid's definition contains nothing else than this simplicity. But now the transition of this quality to the quantitative determination (of the shortest) which is supposed to constitute the synthetic element is wholly analytical. As spatial, the line is quantity in general; the simplest in terms of quantum is the *least;* and this predicated of a line is the *shortest.* Geometry can accept these determinations as a corollary to the definition; but Archimedes in his books on the sphere and cylinder[1] did the appropriate thing in making the said determination of a straight line into an axiom, in just as correct a sense as Euclid included the determination concerning parallel lines among the axioms, for the development of this determination into a definition would have required determinations not immediately spatial in character but of a more abstract qualitative kind, like simplicity, sameness of direction, and the like just mentioned. These ancients gave even to their sciences a plastic character, confining their exposition strictly to the peculiarity of their subject matter and therefore excluding what would have been heterogeneous to it.

Kant's notion of synthetic *a priori* judgements—the notion of something differentiated which equally is inseparable, of an identity which is in its own self an inseparable difference, belongs to what is great and imperishable in his philosophy. Of course, this notion is also present in intuition since it is the Notion itself and everything is implicitly the Notion; but the determinations selected in those examples do not exhibit it. On the contrary, number and counting is an identity and the creating of an identity which is wholly and solely external, only a superficial synthesis, a unity of ones which are, in fact, posited as inherently not identical with one another but as external, each separate on its own account; the determination of the straight line as being the shortest between two points is based rather on the moment of a merely abstract identity possessing no difference within itself.

I return from this digression to addition itself. The negative species of calculation corresponding to it, subtraction, is similarly the wholly analytical separation into numbers which, as in

[1] See Hauber's translation, p. 4.

addition, are determined only as unequal generally relatively to one another.

2. The next determination is the *equality* of the numbers which are to be counted. Through this equality they are a *unit*, and number thus acquires the difference of *unit* and *amount*. Multiplication is the task of counting up an amount of units each of which is itself an amount. Here it is immaterial which of the two numbers is called unit and which amount, whether we say four times three, where four is the amount and three is the unit, or conversely, three times four. We have already indicated above that the original finding of the product is effected by simple counting, that is, counting off on the fingers, etc.; the subsequent ability to state the product immediately rests on the collection of these products, on the multiplication table and on knowing it by heart.

Division is the negative species of calculation with the same determination of the difference. Equally it is immaterial which of the two factors, the divisor or quotient, is taken as unit or amount. The divisor is taken as unit and the quotient as amount when the problem is to find out *how often* (amount) *a* number (unit) is contained in a given number; conversely, the divisor is taken as amount and the quotient as unit, when the problem is to divide a number into a given amount of equal parts and to find the magnitude of such part (of the unit).

3. The two numbers which are determined as being related to each other as unit and amount are, as number, still immediate with respect to each other and therefore simply *unequal*. The further equality is that of unit and amount themselves; with this, the advance to equality of the determinations immanent in the determination of number is completed. On the basis of this complete equality, counting is the raising to a power (the negative version is extraction of the root)—and in the first instance the *squaring* of a number—the complete immanent determinedness of counting where (1) the several numbers to be added are the same, and (2) their plurality or amount is itself the same as the number which is posited a plurality of times, which is unit. There are no other determinations in the Notion of number which could give rise to a difference; nor can there be any further equalizing of the difference immanent in number. Where a number is raised to a higher power than the square the continuation is *formal*:

partly, with even exponents, there is only a repetition of squaring, and partly, with uneven powers, inequality again enters; to take the nearest example of the cube, although formally there is an equality of the new factor with both the amount and the unit, this factor is, as unit, unequal to the amount (the square, 3, against 3 times 3); this is still more evident in the cube of 4 where the amount, 3, which indicates the number of times the unit is to be multiplied by itself, is different from the number itself standing for the unit. We have here in principle those determinations of amount and unit which, as the essential difference of the Notion, have to be equalized before number as a going-out-of-itself has completely returned into itself. The foregoing exposition also contains the reason why first, the solution of higher equations must consist on their reduction to quadratics, and secondly, why equations with odd exponents can only be formally determined and, just when the roots *are* rational they cannot be found otherwise than by an imaginary expression, that is, by the opposite of that which the roots are and express. From what has been said, it is clear that the arithmetical square alone contains an immanent absolute determinedness; for which reason equations with further formal powers must be reduced to it, just as in geometry the right-angled triangle contains an immanent, absolute determinedness which is expounded in Pythagoras' theorem, for which reason, too, all other geometrical figures must be reduced to it for their complete determination.

A graded instruction based on a logically formed division of the subject treats of powers before it treats of proportions; it is true that the latter are connected with the difference of unit and amount which constitutes the determination of the second species of calculation, but they go beyond the one of the *immediate* quantum in which unit and amount are only moments; the further determination in the sphere of the immediate quantum still remains external to the quantum itself. In ratio, number is no longer an *immediate* quantum; it then possesses its determinateness as a mediation. The quantitative[1] relation will be considered in what follows.

It cannot be said that the progressive determination of the species of calculation here given is a philosophy of them or that it exhibits, possibly, their inner significance; and for this reason,

[1] Reading *quantitative* for *qualitative*.

that it is not in fact an immanent development of the Notion. However, philosophy must be able to distinguish what is an intrinsically self-external material; the progressive determining of it by the Notion can then take place only in an external manner, and its moments, too, can be only in the form peculiar to their externality, as here, equality and inequality. It is an essential requirement when philosophizing about real objects to distinguish those spheres to which a specific form of the Notion belongs, that is, spheres in which the Notion has an actual existence; otherwise the peculiar nature of a subject matter which is external and contingent will be distorted by Ideas, and similarly these Ideas will be distorted and made into something merely formal. But here, the externality in which the moments of the Notion appear in this external material, number, is the appropriate form; since these moments exhibit the subject matter in its fixed differences, and since, too, they contain no demand for speculative thinking and therefore appear easy, they deserve to be employed in text books of the elements.

Remark 2: The Employment of Numerical Distinctions for Expressing Philosophical Notions

As we know, Pythagoras represented rational relationships (or philosophemata) by numbers; and more recently, too, numbers and forms of their relations, such as powers and so on, have been employed in philosophy for the purpose of regulating thoughts or expressing them. From an educational point of view, number has been regarded as the most suitable object of inner intuition and arithmetical operations with number have been held to be the mental activity in which mind brings to view its most characteristic relationships and in general, the fundamental relationships of essence. How far number can claim this high worth is evident from its Notion as now before us.

We saw that number is the absolute determinateness of quantity, and its element is the difference which has become indifferent—an implicit determinateness which at the same time is posited as wholly external. Arithmetic is an analytical science because all the combinations and differences which occur in its subject matter are not intrinsic to it but are effected on it in a wholly external manner. It does not have a concrete subject matter possessing inner, intrinsic, relationships which, as at first con-

cealed, as not given in our immediate acquaintance with them, have first to be elicited by the efforts of cognition. Not only does it not contain the Notion and therefore no problem for speculative thought, but it is the antithesis of the Notion. Because of the indifference of the factors in combination to the combination itself in which there is no necessity, thought is engaged here in an activity which is at the same time the extreme externalization of itself, an activity in which it is forced to move in a realm of thoughtlessness and to combine elements which are incapable of any necessary relationships. The subject matter is the abstract thought of *externality* itself.

As this *thought* of externality, number is at the same time the abstraction of the manifoldness of sense, of which it has retained nothing but the abstract determination of externality itself. In number, therefore, sense is brought closest to thought: number is the *pure thought* of thought's own externalization.

The mind which rises above the world of the senses and contemplates its own essence, when it seeks an element for its pure *representation*, for the *expression of its essence*, may therefore happen on *number*, this inner, abstract externality, before it grasps thought itself as this element and wins the purely spiritual expression for the representation of its essence. This is why we see number used for the expression of philosophemata early on in the history of philosophy. It forms the latest stage in that imperfection which contemplates the universal admixed with sense. The ancients were clearly aware that number stands midway between sense and thought. Aristotle quotes Plato as saying that the mathematical determinations of things stand apart from and midway between the world of the senses and the Ideas; they are distinguished from the former by being invisible (eternal) and unmoved, and from the latter by being a many and a like, where as the Idea is purely self-identical and within itself a one.[1] A more detailed and profound reflection on this subject by Moderatus of Cadiz is quoted in *Malchi Vita Pythagorae*. That the Pythagoreans hit on numbers he ascribes to their inability to apprehend *clearly in reason* fundamental ideas and first principles, because these are hard to think and hard to express; numbers serve well as designations in instruction; in this among other things the Pythagoreans imitated the geometers who cannot

[1] *Metaphys.* I.5.

express what is corporeal in thoughts and therefore use figures, saying 'this is a triangle', by which they do not mean that the visible drawing is to be taken for the triangle but only that it is a representation of the thought of it.[1] Thus the Pythagoreans expressed the thought of unity, of self-sameness and equality and the ground of agreement, of connection and the sustaining of everything, of the self-identical, as a *one*. It is superfluous to remark that the Pythagoreans passed on from numbers to thought as a medium of expression, to the express categories of like and unlike, of limit and infinity; even in respect of these numerical expressions it is reported that the Pythagoreans distinguished between the Monas and the one;[2] the Monas they took as the thought, but the one as the number, and similarly, they took two for the arithmetical term and the Dyas (for this is what it seems to mean there) for the thought of the indeterminate. These ancients at the outset perceived quite correctly the inadequacy of number forms for thought determinations and equally correctly they further demanded in place of this substitute for thoughts the characteristic expression; how much more advanced they were in their thinking than those who nowadays consider it praiseworthy, indeed profound, to revert to the puerile incapacity which again puts in the place of thought determinations numbers themselves and number-forms like powers, the infinitely great, the infinitely small, one divided by infinity, and other such determinations, which are themselves often only a perverted mathematical formalism.

In connection with the expression quoted above, that number stands between *sense* and thought, since, like the former, it is in its own self a many and an asunderness, it must be observed that the many itself is sense taken up into thought, is the category of what is in its own self external and so proper to sense. When richer, concrete, veritable *thoughts*, when what is most alive and most active, what is comprehended only in its concrete relationships, when such are transposed into this element of pure self-externality, they become dead, inert determinations. The richer in determinateness and, therefore, in relationships thoughts become, the more confused and also the more arbitrary and meaningless becomes their representation in such forms as

[1] ed. Ritterhus, pp. 30 *sq.*

[2] *Ibid.*, in the Remark to p. 31, l.s., from a life of Pythagoras in *Photius*, p. 722.

numbers. The one, the two, the three, the four, Henas or Monas, Dyas, Trias, Tetractys, have still some resemblance to the wholly *simple abstract* Notions; but when numbers are supposed to represent *concrete* relationships, it is vain to try to retain such resemblance.

But thought is set its hardest task when the determinations for the movement of the Notion through which alone it is the Notion, are denoted by one, two, three, four; for it is then moving in its opposite element, one which is devoid of all relation; it is engaged on a labour of derangement. The difficulty in comprehending that, for example, one is three and three is one, stems from the fact that one is devoid of all relation and therefore does not in its own self exhibit the determination through which it passes into its opposite; on the contrary, one is essentially a sheer excluding and rejection of such a relation. Conversely, understanding makes use of this to combat speculative truth (as, for example, against the truth laid down in the doctrine called the trinity) and it *counts* the determinations of it which constitute one unity, in order to expose them as sheer absurdity—that is, understanding itself commits the same absurdity of making that which is pure relation into something devoid of all relation. When the trinity was so named it was not reckoned that understanding would consider the one and number to be the *essential* determinateness of its content. This name expresses contempt for the understanding, which has nevertheless *confirmed itself in its conceit* of clinging to the one and number as such, and has set it up against reason.

To take numbers and geometrical figures (as the circle triangle etc., have often been taken), simply as *symbols* (the circle, for example, as a symbol of eternity, the triangle, of the trinity), is so far harmless enough; but, on the other hand, it is foolish to fancy that in this way more is expressed than can be grasped and expressed by thought. Whatever profound wisdom may be supposed to lie in such meagre symbols or in those richer products of fantasy in the mythology of peoples and in poetry generally, it is properly for thought alone to make explicit for consciousness the wisdom that lies only *in* them; and not only *in* symbols but *in* nature and *in* mind. In symbols the truth is dimmed and veiled by the sensuous element; only in the form of thought is it fully revealed to consciousness: the *meaning* is only the thought itself.

But the perversity of employing mathematical categories for the determination of what belongs to the method or content of the science of philosophy is shown chiefly by the fact that, in so far as mathematical forms signify thoughts and distinctions based on the Notion, this their meaning has indeed first to be indicated, determined and justified in philosophy. In the concrete philosophical sciences philosophy must take the logical element from logic, not from mathematics; it can only be an expedient of philosophical incapacity which, instead of going to philosophy for the logical element, has recourse to the shapes assumed by the logical element in other sciences, many of which shapes are only adumbrations of that element, others even defective forms of it. Apart from this, the mere application of such borrowed forms is an external procedure; the application itself must be preceded by an awareness not only of their meaning but of their value, too, and such awareness can come only from reflecting on them, not from the authority of mathematics. But logic itself is such awareness and it strips these forms of their particularity which it renders superfluous and unnecessary; it is logic which rectifies these forms and alone procures for them their justification, meaning and value.

As for the supposed primary importance of number and calculation in an *educational* regard, the truth of the matter is clearly evident from what has been said. Number is a non-sensuous object, and occupation with it and its combinations is a non-sensuous business; in it mind is held to communing with itself and to an inner abstract labour, a matter of great though one-sided importance. For, on the other hand, since the basis of number in only an external, thoughtless difference, such occupation is an unthinking, mechanical one. The effort consists mainly in holding fast what is devoid of the Notion and in combining it purely mechanically. The content is the empty one; the substantial content of moral and spiritual life in its various forms on which, as the noblest aliment, education should nurture the young mind, is to be supplanted by the blank one or unit; when such exercise is made the prime interest and occupation, the only possible outcome must be to dull the mind and to empty it of both form and substance. Calculation being so much an external and therefore mechanical business, it has been possible to construct machines which perform arithmetical operations with complete

accuracy. A knowledge of just this one fact about the nature of calculation is sufficient for an appraisal of the idea of making calculation the principal means for educating the mind and stretching it on the rack in order to perfect it as a machine.

B. EXTENSIVE AND INTENSIVE QUANTUM

(*a*) *Their Difference*

1. We have seen that quantum has its determinateness as limit in *amount*. Within itself quantum is discrete, a plurality which has no being distinct from its limit, nor is the limit external to it. Quantum thus then with its limit, which is in its own self a plurality, is *extensive magnitude*.

Extensive and *continuous* magnitude are to be distinguished from each other; the direct opposite of the former is not discrete but *intensive* magnitude. Extensive and intensive magnitudes are determinatenesses of the quantitative *limit* itself, whereas quantum is identical with its limit; continuous and discrete magnitudes, on the other hand, are determinations of magnitude *in itself*, that is, of quantity as such, in so far as in quantum abstraction is made from the limit. Extensive magnitude has the moment of continuity present within itself and in its limit, for its many is altogether continuous; the limit as negation appears, therefore, in this *equality* of the many as a limiting of the oneness. Continuous magnitude is quantity as continuing itself without regard to any limit and in so far as it is conceived as having a limit, this is simply a limitation *free from any posited discreteness*. Quantum as only continuous magnitude is not yet truly determined as being for itself because it lacks the one (in which being-for-selfness is implied) and number. Similarly, a discrete magnitude is immediately only a differentiated many in general; were this as such supposed to have a limit, it would be only an aggregate, that is, would be only indefinitely limited; before it can be a specific quantum, the many must be compressed into a one and thereby posited as identical with the limit. Continuous and discrete magnitude, taken simply as *quanta* have each posited in it only one of the two sides which together make quantum fully determined and a *number*. This latter is immediately an *extensive quantum*—the *simple* determinateness which is essentially an *amount*, but an amount of one and the same *unit*; extensive

quantum is distinguished from number only by this, that in number the determinateness is expressly posited as a plurality.

2. However, the determinateness of something in terms of number does not require it to be distinguished from another numerically determined something, as if both were necessary to the determinateness of the first; and this is because the determinateness of magnitude as such is a limit determinate by itself, indifferent and related simply to itself; and in number the limit is posited as included in the one, which is a being-for-self, and it has *within itself* the externality, the relation to other. Further, this many of the limit itself is, like the many as such, not unequal within itself but continuous; each of the many is the same as the others; consequently, the many as a plural asunderness or discreteness does not constitute the determinateness as such. This many, therefore, spontaneously collapses into its continuity and becomes a simple oneness. Amount is only a moment of number, but *as an aggregate of numerical ones*, it does *not* constitute the determinateness of number; on the contrary, these ones as indifferent and self-external are sublated in number which has returned into itself; the externality which constituted the ones as a plurality vanishes in the one as a relation of number to its own self.

Consequently the limit of quantum, which as extensive had its real determinateness in the self-external amount, passes over into *simple determinateness*. In this simple determination of the limit, quantum is intensive magnitude; and the limit or determinateness which is identical with the quantum is now also thus posited as unitary—*degree*.

The degree is thus a specific magnitude, a quantum; but at the same time it is not an aggregate or plural *within itself;* it is a plurality only in principle (*eine Mehrheit*), for plurality has been brought together into a simple, *unitary* determination, determinate being has returned into being-for-self. The determinateness of degree must, it is true, be expressed by a *number*, the completely determined form of quantum, but the number is not an *amount* but unitary, only a degree. When we speak of ten or twenty degrees, the quantum that has that number of degrees is the tenth or twentieth degree, not the amount and sum of them—as such, it would be an extensive quantum—but it is only *one* degree, the tenth or twentieth. It contains the determinateness

implied in the amount ten or twenty, but does not contain it as a plurality but is number as a *sublated* amount, as a *unitary* determinateness.

3. In number, quantum is posited in its complete determinateness; but as intensive quantum, as in number's being-for-self, it is posited as it is in its Notion or in itself. That is to say, the form of self-relation which it has in degree is at the same time *the externality of the degree to its own self.* Number, as an extensive quantum, is a numerical plurality and so has the externality within itself. This externality, as simply a plurality, collapses into undifferentiatedness and sublates itself in the numerical one, in its self-relation. Quantum, however, has its determinateness as an amount; it contains this, as we have already seen, even though the amount is no longer posited in it. Degree, therefore, which, as in its own self unitary, no longer has *within* itself this external otherness, has it *outside* itself and relates itself to it as to its determinateness. A plurality external to the degree constitutes the determinateness of the simple limit which the degree is for itself. In extensive quantum amount, in so far as it was supposed to be present in the number, was so only as sublated; now it is determined as placed outside the number. Number as a one, being posited as self-relation reflected into itself, excludes from itself the indifference and externality of the amount and is self-relation *as relation through itself to an externality.*

In this, quantum has a reality conformable to its Notion. The *indifference* of the determinateness constitutes its quality, that is, the determinateness which is in its own self a self-external determinateness. Accordingly, degree is a unitary quantitative determinateness *among* a plurality of such intensities which, though differing from each other, each being only a simple self-relation, are at the same time essentially interrelated so that each has its determinateness in this continuity with the others. This relation of degree through itself to its other makes ascent and descent in the scale of degrees a continuous progress, a flux, which is an uninterrupted, indivisible alteration; none of the various distinct degrees is separate from the others but each is determined only through them. As a self-related determination of quantity, each degree is indifferent to the others; but it is just as much implicitly related to this externality, it is only through this externality that it is what it is; its relation to itself is, in

short, the non-indifferent relation to externality, and in this it has its quality.

(*b*) *Identity of Extensive and Intensive Magnitude*

Degree is not external to itself within itself. Nevertheless, it is not the *indeterminate* one, the principle of number as such, which is not amount save in the negative sense of not being any particular amount. Intensive magnitude is primarily a unitary one of a plurality; there are many degrees, but they are determined neither as a simple one nor as a plurality, but only in the relation of this self-externality, or in the identity of the one and the plurality. If, therefore, the many as such are indeed outside the simple, unitary degree, nevertheless the determinateness of the degree consists in its relation to them; it thus contains amount. Just as twenty, as an extensive magnitude, contains the twenty ones as discrete within it, so does the specific degree contain them as the continuity which this determinate plurality simply is; it is the *twentieth* degree, and is the twentieth degree only by virtue of this amount, which as such is outside it.

The determinateness of intensive magnitude is, therefore, to be considered from two sides. Intensive magnitude is determined by *other* intensive quanta and is continuous with its otherness, so that its determinateness consists in this relation to its otherness. Now in the first place, in so far as it is a *simple* determinateness it is determinate *relatively* to other degrees; it excludes them from itself and has its determinateness in this exclusion. But, secondly, it is determinate in its own self; this it is in the amount as its *own* amount, not in the amount as excluded, nor in the amount of other degrees. The twentieth degree contains the twenty within itself; it is not only determined as distinguished from the nineteenth, twenty-first, and so on, but its determinateness is its *own* amount. But in so far as the amount is its own—and the determinateness is at the same time essentially an amount—the degree is an extensive quantum.

Extensive and intensive magnitude are thus one and the same determinateness of quantum; they are only distinguished by the one having amount within itself and the other having amount outside itself. Extensive magnitude passes over into intensive magnitude because its many spontaneously collapse into oneness, outside which the many stand. But conversely, this unitary degree

has its determinateness only in the amount, and that too in its *own* amount; as indifferent to the differently determined intensities it has within itself the externality of the amount; and so intensive magnitude is equally essentially an extensive magnitude.

With this identity, the *qualitative something* makes its appearance, for the identity is the unity which is self-related through the *negation of its differences*; these differences, however, constitute the determinate being of the quantitative determinateness; this negative identity is therefore *a something*, and a something which is indifferent to its quantitative determinateness. *Something* is a quantum; but now the qualitative determinate being, as it is in itself, is posited as indifferent to quantum. Quantum, number as such, and so forth, could be spoken of without any mention of its having a something as substrate. But the something now confronts these its determinations, through the negation of which it is *mediated* with itself, as *existing for itself* and, since it has a quantum, as something which has an extensive and an intensive quantum. Its one determinateness which it has as quantum is posited in the differentiated moments of *unit* and *amount*; this determinateness is not only *in itself* one and the same, but its positing in these differences as extensive and intensive quantum is the return into this unity which, as negative, is the something posited as indifferent to it.

Remark 1: *Examples of This Identity*

Extensive and intensive quantum are usually distinguished in the ordinary conception of them as *kinds of magnitude*, as if some objects had only intensive, others only extensive magnitude. In addition, we have the conception of a philosophical science of Nature in which what is a plurality or *extensive*—for example, in the fundamental property of matter to occupy space, and in other concepts too—is converted into something *intensive*, meaning thereby that the intensive aspect as *dynamic* is the true determination; density, or the specific filling of space, for example, must essentially be understood *not* as a certain *aggregate* and *amount* of material parts in a quantum of space, but as a certain degree of the space-filling *force* of matter.

There are two kinds of determinations to be distinguished here. In what has been called the conversion of the mechanical into the dynamic point of view, there occurs the concept of *separately*

existing, independent parts, which are only externally combined into a whole, and the concept of *force* which is distinct from this. In the occupation of space, what is regarded on the one hand as only an aggregate of atoms external to one another, is on the other hand regarded as the expression of an underlying simple force. But these relations of whole and parts, of force and its expression, which here stand opposed to each other, do not belong in this section; they will be considered in their proper place later on. But this much may be remarked here, that though the relation of force and its expression which corresponds to intensive magnitude is in the first instance truer than that of whole and parts, yet this does not make force, as intensive, any less one-sided; also *expression*, the externality of extensive magnitude, is equally *inseparable* from force; so that *one and the same content* is equally present in the two forms, both in intensive and in extensive magnitude.

The other determinateness which occurs here is the *quantitative* as such, which, as extensive quantum, is sublated and transformed into degree, the supposedly true determination; but it has been shown that degree equally contains the former determinateness, so that the two forms are essential to each other; consequently, every existence exhibits its quantitative character just as much as an extensive as an intensive quantum.

Consequently everything, in so far as it manifests a quantitative character, serves as an example of this. Number itself necessarily has this double form immediately within it. It is an amount in so far as it is an extensive magnitude; but number is also one, a ten, a hundred, and as such it is on the threshold of transition into an intensive magnitude, seeing that in this unity the plurality has become simple. One is *in itself* an extensive magnitude, it can be represented as an arbitrary amount of parts. Thus the tenth, the hundredth, is this simple, intensive magnitude which has its determinateness in the plurality lying outside it, that is, in extensive magnitude. Number is a ten, a hundred and at the same time the tenth, hundredth, in the system of numbers; both are the same determinateness.

In the circle the one is called degree because the determinateness of any part of the circle derives essentially from the many parts outside it; that is, it is determined as only one of a fixed amount of such ones. As a mere spatial magnitude, the degree of the circle is only an ordinary number; taken as degree, it is intensive

magnitude which has a meaning only as determined by the amount of degrees into which the circle is divided, just as number generally has meaning only in the number series.

The magnitude of a more concrete object exhibits its dual aspects of being extensive and intensive, in the dual determinations of its real being, in one of which it appears as an outer being but in the other as an inwardness. Thus, for example, a mass as weight is an extensive magnitude, in so far as it constitutes an amount of pounds, hundredweights, etc., and an intensive magnitude in so far as it exerts a certain pressure; the magnitude of the pressure is a simple number, a degree, which is specified by its place in a scale of degrees. As exerting pressure, mass is manifested as a being-within-self, as a subject to which belongs a difference of intensive magnitude. Conversely, that which exerts this *degree* of pressure is capable of displacing a certain *amount* of pounds, etc., and its magnitude is measured by this.

Again, heat has a degree; this degree, whether it be the tenth, twentieth and so on, is a simple sensation, something subjective. But this degree is equally present as an *extensive* magnitude, as the expansion of a fluid, of mercury in a thermometer, of air, or sound, and so on. A higher degree of temperature expresses itself as a longer column of mercury, or as a narrower sound-cylinder; it heats a larger space in the same way as a lower degree heats only a smaller space.

The higher note is, as more intensive, at the same time a greater number of vibrations, and a louder note, to which we ascribe a higher *degree*, is audible in a larger space. A larger surface can be coloured with a more intensive colour than with a weaker colour used in the same way; or again a *brighter* object (another kind of intensity) is visible at a greater distance than one less bright, and so forth.

Similarly in the spiritual sphere, high intensity of character, of talent or genius, is bound up with a correspondingly far-reaching reality in the outer world, is of widespread influence, touching the real world at many points. The profoundest Notion also has the most universal significance and application.

Remark 2: *The determination of degree as applied by Kant to the soul*

The determinateness of intensive quantum has been applied by

Kant in a peculiar way to a metaphysical determination of the soul. In his criticism of metaphysical propositions about the soul, which he calls paralogisms of pure reason, he comes to consider the inference from the simplicity of the soul to its permanence. He counters this argument by saying 'that even if we admit the simple nature of the soul since, namely, it does not contain a plurality of separate parts and therefore no extensive magnitude, yet we cannot deny to it, any more than to any other existent thing, an intensive magnitude, that is, a *degree* of reality in respect of all its faculties, indeed, in respect of all that constitutes its existence; this *degree* can diminish through all the infinitely many smaller degrees so that although the postulated substance cannot be reduced to nothing by division (into parts), it can be so reduced by a gradual lessening (*remissio*) of its powers. For *consciousness* itself has, at any moment, a degree which can always be diminished, and the same must therefore also be true of its faculty of being aware of itself and thus of all the other faculties'.[1] In rational psychology, which is an abstract metaphysics, the soul is considered not as spirit but as a merely immediate being, as a soul-thing. Kant thus has the right to apply the category of quantum to it 'as to any other existent thing', and in so far as this immediate being is determined as simple, to apply to it the category of intensive quantum. *Being* does, of course, belong to spirit, but its intensity is wholly different from that of intensive quantum; indeed, its intensity is such that in it the form of merely immediate being and all its categories are sublated. What should have been admitted was the elimination not only of the category of extensive quantum but that of quantum altogether. But a further advance has still to be made, namely, to understand how existence, consciousness, finitude, *is* in the eternal nature of spirit and proceeds from it without spirit thereby becoming a thing.

(*c*) *Alteration of Quantum*

The difference between extensive and intensive quantum is indifferent to the determinateness of quantum as such. But in general quantum is determinateness posited as sublated, the indifferent limit, the determinateness which is equally the negation of itself. In extensive magnitude this difference is developed; but intensive magnitude is the *existence* of this externality which

[1] *Critique of Pure Reason*, p. 414.

quantum is within itself. This difference, as internally self-contradictory, is posited as being the simple, *self-related* determinateness which is the negation of itself, having its determinateness not within itself but in another quantum.

A quantum, therefore, in accordance with its quality, is posited in absolute continuity with its externality, with its otherness. Therefore, not only *can* it transcend every quantitative determinateness, not only *can* it be altered, but it is *posited* that it *must* alter. The quantitative determinateness continues itself into its otherness in such a manner that the determination has its being only in this continuity with an other; it is not a *simply affirmative* limit, but a limit which *becomes*.

The one is infinite or self-related negation, hence the repulsion of itself from itself. The quantum, too, is infinite and is posited *as* self-related negativity; it repels itself from itself. But the quantum is a *determinate* one, the one which has passed over into determinate being and limit; it is, therefore, the repulsion of the determinateness from itself, not the producing of that which is the same as itself as in the repulsion of the one, but the producing of its otherness; it is now the express character of quantum to *impel itself beyond itself* and to become an other. In consists in undergoing increase or decrease; it is in its own self the externality of the determinateness.

Thus quantum impels itself beyond itself; this other which it becomes is in the first place itself a quantum; but it is quantum as a limit which does not stay, but which impels itself beyond itself. The limit which again arises in this beyond is, therefore, one which simply sublates itself again and impels itself beyond to a further limit, *and so on to infinity*.

C. QUANTITATIVE INFINITY

(*a*) *Its Notion*

Quantum alters and becomes another quantum; the further determination of this alteration, namely, that it goes on to infinity, lies in the circumstance that quantum is established as being immanently self-contradictory. Quantum becomes an other; but it *continues* itself into its otherness; the other is thus also a quantum. This, however, is not only the other of a particular quantum, but of quantum itself, the negative of quantum as limited; hence it

is the unlimitedness of quantum, its *infinity*. Quantum is an ought-to-be; it is by implication determined as being for itself, and this being-determined-for-itself is rather the being-determined-in-an-other, and, conversely, it is the sublation of being-determined-in-an-other, is an *indifferent* subsisting for itself.

In this way, finitude and infinity each acquire in themselves a dual, and indeed, an opposite meaning. The quantum is *finite*, in the first place simply as limited, and secondly, as impelled beyond itself, as being determined in an other. But the *infinity* of quantum is first, its unlimitedness, and secondly, its returnedness into itself, its indifferent being-for-self. If we now compare these moments with each other, we find that the determination of the finitude of quantum, the impulse to go beyond itself to an other in which its determination lies, is equally the determination of the infinite; the negation of the limit is the same impulsion beyond the determinateness, so that in this negation, in the infinite, quantum possesses its final determinateness. The other moment of infinity is the being-for-self which is indifferent to the limit; but the limiting of quantum itself is such that quantum is explicitly indifferent to its limit, and hence to other quanta and to its *beyond*. In quantum, finitude and infinity (the spurious infinity supposedly separate from the finite) each already has within it the moment of the other.

The difference between the qualitative and quantitative infinite is that in the former the finite and infinite are *qualitatively* opposed and the transition of the finite into the infinite, or the relation of each to the other, lies only in the *in-itself*, in their Notion. Qualitative determinateness, as an immediacy, is related to otherness essentially as to an alien being; it is not *posited* as having its negation, its other *within* it. Quantity, on the other hand, is, as such, *sublated* determinateness; it is *posited* as being unlike and indifferent to itself, consequently as alterable. Therefore the qualitative finite and infinite stand absolutely, that is abstractly, opposed to each other; their unity is their underlying *inner* relation; and therefore the finite continues itself into its other only *implicitly*, not *affirmatively*. The quantitative finite, on the other hand, is *self*-related in its infinite, in which it has its absolute determinateness. This their relation is displayed in the first place in the *quantitative infinite progress*.

(*b*) *The Quantitative Infinite Progress*

The progress to infinity is in general the expression of contradiction, here, of that which is implicit in the quantitative finite, or quantum as such. It is the reciprocal determining of the finite and infinite which was considered in the sphere of quality, with the difference that, as just remarked, in the sphere of quantity the limit in its own self dispatches and continues itself into its beyond and hence, conversely, the quantitative infinite too is posited as having quantum within it; for quantum in its self-externality is also its own self, its externality belongs to its determination.

Now the *infinite progress* is only the *expression* of this contradiction, *not its resolution*; but because the one determinateness is continued into its other, the progress gives rise to the show of a solution in a union of both. As at first posed, it is the *problem* of attaining the infinite, not the actual reaching of it; it is the perpetual *generation* of the infinite, but it does not get beyond quantum, nor does the infinite become positively present. It belongs to the Notion of quantum to have a *beyond* of itself. This beyond is first, the abstract moment of the *non-being* of quantum: the vanishing of quantum is its own act; it is thus related to its *beyond* as to its infinity, in accordance with the *qualitative* moment of the opposition. Secondly, however, quantum is continuous with its beyond; quantum consists precisely in being the other of itself, in being external to itself; this externality is, therefore, no more an other than quantum itself; the *beyond* or the infinite is, therefore, itself *a quantum*. In this way, the beyond is recalled from its flight and the infinite is attained. But because the infinite now affirmatively present is again a quantum, what has been posited is only a fresh limit; this, too, as a quantum, has again fled from itself, is as such beyond itself and has repelled itself into its non-being, into its own beyond, and as it thus repels itself into the beyond, so equally does the beyond perpetually become a quantum.

The continuity of quantum with its other produces the conjunction of both in the expression of an infinitely great or infinitely small. Since both still bear the character of quantum they remain alterable, and the absolute determinateness which would be a being-for-self is, therefore, not attained. This ***self-externality*** of the determination is posited in the dual infinite—which is opposed

to itself as a 'more' and a 'less'—in the infinitely great and infinitely small. In each, the quantum is ***maintained*** in perpetual opposition to its beyond. No matter how much the quantum is increased, it shrinks to insignificance; because quantum is related to the infinite as to its non-being, the opposition is *qualitative*; the increased quantum has therefore gained nothing from the infinite, which is now, as before, the non-being of quantum. In other words, the increase of quantum brings it no *nearer* to the infinite; for the difference between quantum and its infinity is essentially *not* a quantitative difference. The expression 'the infinitely great' only throws the contradiction into sharper relief; it is supposed to be *great*, that is, a quantum, and *infinite*, that is, not a quantum. Similarly, the infinitely small is, as small, a quantum, and therefore remains absolutely, that is, qualitatively, too great for the infinite and is opposed to it. In both, there remains the contradiction of the infinite progress which in them should have reached its goal.

This infinity which is perpetually determined as the beyond of the finite is to be described as the *spurious quantitative infinite*. Like the qualitative spurious infinite, it is the perpetual movement to and fro from one term of the lasting contradiction to the other, from the limit to its non-being, and from this back again to the limit. It is true that in the quantitative progress the movement is not simply towards an abstract *other* in general, but towards an explicitly different quantum; but this remains in the same way opposed to its negation. The progress, too, is therefore not a real advance but a repetition of one and the same thing, a positing, a sublating, and then again a positing and again a sublating, an impotence of the negative, for what it sublates is continuous with it, and in the very act of being sublated returns to it. Thus there are two terms, the bond between which is such that they simply flee from each other; and in fleeing from each other they cannot become separated but are joined together even in their flight from each other.

Remark 1: *The High Repute of the Progress to Infinity*

The spurious infinite, especially in the form of the quantitative progress to infinity which continually surmounts the limit it is powerless to remove, and perpetually falls back into it, is commonly held to be something sublime and a kind of divine worship, while in philosophy it has been regarded as ultimate. This

progression has often been the theme of tirades which have been admired as sublime productions. As a matter of fact, however, this *modern* sublimity does not magnify the *object*—rather does this take flight—but only the *subject* which assimilates such vast quantities. The hollowness of this exaltation, which in scaling the ladder of the quantitative still remains subjective, finds expression in its own admission of the futility of its efforts to get nearer to the infinite goal, the attainment of which must, indeed, be achieved by a quite different method.

In the following tirades of this kind it is also stated what becomes of such exaltation and how it finishes. Kant, for example, at the close of the *Critique of Practical Reason*, represents it as sublime 'when the subject raises himself in thought above the place he occupies in the world of sense, reaching out to infinity, to stars beyond stars, worlds beyond worlds, systems beyond systems, and then also to the limitless times of their periodic motion, their beginning and duration. Imagination fails before this progress into the infinitely remote, where beyond the most distant world there is a still more distant one, and the past, however remote, has a still remoter past behind it, the future, however distant, a still more distant future beyond it; thought fails in the face of this conception of the immeasurable, just as a dream, in which one goes on and on down a corridor which stretches away endlessly out of sight, finishes with falling or fainting.'

This exposition, besides giving a concise yet rich description of such quantitative exaltation, deserves praise mainly on account of the truthfulness with which it states how it fares finally with this exaltation: thought succumbs, the end is falling and faintness. What makes thought succumb, what causes falling and faintness, is nothing else but the wearisome repetition which makes a limit vanish, reappear, and then vanish again, so that there is a perpetual arising and passing away of the one after the other and of the one in the other, of the beyond in the here and now, and of the here and now in the beyond, giving only the feeling of the *impotence* of this infinite or this ought-to-be, which *would* be master of the finite and *cannot*.

Also Haller's description of eternity, called by Kant terrifying, is usually specially admired, but often just not for that very reason which constitutes its true merit:

'I heap up monstrous numbers,
Pile millions upon millions,
I put aeon upon aeon and world upon world,
And when from that awful height
Reeling, again I seek thee,
All the might of number increased a thousandfold
Is still not a fragment of thee.
I remove them and thou liest wholly before me.'

When this heaping and piling up of numbers is regarded as what is valuable in a description of eternity, it is overlooked that the poet himself declares this so-called terrifying journey into the beyond to be futile and empty, and that he closes by saying that only by giving up this empty, infinite progression can the genuine infinite itself become present to him.

There have been astronomers who liked to pride themselves on the sublimity of their science because it had to deal with an innumerable host of stars, with such immeasurable spaces and times in which distances and periods, already vast in themselves, serve as units which, in whatever multiples taken, are again abbreviated to insignificance. The shallow astonishment to which they surrender themselves, the absurd hopes of wandering in another life from one star to another and into immeasurable space to acquire fresh facts *of the same kind*, this they declare to be a cardinal factor in the excellence of their science—a science which is admirable not on account of such quantitative infinitude but, on the contrary, on account of the relations of *measure* and the laws which reason recognizes in these objects and which are the infinite of reason in contrast to that other, irrational infinite.

To the infinity of outer, sensuous intuition, Kant opposes the other infinite, when 'the individual withdraws into his invisible ego and opposes the absolute freedom of his will as a pure ego to all the terrors of fate and tyranny, and starting with his immediate surroundings, lets them vanish before him, and even what seems enduring, worlds upon worlds, collapse into ruins, and, alone, knows *himself as equal to himself*'.

The ego in being thus alone with itself is, it is true, the reached beyond; it has *come to itself*, is *with* itself, here and now; the absolute negativity which in the progress beyond the quantum of sense was only a flight, in pure self-consciousness becomes

affirmative and present. But this pure ego, because it has fixed itself in its abstraction and emptiness, has determinate reality, the fulness of the universe of nature and mind, confronting it as a beyond. We are faced with that same contradiction which lies at the base of the infinite progress, namely a returnedness-into-self which is at the same time immediately an out-of-selfness, a relation to its other as to its non-being; and this relation remains a *longing*, because on the one side is the unsubstantial, untenable void of the ego fixed as such by the ego itself, and on the other, the fulness which though negated remains present, but is fixed by the ego as its beyond.

On these two sublimes Kant remarks 'that admiration (for the first, outer) and reverence (for the second, inner) do indeed stimulate inquiry but cannot be a substitute for their defect'. Thus he declares those exaltations to be unsatisfying for reason, which cannot stop at them and the feelings associated with them, nor can it let the beyond and the void rank as ultimate.

But it is specially in its application to morality that the infinite progress has been taken as ultimate. The just quoted antithesis of finite and infinite in the shape of the manifold world and the ego raised to its freedom, is primarily qualitative. The ego in its self-determining forthwith proceeds to determine nature and to liberate itself therefrom; it thus connects itself through itself with its other which, as an external reality, is manifold and quantitative. The relation to the quantitative becomes itself quantitative; the negative relation of the ego to it, the power of the ego over the non-ego, over sense and outer nature, is consequently so conceived that morality can and ought continually to increase, and the power of sense continually to diminish. But the perfect adequacy of the will to the moral law is placed in the unending progress to infinity, that is, is represented as an *absolutely unattainable* beyond, and this very unattainableness is supposed to be the true sheet-anchor and fitting consolation; for morality is supposed to be a struggle, but such it can be only if the will is inadequate to the moral law which thus becomes a sheer beyond for it.

In this opposition, ego and non-ego or the pure will and the moral law, and nature and the sensuousness of the will, are presupposed as completely self-subsistent and mutually indifferent. The pure will has its own appropriate law which stands

in an essential relationship to the sphere of sense; and nature and sense on *its* side has laws which neither stem from nor are conformable to the will nor, although distinct from it, have they even in principle an essential connection with it but are determined independently, are finished and complete in themselves. At the same time, however, both are moments of *one and the same simple being*, the ego; the will is determined as the negative in relation to nature so that the will only *is* in so far as there is a sphere distinct from it which it sublates, but with which it thereby comes into contact and by which it is itself affected. Nature itself and nature as the sensuous sphere of man, as an independent system of laws, is indifferent to limitation by an other; it preserves itself in this process of limitation, enters into the relation as an independent factor and limits the will of law just as much as this limits it. The two processes comprise a single act: the self-determining of the will with the sublating of the otherness of nature, and the positing of this otherness as continuing itself as a reality in the process of being sublated, so that the otherness is not sublated. The contradiction involved in this is not resolved in the infinite progress: on the contrary, it is represented and affirmed as unresolved and unresolvable; the conflict of morality and sense is represented as the ultimate, absolute relation.

This standpoint which is powerless to overcome the qualitative opposition between the finite and infinite and to grasp the idea of the true will which is substantial freedom, has recourse to *quantity* in order to use it as a mediator, because it is sublated quality, the difference which has become indifferent. But since both members of the antithesis remain implied as qualitatively distinct, the fact is rather that each is straightway made indifferent to this alteration because it is *as quanta* that they are related to each other. Nature is determined by the ego, sense by the will of the good; the alteration produced in sense by the will is only a quantitative difference, one which leaves sense itself unchanged.

In the more abstract exposition of the Kantian philosophy, or at least of its principles, namely in Fichte's *Theory of Science*, the infinite progress in the same way constitutes the foundation and the ultimate. In this exposition, the first axiom, ego = ego, is followed by a second, independent of it, the opposition of the non-ego; the relation between the two is also directly assumed as a

quantitative difference, that is, the non-ego is *partly* determined by the ego, and *partly* not. In this way, the non-ego is continued into its non-being in such wise that in its non-being it remains opposed as something not sublated. Consequently, after the contradictions contained in this have been developed in the system, the final result is that relationship which formed the beginning: the non-ego remains an infinite obstacle [*Anstoss*], an absolute other; the final relation of the non-ego and the ego to each other is the infinite progress, a longing and aspiration—the same contradiction with which the system began.

Because the quantitative is determinateness posited as sublated it was thought that much, or rather everything, had been gained for the unity of the absolute, for the one substantiality, when opposition generally had been reduced to a merely quantitative difference. That all opposition is only quantitative was for some time a cardinal thesis of recent philosophy; the opposed determinations have the same nature, the same content; they are real sides of the opposition in so far as each of them has within it both determinations, both factors of the opposition, only that on one side one of the factors *preponderates*, on the other side the other, that is, one of the factors, a material substance or activity, is present in a *greater quantity* or in an *intenser degree* in one side than in the other. But in so far as substances or activities are presupposed, the quantitative difference rather confirms and completes their externality and indifference to each other and to their unity. The difference of the *absolute* unity is supposed to be only quantitative; the quantitative, it is true, is immediate, sublated determinateness, but only the imperfect, as yet only *first*, negation, not the infinite, not the *negation* of the negation. When *being* and *thought* are represented as quantitative determinations of absolute substance they too, as quanta, become completely external to each other and unrelated as, in a subordinate sphere, do carbon, nitrogen, etc. It is a third, an external reflection, which abstracts from their difference and recognizes their unity, but a unity which is *inner*, *implicit* only, not *for itself*. This unity is, therefore, in fact conceived only as a first, *immediate* unity, or only as *being*, which in its quantitative difference *remains* like itself, but does not of itself *posit* itself as like itself; hence it is not grasped as a negation of the negation, as an infinite unity. Only in the qualitative opposition does the posited infinitude,

being-for-self, emerge and the quantitative determination itself pass over into the qualitative, as we shall presently find.

Remark 2: *The Kantian Antinomy of the Limitation and Non-limitation of the World in Time and Space*

It was remarked above that the Kantian antinomies are expositions of the opposition of finite and infinite in a *more concrete* shape, applied to more specific substrata of conception. The antinomy there considered contained the opposition of qualitative finitude and infinitude. In another, the *first* of the four cosmological antinomies, it is the conflict arising rather from the quantitative limit which is considered. I shall therefore proceed to examine this antinomy here.

It concerns the limitation or non-limitation of the world in time and space. This antithesis could be considered equally well with reference to time and space themselves, for whether time and space are relations of things themselves or are only forms of intuition, the antinomy based on limitation or non-limitation in them is not affected thereby.

The detailed analysis of this antinomy will likewise show that both statements and equally their proofs (which, like those already considered, are conducted apagogically) amount to nothing more than the two simple opposite assertions: (1) there is a limit, and (2) the limit must be transcended.

The thesis is:

'The world has a beginning in time and is also enclosed within spatial limits.'

That part of the proof which concerns time assumes the opposite:

'The world has no beginning in time; therefore, up to *any given point of time*, an eternity has elapsed and consequently an infinite series of successive states of things in the world has *passed away*. Now the infinity of a series consists precisely in the impossibility of ever *completing* it by successive synthesis. Therefore an infinite world series which has passed away is impossible and consequently a beginning of the world is a necessary condition of its existence—which was to be proved.'

The other part of the proof which concerns space is based on time. To comprehend a spatially infinite world would require an infinite time and this time must be regarded as having already

elapsed in so far as the world in space is to be regarded not as gradually coming to be but as completely given. But it was shown of time in the first part of the proof that it is impossible to assume an infinite time as elapsed.

But it is at once evident that it was unnecessary to make the proof apagogical, or even to carry out a proof at all, since the basis of the proof itself is the direct assertion of what was to be proved. Namely, there is assumed some or any *given point of time* up to which an eternity has elapsed (eternity here has only the trivial meaning of a simply endless time). Now a given point of time means nothing else than a definite limit in time. In the proof therefore, a limit of time is *presupposed* as actual; but *that* is just what *was to be proved.* For the thesis is, that the world has a beginning in time.

There is only this difference, that the *assumed* limit of time is a now as end of the time already elapsed, but the limit which is to be proved is a now as beginning of a future. But this difference is immaterial. The now is taken as the point in which an infinite series of successive states of things in the world is supposed to have *passed away*, therefore as end, a *qualitative* limit. If this *now* were considered to be merely a quantitative limit which flows on and which not only must be transcended but *is* only as the transcending of itself, then the infinite time series would not have *passed away* in it, but would continue to flow on, and so the argument of the proof would vanish. On the other hand, if the point of time is assumed as a qualitative limit for the past, in which case it is also a *beginning* for the future (for each point of time is *in itself* the connection of the past and the future), then it is also an *absolute*, that is, abstract *beginning* for the future—and it was this that was to be proved. The fact that its future and this its beginning is already preceded by a past does not affect the argument; because this point of time is a qualitative limit—and that it is to be taken as qualitative is implied in the description of it as *completed*, elapsed, and therefore as not continuing—therefore in it time is *broken off* and the past lacks a connection with this time which could only be called future with reference to that past and, consequently, without such connection is only time as such, which has an absolute beginning. But if—as is, then, the case—it were related to the past through the now, the given point of time, and were thus determined as a future, then this

point of time, too, regarded from the other side, would not be a limit; the infinite time series would continue itself in what was called future and would not be, as was assumed, *completed.*

In truth, time is pure quantity; the *point of time* in which it is supposed to be interrupted, which is employed in the proof, is really only the *self-sublating* being-for-self of the now. All that the proof does is to represent the absolute limit of time asserted in the thesis as a *given point of time*, and then straightway to assume it as a completed, that is, abstract point—a popular determination which sensuous conception readily lets pass as a *limit*, thus allowing as an *assumption* in the proof what had been put forward as the thing to be proved.

The antithesis runs:

'The world has no beginning and no limits in space but is infinite with reference both to time and space.'

The proof likewise assumes the opposite:

'The world has a beginning. Since the beginning is an existence preceded by a time in which the thing is not, there must have been a preceding time in which the world was not, that is, an empty time. Now no originating of anything is possible in an empty time; because no part of such a time possesses *in itself* and in preference to any other, any distinguishing condition of existence or non-existence. In the world, therefore, many groups of things can indeed begin, but the world itself can have no beginning and with respect to past time is infinite.'

This apagogical proof, like the others, contains the direct and unproved assertion of what it was supposed to prove. That is, it first assumes a beyond of the existing world, an empty time; but it also equally *continues* the existence of the world beyond itself into this empty time which is thereby sublated, with the result that the existence of the world is continued into infinity. The world is an existence; the proof *presupposes* that this existence *comes into being* and that the coming-to-be has an *antecedent condition* which is in time. But the antithesis itself consists in the very fact that there is no unconditioned existence, no absolute limit, but that the existence of the world always requires an *antecedent condition.* Thus, what was to be proved is found as an assumption in the proof. Further, the *condition* is sought in empty time, which means in effect that it is taken as temporal and therefore as an existence and as limited. Altogether, then, the

assumption is made that the world as an existence presupposes another conditioned existence in time, and so on, therefore, to infinity.

The proof regarding the infinity of the world in *space* is the same. Apagogically, the spatial finiteness of the world is assumed; 'this (the world) would therefore exist in an empty unlimited space and would stand in a relation to it; but such a relation of the world to *no* object is a nullity'.

Here, too, what was supposed to be proved is directly presupposed in the proof. It is directly assumed that the spatially limited world exists in an empty space and is supposed to stand in a *relation* to it, that is, there must be a movement out beyond it—on the one hand into the void, into the beyond and non-being of the world, but on the other hand, in order that it be in *relation* with its beyond, that is, *continue* itself into it, the beyond must be imagined as filled with the existence of the world. The infinity of the world in space which is asserted in the antithesis is nothing else than, on the one hand, empty space, and on the other the *relation* of the world to it, that is, the continuity of the world in empty space or the filling of space—which contradiction, namely, space as simultaneously empty and also filled, is the infinite progress of existence in space. This very contradiction, the relation of the world to empty space, is directly made the basis of the proof.

The thesis and antithesis and their proofs therefore represent nothing but the opposite assertions, that a *limit is*, and that the limit equally is only a *sublated* one; that the limit has a beyond, with which however it stands in *relation*, and beyond which it must pass, but that in doing so there arises another such limit, which is no limit.

The *solution* of these antinomies, as of those previously mentioned, is transcendental, that is, it consists in the assertion of the ideality of space and time as forms of intuition—in the sense that the world is *in its own self* not self-contradictory, not self-sublating, but that it is only *consciousness* in its intuition and in the relation of intuition to understanding and reason that is a self-contradictory being. It shows an excessive tenderness for the world to remove contradiction from it and then to transfer the contradiction to spirit, to reason, where it is allowed to remain unresolved. In point of fact it is spirit which is so strong that it can endure

contradiction, but it is spirit, too, that knows how to resolve it. But the so-called world (whether it be called an objective, real world or, according to transcendental idealism, a subjective intuition and a sphere of sense determined by the categories of the understanding) is never and nowhere without contradiction, but it is unable to endure it and is, therefore, subject to coming-to-be and ceasing-to-be.

(*c*) *The Infinity of Quantum*

1. The infinite quantum as infinitely great or infinitely small is itself implicitly the infinite progress; as great or small it is a quantum and at the same time it is the non-being of quantum. The infinitely great and infinitely small are therefore pictorial conceptions which, when looked at more closely, turn out to be nebulous shadowy nullities. But in the infinite progress, this contradiction is explicitly present and with it that which is the nature of quantum which, as an intensive magnitude, has attained its reality and now in its determinate being is *posited* as it is in its *Notion*. It is this identity which we have now to consider.

Quantum as degree is unitary, self-related and determinate within itself. Through this unitary nature, the otherness and determinateness in quantum are sublated, so that the determinatensess is external to it; it has it determinateness outside it. This its self-externality is in the first place the *abstract non-being* of quantum generally, the spurious infinity. But, further, this non-being is also quantitative and this continues itself into its non-being, for it is in its externality that quantum has its determinateness; this its externality is, therefore, itself equally a quantum; this non-being of quantum, infinity, is thus limited, that is, this beyond is sublated, is itself determined as quantum which, therefore, in its negation is with itself.

But this is what quantum as such is *in itself*. For it is *itself* just by being external to itself; externality constitutes that whereby it is quantum and is with itself. In the infinite progress, therefore, the *Notion* of quantum is *posited*.

Let us take the progress at first in its abstract determinations as we find them; then in it we have the sublating of quantum, but equally too of its beyond, therefore the negation of quantum as well as the negation of this negation. Its truth is their unity in which they are, but only as moments. It is the resolution of the contra-

diction of which it is the expression, and its immediate significance is, therefore, the restoration of the Notion of quantity, namely, that quantity is an indifferent or external limit. In the infinite progress as such, the only reflection usually made is that every quantum, however great or small, must be capable of vanishing, of being surpassed; but not that this self-sublating of quantum, the beyond, the spurious infinite itself also vanishes.

Even the *first* sublation, the negation of quality as such whereby quantum is posited, is *in principle* [*an sich*] the sublating of the negation—the quantum is sublated qualitative limit, hence sublated negation—but at the same time it is this only *in principle*; it is *posited* as a determinate being, and then its negation is fixed as the infinite, as the beyond of quantum, which remains on this side as an *immediate*; thus the infinite is determined only as a *first* negation and it appears as such in the infinite progress. But we have seen that in this something more is present, the negation of the negation, or that which the infinite in truth is. We regarded this previously as the restoration of the *Notion* of quantity; this restoration means in the first place, that its determinate being has received a more precise determination; we now have quantum determined in conformity with its Notion, which is different from quantum in its immediacy; externality is now the opposite of itself, posited as a moment of quantity itself—quantum is posited as having its determinateness in another quantum by means of its non-being, of infinity; that is, it is *qualitatively* that which it is. However, this comparison of the *Notion* of quantum with its determinate being belongs more to our reflection, to a relationship which is not yet present here. The immediately following determination is that the quantum has reverted to *quality*, is from now on qualitatively determined. For its peculiarity, its quality, is the externality, the indifference of the determinateness; and quantum is now posited as being in fact *itself* in its externality, as self-related therein, in simple unity with itself, that is, *qualitatively* determined. This qualitative moment is still more closely determined, namely as being-for-itself; for the self-relation to which it has attained has proceeded from mediation, from the negation of the negation. Quantum has infinity, self-determinedness, no longer outside it but within itself.

The infinite, which in the infinite progress has only the empty meaning of a non-being, of an unattained but sought beyond, is

in fact nothing else than *quality*. Quantum as an indifferent limit goes out beyond itself to infinity; in doing so it seeks nothing else than to be determined for itself, the qualitative moment, which, however, is thus only an ought-to-be. Its indifference to limit, and hence its lack of an explicit determinateness of its own and its passage away from and beyond itself, is that which makes quantum what it is; this its passage into the beyond is to be negated and quantum is to find in the infinite its absolute determinateness.

Quite generally: quantum is sublated quality; but quantum is infinite, goes beyond itself, is the negation of itself. Thus its passage beyond itself is, therefore, *in itself* the negation of the negated quality, the restoration of it; and thus quantum is explicitly determined as possessing as its *own moment*, the externality which formerly appeared as a beyond.

Quantum is thus posited as repelled from itself, with the result that there are two quanta which, however, are sublated, are only as moments of *one unity*, and this unity is the determinateness of quantum. Quantum as thus *self-related* as an indifferent limit in its externality and therefore posited as qualitative, is *quantitative ratio*. In the ratio, quantum is external to itself, is distinguished from itself; this its externality is the relation of one quantum to another, each of which has meaning only in this its relation to its other; and this relation constitutes the determinateness of the quantum, which is as such a unity. It has in this unity not an indifferent, but a qualitative, determination; in this its externality it has returned into itself, and in it quantum is that which it is.

Remark 1: *The Specific Nature of the Notion of the Mathematical Infinite*

The mathematical infinite has a twofold interest. On the one hand its introduction into mathematics has led to an expansion of the science and to important results; but on the other hand it is remarkable that mathematics has not yet succeded in justifying its use of this infinite by the Notion (Notion taken in its proper meaning). Ultimately, the justifications are based on the *correctness of the results* obtained with the aid of the said infinite, which correctness is proved on quite other grounds: but the justifications are not based on the clarity of the subject matter and on the

operation through which the results are obtained, for it is even admitted that the operation itself is incorrect.

This alone is in itself a bad state of affairs; such a procedure is unscientific. But it also involves the drawback that mathematics, being unaware of the nature of this its instrument because it has not mastered the metaphysics and critique of the infinite, is unable to determine the scope of its application and to secure itself against the misuse of it.

But in a philosophical respect the mathematical infinite is important because underlying it, in fact, is the notion of the genuine infinite and it is far superior to the ordinary so-called *metaphysical infinite* on which are based the objections to the mathematical infinite. Often, the science of mathematics can only defend itself against these objections by denying the competence of metaphysics, asserting that it has nothing to do with that science and does not have to trouble itself about metaphysical concepts so long as it operates consistently within its own sphere. Mathematics has to consider not what is true in itself but what is true in its own domain. Metaphysics, though disagreeing with the use of the mathematical infinite, cannot deny or invalidate the brilliant results obtained from it, and mathematics cannot reach clearness about the metaphysics of its own concept or, therefore, about the derivation of the modes of procedure necessitated by the use of the infinite.

If it were solely the difficulty of the Notion as such which troubled mathematics, it could ignore it without more ado since the Notion is more than merely the statement of the essential determinatenesses of a thing, that is, of the determinations of the understanding: and mathematics has seen to it that these determinatenesses are not lacking in precision; for it is not a science which has to concern itself with the concepts of its objects and which has to generate their content by explicating the concept, even if this could be effected only by ratiocination. But mathematics, in the method of its infinite, finds a radical contradiction to that very method which is peculiar to itself and on which as a science it rests. For the infinitesimal calculus permits and requires modes of procedure which mathematics must wholly reject when operating with finite quantities, and at the same time it treats these infinite quantities as if they were finite and insists on applying to the former the same modes of operation which are valid

for the latter; it is a cardinal feature in the development of this science that it has succeeded in applying to *transcendental* determinations and their treatment the form of ordinary calculation.

Mathematics shows that, in spite of the clash between its modes of procedure, results obtained by the use of the infinite completely agree with those found by the strictly mathematical, namely, geometrical and analytical method. But in the first place, this does not apply to every result and the introduction of the infinite is not for the sole purpose of shortening the ordinary method but in order to obtain results which this method is unable to secure. Secondly, success does not by itself justify the mode of procedure. This procedure of the infinitesimal calculus shows itself burdened with a seeming inexactitude, namely, having increased finite magnitudes by an infinitely small quantity, this quantity is in the subsequent operation in part retained and in part ignored. The peculiarity of this procedure is that in spite of the admitted inexactitude, a result is obtained which is not merely fairly close and such that the difference can be ignored, but is perfectly exact. In the operation itself, however, which precedes the result, one cannot dispense with the conception that a quantity is not equal to nothing, yet is so inconsiderable that it can be left out of account. However, what is to be understood by mathematical determinateness altogether rules out any distinction of a greater or lesser degree of exactitude, just as in philosophy there can be no question of greater or less probability but solely of Truth. Even if the method and use of the infinite is justified by the result, it is nevertheless not so superfluous to demand its justification as it seems in the case of the nose to ask for a proof of the right to use it. For mathematical knowledge is scientific knowledge, so that the proof is essential; and even with respect to results it is a fact that a rigorous mathematical method does not stamp all of them with the mark of success, which in any case is only external.

It is worth while considering more closely the mathematical concept of the infinite together with the most noteworthy of the attempts aimed at justifying its use and eliminating the difficulty with which the method feels itself burdened. The consideration of these justifications and characteristics of the mathematical infinite which I shall undertake at some length in this Remark will at the same time throw the best light on the nature of the true Notion

itself and show how this latter was vaguely present as a basis for those procedures.

The usual definition of the mathematical infinite is that it is a magnitude than which there is no greater (when it is defined as the infinitely great) or no smaller (when it is defined as the infinitely small), or in the former case is greater than, in the latter case smaller than, any given magnitude. It is true that in this definition the true Notion is not expressed but only, as already remarked, the same contradiction which is present in the infinite progress; but let us see what is implicitly contained in it. In mathematics a magnitude is defined as that which can be increased or diminished; in general, as an indifferent limit. Now since the infinitely great or small is that which cannot be increased or diminished, it is in fact no longer a quantum as such.

This is a necessary and direct consequence. But it is just the reflection that quantum (and in this remark quantum as such, as we find it, I call finite quantum) is sublated, which is usually not made, and which creates the difficulty for ordinary thinking; for quantum in so far as it is infinite is required to be thought as sublated, as something which is not a quantum but yet retains its quantitative character.

To quote Kant's opinion of the said definition which he finds does not accord with what is understood by an infinite whole: 'According to the usual concept, a magnitude is infinite beyond which there can be no greater (i.e. greater than the *amount* contained therein of a given unit); but there can be no greatest amount because one or more units can always be added to it. But our concept of an infinite whole does not represent *how great* it is and it is not therefore the concept of a *maximum* (or a minimum); this concept rather expresses only the *relation* of the whole to an arbitrarily assumed unit, with respect to which the relation is greater than any number. According as this assumed unit is greater or smaller, the infinite would be greater or smaller. The infinity, however, since it consists solely in the *relation* to this given unit, would always remain the same, although of course the absolute magnitude of the whole would not be known through it.'[1]

Kant objects to infinite wholes being regarded as a maximum, as a *completed* amount of a given unit. The maximum or minimum

[1] Observation on the 'Thesis of the First Cosmological Antinomy', in the *Critique of Pure Reason.*

as such still appears as a quantum, an amount. Such a conception cannot avert the conclusion, adduced by Kant, which leads to a greater or lesser infinite. And in general, so long as the infinite is represented as a quantum, the distinction of greater or less still applies to it. This criticism does not however apply to the Notion of the genuine mathematical infinite, of the infinite difference, for this is no longer a finite quantum.

Kant's concept of infinite, on the other hand, which he calls truly transcendental is 'that the successive synthesis of the unit in the measurement of a quantum *can never* be completed'. A quantum as such is presupposed as given; by synthetizing the *unit* this is supposed to be converted into an amount, into a definite assignable quantum; but this synthesis, it is said, can never be completed. It is evident from this that we have here nothing but an expression of the progress to infinity, only represented *transcendentally*, i.e. properly speaking, subjectively and psychologically. True, in itself the quantum is supposed to be completed; but transcendentally, namely in the *subject* which gives it a *relation* to a unit, the quantum comes to be determined only as incomplete and as simply burdened with a *beyond*. Here, therefore, there is no advance beyond the contradiction contained in quantity; but the contradiction is distributed between the object and the subject, limitedness being ascribed to the former, and to the latter the progress to infinity, in its spurious sense, beyond every assigned determinateness.

On the other hand, it was said above that the character of the mathematical infinite and the way it is used in higher analysis corresponds to the Notion of the genuine infinite; the comparison of the two determinations will now be developed in detail. In the first place, as regards the true infinite quantum, it was characterized as *in its own self* infinite; it is such because, as we have seen, the finite quantum or quantum as such and its *beyond*, the spurious infinite, are *equally* sublated. Thus the sublated quantum has returned into a simple unity and self-relation; but not merely like the extensive quantum which, in passing into intensive quantum, has its determinateness only *in itself* [or *implicitly*] in an external plurality, towards which, however, it is indifferent and from which it is supposed to be distinct. The infinite quantum, on the contrary, contains within itself first externality and secondly the negation of it; it is thus no longer any finite quantum, not a

quantitative determinateness which would have a *determinate being* as *quantum*; it is simple, and therefore only a *moment*. It is a quantitative determinateness in *qualitative* form; its infinity consists in its being a *qualitative determinateness*. As such moment, it is in essential unity with its other, and *is* only as determined by this its other, i.e. it has meaning solely with reference to that which stands in *relation* to it. *Apart from this relation* it is a *nullity*—simply because quantum as such is indifferent *to the relation*, yet in the relation is supposed to be an *immediate*, inert determination. As only a moment, it is, *in the relation*, not an independent, indifferent something; the quantum in its infinity is a *being-for-self*, for it is at the same time a quantitative determinateness only in the form of a *being-for-one*.

The Notion of the infinite as abstractly expounded here will show itself to be the basis of the mathematical infinite and the Notion itself will become clearer if we consider the various stages in the expression of a quantum *as moment of a ratio*, from the lowest where it is still also a quantum as such, to the higher where it acquires the meaning and the expression of a properly infinite magnitude.

Let us then first take quantum in the relation where it is a fractional number. Such fraction, $\frac{2}{7}$ for example, is not a quantum like 1, 2, 3, etc.; although it is an ordinary finite number it is not an immediate one like the whole numbers but, as a fraction, is directly determined by two other numbers which are related to each other as amount and unit, the unit itself being a specific amount. However, if we abstract from this more precise determination of them and consider them solely as quanta in the qualitative relation in which they are here, then 2 and 7 are indifferent quanta; but since they appear here only as *moments*, the one of the other, and consequently of a third (of the quantum which is called the exponent), they directly count no longer simply as 2 and 7 but only according to the *specific relationship* in which they stand to each other. In their place, therefore, we can just as well put 4 and 14, or 6 and 21, and so on to infinity. With this, then, they begin to have a qualitative character. If 2 and 7 counted as mere quanta, then 2 is just 2 and nothing more, and 7 is simply 7; 4, 14, 6, 21 etc., are completely different from them and, as only immediate quanta, cannot be substituted for them. But in so far as 2 and 7 are not to be taken as such immediate

quanta their indifferent limit is sublated; on this side therefore they contain the moment of infinity, since not only are they no longer merely 2 and 7, but their quantitative determinateness remains—but as one which is in itself qualitative, namely in accordance with their significance as moments in the ratio. Their place can be taken by infinitely many others without the value of the fraction being altered, by virtue of the determinateness possessed by the ratio.

However, the representation of infinity by a fractional number is still imperfect because the two sides of the fraction, 2 and 7, can be taken out of the relation and are ordinary, indifferent quanta; their connection as moments of the ratio is an external circumstance which does not directly concern them. Their relation, too, is itself an ordinary quantum, the exponent of the ratio.

The *letters* with which general arithmetic operates, the next universality into which numbers are raised, do not possess the property of having a specific numerical value; they are only general symbols and indeterminate possibilities of any specific value. The fraction $\frac{a}{b}$ seems, therefore, to be a more suitable expression of the infinite, since a and b, taken out of their relation to each other, remain undetermined, and taken separately, too, have no special peculiar value. However, although these letters are posited as indeterminate magnitudes their meaning is to be some finite quantum. Therefore, though they are the general representation of number, it is only of a *determinate number*, and the fact that they are in a ratio is likewise an inessential circumstance and they retain their value outside it.

If we consider more closely what is present in the ratio we find that it contains the following two determinations: first it is a quantum, secondly, however, this quantum is not immediate but contains qualitative opposition; at the same time it remains therein a determinate, indifferent quantum by virtue of the fact that it returns into itself from its otherness, from the opposition, and so also is infinite. These two determinations are represented in the following familiar form developed in their difference from each other.

The fraction $\frac{2}{7}$ can be expressed as $0{\cdot}285714\ldots$ $\frac{1}{1-a}$ as $1 + a + a^2 + a^3$ etc. As so expressed it is an infinite series; the

fraction itself is called the sum, or *finite expression* of it. A comparison of the two expressions shows that one of them, the infinite series, represents the fraction no longer as a ratio but from that side where it is a quantum as an *aggregate* of units added together, as an amount. That the magnitudes of which it is supposed to consist as amount are in turn decimal fractions and therefore are themselves ratios, is irrelevant here; for this circumstance concerns the particular kind of *unit* of these magnitudes, not the magnitudes as constituting an *amount.* Just as a multi-figured integer in the decimal system is reckoned essentially as an *amount*, and the fact that it consists of *products* of a number and of the number ten and powers of ten is ignored. Similarly here, it is irrelevant that there are fractions other than the example taken of $\frac{2}{7}$ which, when expressed as decimal fractions, do not give an infinite series; but they can all be so expressed in a numerical system based on another unit.

Now in the infinite series, which is supposed to represent the fraction as an amount, the aspect of the fraction as a ratio has vanished and with it there has vanished too the aspect which, as we have already shown, makes the fraction *in its own self* infinite. But this infinity has entered in another way; the series, namely, is itself infinite.

Now the nature of this infinity of the series is self-evident; it is the spurious infinity of the progression. The series contains and exhibits the contradiction of representing that which is a relation possessing a *qualitative* nature, as devoid of relation, as a mere *quantum*, as an amount. The consequence of this is that the amount which is expressed in the series always lacks something, so that in order to reach the required determinateness, we must always go further than the terms already posited. The law of the progression is known, it is implicit in the determination of the quantum contained in the fraction and in the nature of the form in which it is supposed to be expressed. By continuing the series the amount can of course be made as accurate as *required*; but representation by means of the series continues to remain only an *ought-to-be*; it is burdened with a *beyond* which cannot be sublated, because to express as an *amount* that which rests on a *qualitative* determinateness is a *lasting contradiction.*

In this infinite series, this inexactitude is actually present,

whereas in the genuine mathematical infinite there is only an appearance of inexactitude. These two kinds of mathematical infinite are as little to be confounded as are the two kinds of philosophical infinite. In representing the genuine mathematical infinite, *the form of series* was used originally and it has recently again been invoked; but this form is not necessary for it. On the contrary, the infinite of the infinite series is essentially different from the genuine infinite as the sequel will show. Indeed the form of infinite series is even inferior to the fractional expression.

For the infinite series contains the spurious infinity, because what the series is meant to express remains an *ought-to-be* and what it does express is burdened with a beyond which does not vanish and *differs* from what was meant to be expressed. It is infinite not because of the terms actually expressed but because they are incomplete, because the *other* which essentially belongs to them is beyond them; what is really present in the series, no matter how many terms there may be, is only something finite, in the proper meaning of that word, posited as finite, i.e., as something *which is not what it ought to be*. But on the other hand, what is called the *finite expression* or the *sum* of such a series lacks nothing; it contains that complete value which the series only seeks; the *beyond* is recalled from its flight; what it is and what it ought to be are not separate but the same.

What distinguishes these two is more precisely this, that in the infinite series the *negative* is *outside* its terms which are present only *qua* parts of the *amount*. On the other hand, in the finite expression which is a ratio, the *negative* is immanent as the *reciprocal* determining of the sides of the ratio and this is an accomplished return-into-self, a self-related unity as a negation of the negation (*both* sides of the ratio are only *moments*), and consequently has *within it* the determination of infinity. Thus the usually so-called *sum*, the $\frac{2}{7}$ or $\frac{1}{1-a}$ is in fact a ratio; and this so-called *finite expression* is the truly *infinite expression*. The infinite *series*, on the other hand, is in truth a *sum;* its purpose is to represent in the form of a sum what is in itself a ratio, and the existing terms of the series are not terms of a ratio but of an aggregate. Furthermore, the series is in fact the *finite expression;* for it is the incomplete aggregate and remains essentially deficient. According to what is really present in it, it is a specific quantum,

but at the same time it is less than what it ought to be; and then, too, what it lacks is itself a specific quantum; this missing part is in fact that which is called infinite in the series, from the merely formal point of view that it is something lacking, a *non-being;* with respect to its content it is a finite quantum. Only what is actually present in the series, plus what is lacking, together constitute the amount of the fraction, the specific quantum which the series also *ought* to be but is not capable of being. The word *infinite*, even as used in infinite series, is commonly fancied to be something lofty and exalted; this is a kind of superstition, the superstition of the understanding; we have seen how, on the contrary, it indicates only a deficiency.

We may further remark that the existence of infinite series which cannot be summed is an external and contingent circumstance with respect to the form of series as such. They contain a higher kind of infinity than do those which can be summed, namely an incommensurability, or the impossibility of representing the quantitative ratio contained in them as a quantum, even in the form of a fraction; but the *form of series* as such which they have contains the same determination of spurious infinity that is present in the series capable of summation.

The terminological inversion we have just noticed in connection with the fraction and its expression as a series, also occurs when the *mathematical* infinite—not the one just named but the genuine infinite—is called the *relative* infinite, while the ordinary *metaphysical*—by which is understood the abstract, spurious infinite—is called *absolute*. But in point of fact it is this metaphysical infinite which is merely relative, because the negation which it expresses is opposed to a limit only in such a manner that this limit *persists* outside it and is not sublated by it; the mathematical infinite, on the contrary, has within itself truly sublated the finite limit because the *beyond* of the latter is united with it.

It is primarily in this sense, in which it has been demonstrated that the so-called sum or finite expression of an infinite series is rather to be regarded as the infinite expression, that Spinoza opposes the concept of true infinity to that of the spurious and illustrates it by examples. It will shed most light on his concept if I follow up this exposition with what he says on the subject.

He starts by defining the infinite as the absolute affirmation of any kind of natural existence, the finite on the contrary as a

determinateness, as a negation. That is to say, the absolute affirmation of an existence is to be taken as its relation to itself, its not being dependent on an *other*; the finite, on the other hand, is negation, a ceasing-to-be in the form of a *relation* to an *other* which begins *outside* it. Now the absolute affirmation of an existence does not, it is true, exhaust the notion of infinity; this implies that infinity is an affirmation, not as immediate, but only as restored by the reflection of the *other* into itself, or as negation of the negative. But with Spinoza, substance and its absolute unity has the form of an inert [*unbewegte*] unity, i.e. of a unity which is not self-mediated, of a fixity or rigidity in which the Notion of the negative unity of the self, i.e. subjectivity, is still lacking.

The mathematical example with which he illustrates the true infinite[1] is a space between two unequal circles which are not concentric, one of which lies inside the other without touching it. It seems that he thought highly of this figure and of the concept which it was used to illustrate, making it the motto of his *Ethics*. 'Mathematicians conclude', he says, 'that the inequalities possible in such a space are infinite, not from the infinite *amount* of parts, for its size is *fixed* and *limited* and I can assume larger and smaller such spaces, but because the nature of the fact surpasses every determinateness.' It is evident that Spinoza rejects that conception of the infinite which represents it as an amount or as a series which is not completed, and he points out that here, in the space of his example, the infinite is not beyond, but actually present and complete; this space is bounded, but it is infinite 'because the nature of the fact surpasses every determinateness', because the determination of magnitude contained in it cannot at the same time be represented as a quantum, or in Kant's words already quoted, the *synthesis* cannot be completed to form a (discrete) quantum. How in general the opposition of continuous and discrete quantum leads to the infinite, will be shown in detail in a later Remark. Spinoza calls the infinite of a series the infinite of the imagination; on the other hand, the infinite as self-relation he calls the infinite of thought, or *infinitum actu*. It is, namely, *actu, actually* infinite because it is complete and present within itself. Thus the series $0{\cdot}285714\ldots$ or $1 + a + a^2 + a^3 \ldots$ is the infinite merely of imagination or supposition; for it has no

[1] *Epist. XXIX.*

actuality, it definitely lacks something; on the other hand $\frac{2}{7}$ or $\frac{1}{1-a}$ is *actually* not only what the series is in its developed terms, but is, in addition, what the series lacks, what it only *ought to be*. The $\frac{2}{7}$ or $\frac{1}{1-a}$ is equally a finite magnitude like Spinoza's space enclosed between two circles, with its inequalities, and can like this space be made larger or smaller. But this does not involve the absurdity of a larger or smaller infinite; for this quantum of the whole does not concern the relation of its moments, *the nature of the fact*, i.e. the qualitative determination of magnitude; what is *actually present* in the infinite series is equally a finite quantum, but it is also still deficient. *Imagination* on the contrary stops short at quantum as such and does not reflect on the qualitative relation which constitutes the ground of the existing incommensurability.

The incommensurability which lies in Spinoza's example embraces in general the functions of curved lines and more precisely, leads to the infinite which mathematics has introduced with such functions, in general, with the *functions of variable magnitudes*. This infinite is the *genuine mathematical* qualitative infinite which Spinoza also had in mind. We shall now consider this determination here in detail.

First of all, as regards the category of *variability* which is accorded such importance and which embraces the magnitudes related in these functions, it is to be noted that these magnitudes are not supposed to be variable in the way that the two numbers 2 and 7 are in the fraction $\frac{2}{7}$: their place can equally well be taken by 4 and 14, 6 and 21, and by other numbers *ad infinitum* without altering the value of the fraction; and still more in $\frac{a}{b}$ can a and b be replaced by any arbitrary number without altering what $\frac{a}{b}$ is intended to express. Now in the sense that in the place, too, of x and y of a function, there can be put an infinite, i.e. inexhaustible, multitude of numbers, a and b are just as much variable magnitudes as the said x and y. The expression 'variable magnitudes' is therefore very vague and ill-chosen for those determinations of

magnitude whose interest and manner of treatment lie in something quite distinct from their mere variability.

In order to make clear wherein lies the true character of those moments of a function with which higher analysis is concerned, we must once more run through the stages to which we have already drawn attention. In $\frac{2}{7}$ or $\frac{a}{b}$, 2 and 7 are each independent determinate quanta and the relation is not essential to them; a and b likewise are intended to represent quanta which remain what they are even outside the relation. And further, $\frac{2}{7}$ and $\frac{a}{b}$ are each a fixed quantum, a quotient; the ratio constitutes an amount, the unit of which is expressed by the denominator and the amount of these units by the numerator, or conversely; even if 4 and 14, and so on, are substituted for 2 and 7, the ratio, also as a quantum, remains the same. But now in the function $\frac{y^2}{x} = p$, for example, this is essentially changed; here, it is true that x and y can stand for definite quanta, but only x and y^2 have a determinate quotient, not x and y. Hence not only are these *sides* of the ratio x and y, not any determinate quanta, but, secondly, their ratio is not a fixed quantum (nor is such a quantum meant as in the case of a and b), not a fixed quotient, but this quotient is, *as a quantum*, absolutely *variable*. But this is solely because x has a relation, not to y, but to the *square* of y. The relation of a magnitude to a *power* is not a *quantum*, but essentially a *qualitative* relation; the power-relation is the feature which is to be regarded as the fundamental determination. But in the function of the straight line $y = ax$, $\frac{y}{x} = a$ is an ordinary fraction and quotient; consequently this function is only *formally* a function of variable magnitudes, or x and y here are what a and b are in $\frac{a}{b}$, that is, they are not in that determination in which the differential and integral calculus considers them. On account of the *special* nature of the variable magnitudes in this mode of consideration, it would have been fitting to have introduced both a special name for them and other *symbols* than those generally used for *unknown quantities* in any finite equation, determinate or indeterminate; for there is an essential difference between those magnitudes and such quanta

which are merely unknown, but are in themselves completely determined or are a definite range of determinate quanta. It is, too, only because of a lack of awareness of what constitutes the peculiar interest of higher analysis and of what has led to the need for and invention of the differential calculus, that functions of the first degree and the equation of the straight line are themselves included in the treatment of this calculus; such formalism originates partly, too, in the mistake of imagining that the intrinsically correct demand for the *generalization* of a method has been fulfilled when the *specific* determinateness on which the need for the calculus is based is omitted, as if in this domain we were concerned only with *variable magnitudes*. A great deal of formalism would, indeed, have been avoided if it had been perceived that the calculus is concerned not with variable magnitudes as such but with *relations of powers*.

But there is still another stage where the peculiar character of the mathematical infinite becomes prominent. In an equation in which x and y are determined primarily by a power-relation, x and y as such are still supposed to signify quanta; now this significance is altogether and completely lost in the so-called *infinitesimal differences*. Dx, dy, are no longer quanta, nor are they supposed to signify quanta; it is solely in their relation to each other that they have any meaning, *a meaning merely as moments*. They are no longer *something* (something taken as a quantum), not finite differences; but neither are they *nothing;* not empty nullities. Apart from their relation they are pure nullities, but they are intended to be taken only as moments of the relation, as *determinations* of the differential co-efficient $\frac{dx}{dy}$.

In this concept of the infinite, the quantum is genuinely completed into a qualitative reality; it is posited as actually infinite; it is sublated not merely as this or that quantum but as quantum generally. But the quantitative determinateness *remains* as *element* of the principle of the quanta, or, as has also been said, the quanta remain *in their first concept*.

It is this concept which has been the target for all the attacks made on the fundamental determination of the mathematics of this infinite, i.e. of the differential and integral calculus. Failure to recognize it was the result of incorrect ideas on the part of mathematicians themselves; but it is the inability to justify the

object as *Notion* which is mainly responsible for these attacks. But mathematics, as we remarked above, cannot evade the Notion here; for, as mathematics of the infinite, it does not confine itself to the *finite* determinateness of its objects (as in ordinary mathematics, which considers and relates space and number and their determinations only according to their finitude); on the contrary, when it treats a determination taken from ordinary mathematics, it converts it into an identity with its opposite, e.g. converting a curved line into a straight one, the circle into a polygon, etc. Consequently, the operations which it allows itself to perform in the differential and integral calculus are in complete contradiction with the nature of merely finite determinations and their relations and would therefore have to be justified solely by the *Notion*.

Although the mathematics of the infinite maintained that these quantitative determinations are vanishing magnitudes, i.e. magnitudes which are no longer any particular quantum and yet are not nothing but are still a *determinateness* relatively to an other, it seemed perfectly clear that such an *intermediate state*, as it was called, between being and nothing does not exist. What we are to think of this objection and the so-called intermediate state, has already been indicated above in Remark 4 to the category of becoming. The unity of being and nothing is, of course, not a *state*; a state would be a determination of being and nothing into which these moments might be supposed to have lapsed only by accident, as it were, into a diseased condition externally induced through erroneous thinking; on the contrary, this mean and unity, the vanishing or equally the becoming is alone their *truth*.

Further, it has been said that what is infinite is not *comparable* as something greater or smaller; therefore there cannot be a relation between infinites according to orders or dignities of the infinite, although in the science of infinitesimals these distinctions do occur. Underlying this objection already mentioned is always the idea that here we are supposed to be dealing with *quanta* which are compared *as* quanta, that determinations which are no longer quanta no longer have any relationship to each other. But the truth is rather that that which has being *solely* in the ratio is not a quantum; the nature of quantum is such that it is supposed to have a completely indifferent existence apart from its ratio, and its difference from another quantum is supposed not to

concern its own determination; on the other hand the qualitative is what it is only in its distinction from an other. The said infinite magnitudes, therefore, are not merely comparable, but they exist only as moments of comparison, i.e. of the ratio.

I will adduce the most important definitions of this infinite which have been given in mathematics. From these it will be clear that the thought underlying them accords with the Notion developed here, but that the originators of the definitions did not establish the thought as Notion and found it necessary in the application to resort again to expedients which conflict with their better cause.

The thought cannot be more correctly determined than in the way Newton has stated it. I eliminate here those determinations which belong to the idea of motion and velocity (from which, mainly, he took the name of *fluxions*) because in them the thought does not appear in its proper abstraction but as concrete and mixed with non-essential forms. Newton explains that he understands by these fluxions not *indivisibles* (a form which was used by earlier mathematicians, Cavalieri[1] and others and which involves the concept of an intrinsically determinate quantum), but *vanishing divisibles;* also not sums and ratios of determinate parts but the *limits* (*limites*) of *sums* and *ratios*.[2] It may be objected that vanishing magnitudes do not have a *final ratio*, because the ratio before it vanishes is not final, and when it has vanished is no longer a ratio. But by the ratio of vanishing magnitudes is to be understood not the ratio *before which* and *after which* they vanish, but *with which* they vanish (*quacum evanescunt*). Similarly, the *first* ratio of nascent magnitudes is that *with which* they become.

Newton did what the scientific method of his time demanded, he only explained what was to be understood by an expression; but that such and such is to be understood by it is, properly speaking, a subjective presumption, or a historical demand, without any indication that such a concept is in itself absolutely necessary or that there is truth in it. However, what has been quoted shows that the concept put forward by Newton corresponds to the way in which infinite quantity resulted from the reflection of quantum

[1] Cavalieri, Francesco Bonaventura, 1598–1647, Professor of Mathematics at Bologna: *Geometria Indivisibilium Continuorum Nova*, 1635; *Exercitationes Geometricae*, 1647.

[2] *Princ. Mathem. Phil. Nat.* L.1. Lemma XI. Schol.

into itself in the exposition above. By magnitudes is understood magnitudes in their vanishing, i.e. which are no longer quanta; also, not ratios of determinate parts, but the *limits of the ratio.* The meaning is, therefore, that with the vanishing of the quanta individually, the sides of the ratio, there also vanishes the ratio itself in so far as it is a quantum; the limit of the quantitative ratio is that in which it both is and is not, or, more precisely, in which the quantum has vanished, with the result that the ratio and its sides are preserved, the former only as a qualitative relation of quantity and the latter similarly as qualitative moments of quantity. Newton goes on to add that from the fact that there are final ratios of vanishing magnitudes, it must not be inferred that there are final magnitudes, *indivisibles.* For this would mean a leap again from the abstract ratio to its sides as supposedly having an independent value of their own as indivisibles outside their relation, as something which would be a one, something devoid of any relation at all.

To prevent such a misunderstanding, he again points out that *final ratios* are not ratios of *final magnitudes*, but are limits to which the ratios of the magnitudes decreasing without limit are nearer than any *given*, i.e. finite, difference; the ratios, however, do not exceed these limits, for if they did they would become nullities. In other words, *final magnitudes* could have been taken to mean, as already said, indivisibles or ones. But the definition of the final ratio excludes the conception both of the indifferent one which is devoid of any relation, and of the finite quantum. If the required determination had been developed into the Notion of a quantitative determination which is purely a moment of the ratio, there would have been no need for the *decreasing without limit* into which Newton converts the quantum and which only expresses the progress to infinity, or for the determination of divisibility which no longer has any immediate meaning here.

As regards the preservation of the ratio in the vanishing of the quanta, there is found elsewhere, as in Carnot,[1] the expression that *by virtue of the law of continuity*, the vanishing magnitudes still retain the ratio from which they come, before they vanish.

[1] Carnot, Lazare Nicolas Marguerite, Count, 1753–1823, 'organizer of victory' in the republican armies, equally important as politician and soldier until his banishment in 1815, died at Magdeburg: *Réflexions sur la Métaphysique du Calcul Infinitésimal*, 1797.

This conception expresses the true nature of the matter, if the continuity of the quantum is not understood to be the continuity which it has in the infinite progress where the quantum is continued in its vanishing in such a manner that in its *beyond* there arises only a finite quantum again, only a fresh term of the series; but a *continuous* progress is always imagined as one in which values are passed through, values which are still finite quanta. On the other hand, where the transition is made into the true infinite it is the ratio that is continuous; so *continuous* is it, so completely is it preserved, that the transition may be said to consist solely in throwing into relief the pure ratio and causing the non-relational determination—i.e. that a quantum which is a side of the ratio is still a quantum outside this relation—to vanish. This purification of the quantitative ratio is thus analogous to grasping an empirical reality in terms of its Notion. The empirical reality is thereby raised above itself in such a way that its Notion contains the same characteristic features as it has itself, but these are grasped in their essentiality and are taken *into the unity* of the Notion in which they have lost their indifferent, Notion-less existence.

The other form of Newton's exposition of the magnitudes in question is equally interesting, namely, as generative magnitudes or principles. A *generated magnitude* (*genita*) is a product or quotient, such as a root, rectangle, square, also the sides of rectangles and squares—in general, a *finite magnitude*. 'Such a magnitude being considered as variable, increasing or decreasing in ceaseless motion and flux, he gives its momentary increments or decrements the name of moments. But these are not to be taken for particles of a definite magnitude (*particulae finitae*): such would not themselves be *moments* but magnitudes *generated* from moments. Rather are they to be understood as the nascent *principles* or *beginnings* of finite magnitudes.' Here the quantum is distinguished from itself: as a product or a real being [*Daseiendes*], and in its becoming (or as nascent), in its beginning and principle, that is to say, in its Notion, or, what is here the same thing, in its qualitative determination: in the latter the quantitative differences, the infinite increments or decrements, are only moments; only that which has becoming at its back has passed over into the indifference of determinate being and into externality, i.e. is quantum. But if on the one hand the philosophy of the

true Notion must acknowledge these determinations of the infinite with respect to increments or decrements, on the other hand it must be observed that the very forms of increments etc. fall *within* the category of immediate quantum and of the continuous progress to which we have referred; in fact the conceptions of increment, growth or increase of x by dx or i, and so on, are to be regarded as the fundamental vice in these methods—the permanent obstacle to disengaging the determination of the qualitative moment of quantity in its purity from the conception of the ordinary quantum.

The conception of *infinitesimals* which is implicit, too, in the increment or decrement itself, is much inferior to the above determinations. The nature of these magnitudes is supposed to be such that they may be *neglected*, not only in comparison with finite magnitudes, but also their higher orders in comparison with their lower, and even the products of several in comparison with a single one. With Leibniz, this demand to neglect is more strikingly prominent than with previous inventors of methods relating to these infinitesimals in which this call to neglect also occurs. It is chiefly this call to neglect which, along with a gain in facility, has given this calculus the appearance of inexactitude and express incorrectness in its method of procedure. Wolf has tried to make this neglect intelligible in his own way of popularizing things, i.e. by polluting the pure Notion and setting in its place incorrect sensuous conceptions. For example, he compares the neglect of infinitesimals of higher orders relatively to lower with the procedure of a surveyor who, in measuring the height of a mountain is no less accurate if meanwhile the wind has blown away a grain of sand from the top; or with the neglect of the height of houses or towers when calculating lunar eclipses.[1]

Even if ordinary common sense in fairness allows such inexactitude, all geometricians reject this conception. It is quite obvious that in the science of mathematics there cannot be any question of such empirical accuracy; mathematical measuring by operations of the calculus or by geometrical constructions and proofs is altogether different from land-surveying, from the measuring of empirical lines, figures etc. Besides, by comparing the result obtained by a strictly geometrical method with that

[1] *Element. Mathes. univ. Tom. I. El. Analys. math. P. II. C.I. s. Schol.*

obtained by the method of infinite differences, analysts demonstrate that the one is the same as the other and that there is absolutely no question of a greater or lesser degree of exactness. And it is self-evident that an absolutely exact result could not emerge from an inexact method. Yet on the other hand again, the *method itself* cannot do without this omission of what is regarded as insignificant, despite its protestations against the way this omission is justified. And this is the difficulty which engages the efforts of the analysts to make intelligible and to remove the inherent inconsistency.

It is especially Euler's[1] conception of the matter which must be cited here. He adopts the general Newtonian definition and insists that the differential calculus considers the *ratios of the increments* of a magnitude, but that the *infinite difference* as such is to be considered as wholly *nil.*[2] How this is to be understood is clear from the foregoing; the infinite difference is a nil only of quantum, not a qualitative nil, but as a nil of quantum it is a pure moment of the ratio only. It is not a *quantitative* difference; but for that reason it is, on the one hand, altogether wrong to speak of those moments which are called infinitesimals, also as increments or decrements and as *differences*. This description implies that something is *added to* or *subtracted from* the initially given finite magnitude, that a subtraction or addition, an arithmetical, external operation takes place. But it is to be noticed that the transition of the function of the variable magnitude into its differential is of a quite different nature; as we have made clear, it is to be considered as a reduction of the finite function to the qualitative relation of its quantitative determinations. On the other hand, the error becomes obvious when it is said that the increments by themselves are zeros, that only their ratios are considered; for a zero no longer has any determinateness at all. This conception then, does get as far as the negative of the quantum and gives definite expression to it, but at the same time it does not grasp this negative in its positive significance of qualitative determinations of quantity which, if they were torn out of the ratio and regarded as quanta, would be only zeros.

[1] Euler, Leopold, 1707–1783, Professor at St Petersburg, at Berlin, and again at St Petersburg: *Introductio in Analysin Infinitorum*, 1748; *Institutiones Calculi Differentialis*, 1755; *Institutiones Calculi Integralis*, 1768–1794.

[2] *Institut. Calc. different. P.I.C. III.*

The opinion of Lagrange[1] on the idea of limits or final ratios is that although one can well imagine the ratio of two magnitudes so long as they remain finite, this ratio does not present any clear and definite concept to the intellect as soon as its terms become simultaneously zero. And the understanding must, indeed, transcend this merely negative side on which the terms of the ratio are quantitatively zero, and must grasp them positively, as qualitative moments. But we cannot regard as satisfactory Euler's further remarks[2] with regard to this conception of his in which he tries to show that two so-called infinitesimals which are supposed to be nothing else but zeros, nevertheless stand in a relation to each other, for which reason they are denoted by symbols other than zero. He tries to base this on the difference between the arithmetical and geometrical ratio: in the former, we have an eye to the difference, in the latter, to the quotient, so that although in the former there is no difference between two zeros, this is not so in the geometrical ratio; if $2 : 1 = 0 : 0$ then from the nature of proportion it follows that, because the first term is twice as great as the second, the third is also twice as great as the fourth; thus according to proportion, $0 : 0$ is to be taken as the ratio of $2 : 1$. Even in common arithmetic $n \cdot 0 = 0$ and therefore $n : 1 = 0 : 0$. But it is just because $2 : 1$ or $n : 1$ is a relation of quanta that there cannot be any corresponding ratio or expression of $0 : 0$.

I refrain from citing any further instances since those already considered show clearly enough that the genuine Notion of the infinite is, in fact, implied in them, but that the specific nature of that Notion has not been brought to notice and grasped. Consequently, in the actual application of the method of infinitesimals, the genuine Notion of the infinite cannot exercise any influence; on the contrary, there is a return of the finite determinateness of quantity and the operation cannot dispense with the conception of a quantum which is merely *relatively small.* The calculus makes it necessary to subject the so-called infinitesimals to ordinary arithmetical operations of addition and so on, which are based on the nature of finite magnitudes, and therefore to

[1] Lagrange, Jos. Louis, 1736–1812, Euler's successor at Berlin, then Professor at the *École Polytechnique* in Paris: *Théorie des Fonctions Analytiques*, 1797, *Introduction*.

[2] *op. cit.*, Section 84 *sqq.*

regard them momentarily as finite magnitudes and to treat them as such. It is for the calculus to justify its procedure in which it first brings them down into this sphere and treats them as increments or differences, and then neglects them as quanta after it had just applied forms and laws of finite magnitudes to them.

I will proceed to cite the main features of the attempts of the geometricians to remove these difficulties.

The older analysts had little scruples in the matter, but the moderns directed their efforts mainly towards bringing the differential calculus back to the evidence of a *strictly geometrical method* and in it to attain to the *rigour of the proofs of the ancients* (Lagrange's expressions) in mathematics. But since the principle of infinitesimal analysis is of a higher nature than the principle of the mathematics of finite magnitudes, that kind of *evidence* had perforce to be dispensed with, just as philosophy, too, cannot lay claim to that obviousness which belongs to the natural sciences, e.g. natural history—and just as eating and drinking are reckoned a more intelligible business than thinking and understanding. Accordingly, we shall deal only with the efforts to attain to the rigour of proof of the ancients.

Some have attempted to dispense altogether with the concept of the infinite, and without it to achieve what seemed to be bound up with its use. Lagrange speaks, e.g., of the method devised by Landen,[1] saying that it is purely analytical and does not employ infinitesimal differences, but starts with different values of variable magnitudes and subsequently equates them. He also gives it as his opinion that in this method, the differential calculus loses its own peculiar advantages, namely simplicity of method and facility of operation. This is, indeed, a procedure which in some measure corresponds to the starting-point of Descartes' tangential method of which detailed mention will be made later. This much, we may remark here, is generally evident, that the general procedure in which different values of variable magnitudes are assumed and subsequently equated, belongs to another department of mathematical treatment than that to which the method of the differential calculus itself belongs; and that the peculiar nature of the simple relation (to be considered in detail further on) to which its actual, concrete determination reduces, namely,

[1] Landen, John, English mathematician, 1719–1790: *Mathematical Lucubrations*, 1755, etc.

of the derived function to the original, is not brought into prominence.

The earlier of the moderns, Fermat,[1] Barrow,[2] and others for example, who at first used infinitesimals in that application which was subsequently developed into the differential and integral calculus, and then Leibniz, too, and those following him including Euler, always frankly believed that they were entitled to omit the products of infinitesimal differences and their higher powers, solely on the ground that they *vanish relatively* to the lower order. This is for them the sole basis of the fundamental principle, namely the determination of that which is the differential of a product or a power, *for the entire theoretical teaching reduces to this*. The rest is partly the mechanism of development and partly application, in which however as we shall later on see, the more important, or rather the sole, interest is to be found. With respect to the present topic, we need only mention here what is elementary, that on the same ground of *insignificance*, the cardinal principle adopted in relation to curves is that the elements of the curves, namely the *increments* of abscissa and ordinate, have the *relation* to each other of *subtangent* and *ordinate*; for the purpose of obtaining similar triangles, the arc which forms the third side of a triangle to the two increments of the *characteristic* triangle (as it rightly used to be called), is regarded as a straight line, as part of the tangent and one of the increments therefore as reaching to the tangent. By these assumptions those determinations are, on the one hand, raised above the nature of finite magnitudes, but on the other hand, a method which is valid only for finite magnitudes and which does not permit the omission of anything on the ground of insignificance, is applied to moments now called infinitesimal. With such a mode of procedure, the difficulty which encumbers the method remains in all its starkness.

We must mention here a remarkable procedure of Newton[3]—the invention of an ingenious device to remove the arithmetically incorrect omission of the products of infinitesimal differences or higher orders of them in the finding of differentials. He finds the differentials of products—from which the differentials of quotients,

[1] Fermat, Pierre de, 1601–1665: *Varia Opera Mathematica*, 1679.

[2] Barrow, Isaac, 1630–1677, Professor at Cambridge: *Lectiones Geometricae*, 1669; *Lectiones Opticae*, 1674.

[3] *Princ. Math. phil. nat.* Lib. II Lemma II, after Propos. VII.

powers, etc., can then be easily derived—in the following way. The product of x and y, when each is taken as reduced by *half* of its infinitesimal difference, becomes $xy - \frac{xdy}{2} - \frac{ydx}{2} + \frac{dxdy}{4}$; but if x and y are made to increase by the same amount, it becomes $xy + \frac{xdy}{2} + \frac{ydx}{2} + \frac{dxdy}{4}$. Now when the first product is subtracted from the second, $ydx + xdy$ remains as a surplus and this is said to be the surplus of the increase by a whole dx and dy, for this increase is the difference between the two products; it is therefore the differential of xy. Clearly, in this procedure, the term which forms the chief difficulty, the product of the two infinitesimal differences, cancels itself out. But in spite of the name of Newton it must be said that such an operation although very elementary, is incorrect; it is not true that $\left(x + \frac{dx}{2}\right)\left(y + \frac{dy}{2}\right) - \left(x - \frac{dx}{2}\right)\left(y - \frac{dy}{2}\right) = (x + dx)(y + dy) - xy$. It can only have been the need to establish the all-important fluxional calculus which could bring a Newton to deceive himself with such a proof.

Other forms which Newton employed in the derivation of differentials are bound up with concrete meanings of the elements and their powers, meanings relating to motion. About the use of the *serial form* which also characterizes his method, it suggests itself to say that it is always possible to obtain the required degree of accuracy by adding more terms and that the omitted terms are *relatively insignificant*, in general, that the result is only an *approximation;* though here too he would have been satisfied with this ground for omission as he is in his method of solving equations of higher degree by approximation, where the higher powers arising from the substitution in the given equation of any ascertained, still inexact term, are omitted on the crude ground of their relative smallness.[1]

The error into which Newton fell in solving a problem by omitting essential, higher powers, an error which gave his opponents the occasion of a triumph of their method over his, and the true origin of which has been indicated by Lagrange in his recent investigation of it[2] demonstrates the formalism and

[1] See Lagrange: *Équations Numériques*, p. 125.
[2] *Théorie des fonct. analyt.* 3me P., Ch. IV.

uncertainty which still prevailed in the use of this instrument. Lagrange shows that Newton made the mistake because he omitted the term of the series containing that power on which the specific problem turned. Newton had kept to the formal, superficial principle of omitting terms on account of their relative smallness. For example, it is well known that in mechanics the terms of the series in which the function of a motion is developed are given a *specific meaning*, so that the first term or the first function refers to the moment of velocity, the second to the accelerating force and the third to the resistance of forces. Here, then, the terms of the series are not to be regarded merely as *parts* of a sum, but rather as *qualitative moments of a whole determined by the concept*. In this way, the omission of the rest of the terms belonging to the spuriously infinite series acquires an altogether different meaning from omission on the ground of their relative smallness.[1] The error in the Newtonian solution

[1] Both considerations are found set simply side by side in the application by Lagrange of the theory of functions to mechanics in the chapter on rectilinear motion (*Théorie des fonct. 3me P. Ch. I, art. 4*). The space passed through, considered as a function of the time elapsed, gives the equation $x = ft$; this, developed as $f(t + \delta)$ gives $ft + \delta f't + \frac{\delta 2}{2} f''t +$, etc.

Thus the space traversed in the period of time is represented in the formula as $= \delta f't + \frac{\delta 2}{2} f''t + \frac{\delta 3}{2 \cdot 3} f'''t +$, etc. The motion by means of which this space has been traversed is (it is said) *therefore*—i.e. because the analytical development gives several, in fact infinitely, many terms—*composed* of various partial motions, of which the spaces corresponding to the time will be $\delta f't$, $\frac{\delta^2}{2} f''t$, $\frac{\delta^3}{2 \cdot 3} f'''dt$, etc. The first partial motion is, in known motion, the formally uniform one with a velocity designated by $f't$, the second is uniformly accelerated motion derived from an accelerative force proportional to $f''t$. 'Now since the remaining terms do not refer to any simple known motion, it is not necessary to take them specially into account and we shall show that they may be abstracted from in determining the motion at the beginning of the point of time.' This is now shown, but of course only by *comparing* the series *all* of whose terms belonged to the determination of the magnitude of the space traversed in the period of time, with the equation given in *art.* 3 for the motion of a falling body, namely $x = at + bt^2$ in which only these two terms occur. But this equation has itself received this form only because the *explanation given* to the terms produced by the *analytical development* is presupposed; this presupposition is that the uniformly accelerated motion is *composed* of a formally uniform motion continued with the velocity attained in the preceding period of time, and of an increment (the a in $s = at^2$, i.e. the empirical co-efficient) which is

arose, *not* because terms of the series were neglected only as *parts of a sum*, but because the *term containing the qualitative determination*, which is the essential point, was ignored.

In this example, the procedure is made to depend on the qualitative *meaning*. In this connection the general assertion can at once be made that the whole difficulty of the principle would be removed if the *qualitative* meaning of the principle were stated and the operation were made to depend on it—in place of the formalism which links the determination of the differential only to that which gives the problem its *name*, to the *difference* as such between a function and its variation after its variable magnitude has received an *increment*. In this sense, it is obvious that the differential of x^n is completely exhausted by the first term of the series which results from the expansion of $(x + dx)^n$. Thus the omission of the rest of the terms is not on account of their relative smallness; and so there is no assumption of an inexactitude, an error or mistake which could be compensated or rectified by another error—a point of view from which Carnot in particular justifies the ordinary method of the infinitesimal calculus. Since what is involved is not a *sum* but a *relation*, the differential is completely given *by the first term*; and where further terms, the differentials of higher orders, are required, their determination involves not the continuation of a series as a *sum*, but the *repetition* of one and the same *relation* which alone is desired and which is thus already *completely* given in the *first term*. The need for the *form* of a *series*, its summation and all that is connected with it, must then be wholly separated from the said *interest of the relation*.

The explanations of the methods of infinitesimal magnitudes given by Carnot, contain a most lucid exposition of what is essential in the ideas referred to above. But in passing to the practical application itself, there enter more or less the usual ideas about the infinite smallness of the omitted terms *relatively* to the others. He justifies the method, not by the nature of the procedure itself, but by the fact that the *results* are correct, and by the *advantages* of a simplification and shortening of the calculus which follow the introduction of *imperfect* equations, as he calls

ascribed to the force of gravity—a distinction which has no existence or basis whatever in the nature of the thing itself, but is only the falsely physicalized expression of what issues from the assumed analytical treatment.

them, i.e. those in which such an arithmetically incorrect omission has occurred.

Lagrange, as is well known, reverted to Newton's original method, that of series, in order to be relieved of the difficulties inherent in the idea of the infinitely small and in the method of first and final ratios and limits. The advantages of his functional calculus as regards precision, abstraction and generality, are sufficiently recognized; we need mention only what is pertinent here, that it rests on the fundamental principle that the difference, without becoming zero, can be assumed so small that each term of the series is greater than the sum of all the following terms. This method, too, starts from the categories of *increment* and *difference* of the function, the variable magnitude of which receives the *increment*, thereby bringing in the troublesome series of the original function; also in the sequel the terms to be omitted are considered only as constituting a *sum*, while the reason for omitting them is made to consist in the relativity of their *quantum*. And so here, too, on the one hand, the principle of the omission is not brought back to the point of view exemplified in some applications, where (as was remarked above) terms of the series are supposed to have a specific *qualitative* significance, and terms are neglected *not* because of their quantitative insignificance but because they are not qualitatively significant; and then, on the other hand, the omission itself has no place in the essential point of view which, as regards the so-called differential coefficient, only becomes specifically prominent with Lagrange, in the so-called *application* of the calculus, as will be more fully considered in the following remark.

The demonstrated *qualitative character as such* of the form of magnitude here under discussion in what is called the infinitesimal, is found most directly in the category of *limit of the ratio* referred to above and the carrying out of which in the calculus has been developed into a characteristic method. Lagrange criticizes this method as lacking ease in application and he claims that the expression *limit* does not present any definite idea; this second point we will take up here and examine more closely what is stated about its analytical meaning. Now the idea of limit does indeed imply the true category of the *qualitatively* determined relation of variable magnitudes above-mentioned; for the forms of it which occur, dx and dy, are supposed to be taken simply

and solely as moments of $\frac{dy}{dx}$, and $\frac{dy}{dx}$ itself must be regarded as a single indivisible symbol. That the mechanism of the calculus, especially in its application, thus loses the advantage it derived from the separation of the sides of the differential coefficient, this we will pass over here. Now the said limit is to be *limit* of a given function; it is to assign to this function a certain value determined by its mode of derivation. But with the mere category of limit we should not have advanced beyond the scope of this Remark, which is to demonstrate that the infinitely small which presents itself in the differential calculus as dx and dy, does not have merely the negative, empty meaning of a non-finite, non-given magnitude, as when one speaks of 'an infinite multitude', 'to infinity', and the like, but on the contrary has the specific meaning of the qualitative nature of what is quantitative, of a moment of a ratio as such. This category, however, merely as such, still has no relation to that which is a given function and does not itself enter into the treatment of such a function or into the use to be made of that determination; thus the idea of limit, too, confined to this its demonstrated character, would lead nowhere. But the very expression 'limit' implies that it is a limit of *something*, i.e. that it expresses a certain value which lies in the function of a variable magnitude; and we must examine the nature of this concrete rôle. It is supposed to be the limit of the *ratio* between the two *increments* by which the two variable magnitudes connected in an equation (one of which is regarded as a function of the other), are supposed to have been *increased;* the increase is taken here as quite undetermined and so far no use is made of the infinitely small. But the way in which this limit is found involves the same inconsistencies as are contained in the other methods. This way is as follows: if $y = fx$, then when y becomes $y + k$, fx is to change into $fx + ph + qh^2 + rh^3$ and so on; thus $k = ph + qh^2$, etc., and $\frac{k}{h} = p + qh + rh^2$, etc. Now if k and h vanish, the right-hand side of the equation also vanishes with the exception of p; now p is supposed to be the limit of the ratio of the two increments. It is clear that while h, as a quantum, is equated with o, $\frac{k}{h}$ nevertheless is not at the same time equal to $\frac{o}{o}$ but is supposed still to remain a ratio. Now the idea of *limit* is supposed to have

the advantage of avoiding the inconsistency here involved; p is, at the same time, supposed to be not the actual ratio, which would be $\frac{o}{o}$, but only that specific value to which the ratio can *infinitely approximate*, i.e. can approach so near that the difference can be smaller than any given difference. The more precise meaning of *approximation* with respect to the terms which are supposed really to approach each other will be considered later. But that a quantitative difference, the definition of which is that it not only *can*, but *shall* be smaller than any given difference, is no longer a quantitative difference, this is self-evident, as self-evident as anything can be in mathematics; but we still have not got away from $\frac{dy}{dx} = o$. If on the other hand $\frac{dy}{dx} = p$, i.e. is assumed to be a definite quantitative ratio as in fact it is, then conversely there is a difficulty about the presupposition which equates h with o, a presupposition which is indispensable for obtaining the equation $\frac{k}{h} = p$. But if it be granted that $\frac{k}{h} = o$, (and when $h = o$, k is in fact automatically $= o$, for k, the increment of y, depends entirely on the existence of the increment h), then the question would arise, what p—which is a quite definite quantitative value—is supposed to be. To this there is at once an obvious answer, the simple, meagre answer that it is a coefficient derived in such and such a way—the first function, derived in a certain specific manner, of an original function. If we content ourselves with this—and Lagrange did, in fact, do so in practice—then the general part of the science of the differential calculus, and straightway this one particular form of it called the *theory of limits* would be rid of the increments and of their infinite or arbitrary smallness—spared too, the difficulty of getting rid again of all the terms of a series other than the first, or rather only the coefficient of the first, which inevitably follow on the introduction of these increments; in addition it would also be purged of those formal categories connected with them, especially of the infinite, of infinite approximation and, too, the categories, here equally empty, of continuous magnitude[1] which, moreover,

[1] The category of continuous or fluent magnitude enters with the consideration of the external and empirical variation of magnitudes—which are brought by an equation into the relation in which one is a function of the other; but

like *nisus*, becoming, occasion of a variation, are deemed necessary. But it would then be required to show what other *meaning* and *value p* has—apart from the meagre definition, quite adequate for the theory, that it is simply a function derived from the expansion of a binomial—i.e. what *relationships* it embodies and what further *use* can be made of them mathematically; this will be the subject of Remark 2. But first we shall proceed to discuss the confusion which the conception of *approximation* currently used in expositions of the calculus, has occasioned in the understanding of the true, qualitative determinateness of the relation which was the primary interest concerned.

It has been shown that the so-called infinitesimals express the vanishing of the sides of the ratio as quanta, and that what remains is their quantitative relation solely as qualitatively determined; far from this resulting in the loss of the qualitative relation, the fact is that it is just this relation which results from the conversion of finite into infinite magnitudes. As we have seen, it is in this that the entire nature of the matter consists. Thus in the *final ratio*, for example, the quanta of abscissa and ordinate vanish; but the sides of this ratio essentially remain, the one an element of the ordinate, the other an element of the abscissa. This vanishing being represented as an infinite approximation, the previously distinguished ordinate is made to pass over into the other ordinate, and the previously distinguished abscissa into the other abscissa; but essentially this is not so, the ordinate does not pass over into the abscissa, neither does the abscissa pass into the ordinate. To continue with this example of variable magnitudes, the element

since the scientific object of the differential calculus is a *certain relation* (usually expressed by the differential co-efficient), the specific nature of which may equally well be called a *law*, the mere continuity is a heterogeneous aspect of this specific nature, and besides is in any case an abstract and here empty category seeing that nothing whatever is said about the law of continuity. Into what formal definitions one may be led in these matters can be seen from the penetrating exposition by my respected colleague, Prof. Dirksen (Dirksen, Enno Heeren, 1792–1850, Professor of Mathematics at Berlin: *Analytische Darstellung der Variationsrechnung*, 1823) of the fundamental determinations used in the deduction of the differential calculus, which forms an appendix to the criticism of some recent works on this science (*Jahrb. f. wissensch. Kritik*, 1827, No. 153 *sqq.*). The following definition is actually quoted (*op. cit.*, p. 1251): 'A continuous magnitude, a continuum, is any magnitude thought of as in a state of becoming such that this becoming takes place not by leaps but by an uninterrupted progress.' This is surely tautologically the same as what was to be defined.

of the ordinate is not to be taken as the *difference of one ordinate from another ordinate*, but rather as the difference or *qualitative* determination of magnitude relatively to the *element of the abscissa*; the principle of the one variable magnitude relatively to that of the other is in reciprocal relation with it. The difference, as no longer a difference of finite magnitudes, has ceased to be manifold within itself; it has collapsed into a simple intensity, into the determinateness of one qualitative moment of a ratio relatively to the other.

This is the nature of the matter but it is obscured by the fact that what has just been called an element, for example, of the ordinate, is grasped as a difference or increment in such a way that it is only the difference between the quantum of one ordinate and the quantum of another ordinate. And so the limit here does not have the meaning of ratio; it counts only as the final value to which another magnitude of a similar kind continually approximates in such a manner that it can differ from it by as little as we please, and that the final *ratio* is a ratio of *equality*. The infinite difference is thus the fluctuation of a difference of one quantum from another quantum, and the qualitative nature according to which dx is essentially *not* a determination of the ratio relatively to x, but to dy, comes to be overlooked. Dx^2 is permitted to vanish relatively to dx, but even more does dx vanish relatively to x; but this means in truth: it has a *relation only to dy*. In such expositions, geometricians are mainly concerned to make intelligible the *approximation* of a magnitude to its limit and to keep to this aspect of the difference of quantum from quantum, how it is no difference and yet still is a difference. But all the same, approximation is a category which of itself says nothing and explains nothing; dx already has approximation behind it; it is neither near nor nearer; and 'infinitely near', itself means the negation of nearness and approximation.

Now since this implies that the increments or infinitesimals have been considered only from the side of the quantum which vanishes in them, and only as a limit, it follows that they are grasped as *unrelated* moments. From this would follow the inadmissible idea that it is allowed in the final ratio to equate, say abscissa and ordinate, or even sine, cosine, tangent, versed sine, and what not. This idea seems at first to prevail when the arc is treated as a tangent; for the *arc*, too, is certainly *incommensurable* with the *straight line*, and its element is, in the first

place, of another *quality* than the element of the straight line. It seems even more absurd and inadmissible than the confusing of abscissa, ordinate, versed sine, cosine, etc., when *quadrata rotundis*, when part of an arc, even though an infinitely small part, is taken to be a part of the tangent and so treated as a straight line. However, this treatment differs essentially from the confusion we have decried; it is justified by the circumstance that in the triangle which has for its sides the element of an arc and the elements of its abscissa and ordinate, the *relation is the same* as if this element of the arc were the element of a straight line, of the tangent; the *angles* which constitute the *essential relation*, i.e. that which remains to these elements when abstraction is made from the finite magnitudes belonging to them, are the same. This can also be expressed as the transition of straight lines which are infinitely small, into curved lines, and their relation in their infinity as a relation of curves. Since, according to its definition, a straight line is the *shortest* distance between two points, its difference from the curved line is based on the determination of *amount*, on the *smaller* amount of what is differentiated in this manner, a determination, therefore, of a *quantum*. But this determination vanishes in the line when it is taken as an intensive magnitude, as an infinite moment, as an element, and with it, too, its difference from the curved line which rested merely on the difference of quantum. As infinite, therefore, the straight line and arc no longer retain any quantitative relation nor consequently, on the basis of the assumed definition, any qualitative difference from each other either; on the contrary, the former passes into the latter.

Analogous, although also distinct from, the equating of heterogeneous forms is the assumption that *infinitely small parts* of the same whole are *equal* to each other; an assumption in itself indefinite and completely indifferent, but which, applied to an object heterogeneous within itself, i.e. an object whose quantitative determination is essentially non-uniform, produces the peculiar inversion contained in that proposition of higher mechanics which states that infinitely small parts of a curve are traversed in *equal*, infinitely small times in a *uniform* motion, inasmuch as this is asserted of a motion in which in equal *finite*, i.e. existent, parts of time, *finite*, i.e. existent, *unequal* parts of the curve are traversed, of a motion therefore which exists as non-uniform and is assumed as such. This proposition is the expression in words of what is

supposed to be the significance of an analytical term obtained in the above-mentioned development of the formula relating to a motion which is non-uniform but otherwise conforms to a law. Earlier mathematicians sought to express in words and propositions and to exhibit in geometrical tables the results of the newly invented infinitesimal calculus (which moreover always had to do with concrete objects), chiefly in order to use them for theorems susceptible of the ordinary method of proof. The terms of a mathematical formula into which analytical treatment resolved the *magnitude* of the object, of motion, for instance, acquired there an *objective* significance, such as velocity, force of acceleration, and so on; in accordance with this meaning they were supposed to furnish correct propositions, physical laws; their objective connections and relations, too, were supposed to be determined in accordance with the analytical connection. A particular example is that in a uniformly accelerated motion there is supposed to exist a special velocity proportional to the times, but that to this velocity there constantly accrues an increment from the force of gravity. In the modern, analytical form of mechanics such propositions are put forward simply as results of the calculus, without enquiry whether by themselves and in themselves they have a *real* significance, i.e. one to which there is a corresponding physical existence and whether such meaning can be demonstrated. The difficulty of making intelligible the connection of such forms when they are taken in the real meaning alluded to, for example the transition from said simply uniform velocity to a uniformly accelerated velocity, is held to be completely eliminated by the analytical treatment in which such connection is a simple result of the authority now established once and for all of the operations of the calculus. It is announced as a triumph of science that by means of the calculus alone, laws are found *transcending experience*, that is, propositions about existence which have no existence. But in the earlier, still naïve period of the infinitesimal calculus, the aim was to assign to those forms and propositions represented in geometrical diagrams a real meaning of their own and to make that meaning plausible, and to apply the forms and propositions bearing such meaning in the proof of the main propositions concerned.[1]

[1] See Newton's proof of his fundamental proposition of the theory of gravitation in *Princ. mathem. philosophiae naturalis*, lib. I, Sect. II, Prop. I, and

It cannot be denied that in this field much has been accepted as proof, especially with the aid of the nebulous conception of the infinitely small, for no other reason than that the result was always already known beforehand, and that the proof which was so arranged that the result did emerge, at least produced the illusion of a framework of proof, an illusion which was still preferred to mere belief or knowledge from experience. But I do not hesitate to regard this affectation as nothing more than mere jugglery and window-dressing, and I include in this description even Newton's proofs, especially those belonging to what has just been quoted, for which Newton has been extolled to the skies and exalted above Kepler, namely that *he* demonstrated mathematically what Kepler had discovered *merely empirically*.

The empty scaffolding of such proofs was erected in order to prove physical laws. But mathematics is altogether incapable of proving quantitative determinations of the physical world in so far as they are laws based on the *qualitative nature* of the moments [of the subject matter]; and for this reason, that this science is not philosophy, does *not start from the Notion*, and therefore the qualitative element, in so far as it is not taken lemmatically from experience, lies outside its sphere. The desire to uphold the honour of mathematics, that all its propositions ought to be *rigorously proved*, has often caused it to forget its limits; thus it seemed against its honour to acknowledge simply experience as the source and sole proof of empirical propositions. Consciousness has since then developed a more instructed view of the matter; so long, however, as consciousness is not clearly aware of the distinction between what is mathematically demonstrable and what can come only from another source, between what are only terms of an analytical expansion and what are physical existences, scientific method cannot be developed into a rigorous and pure attitude in this field. Without doubt, however, the same justice will be done to that framework of Newtonian proof as was done to another baseless and artificial Newtonian structure of optical experiments and conclusions derived from them. Applied mathematics is still full of a similar concoction of experiment and

cf. Schubert's *Astronomy*—1st ed., vol. III. B, Section 20, where it is admitted that the position is not *exactly* as Newton assumes, i.e. at that point which is the nerve of the proof. (Schubert, Friedrich Theodor von, 1758–1825, Director of the Observatory at St Petersburg: *Lehrbuch der theoretischen Astronomie*, 1798; *Populäre Astronomie*, 3 vols., 1804–1810.)

reflection; but just as one part after another of Newtonian optics long since began to be ignored in practice by the science—with the inconsistency however that all the rest although in contradiction was allowed to stand—so, too, it is a fact that already some of those illusory proofs have fallen into oblivion or have been replaced by others.

Remark 2: *The Purpose of the Differential Calculus Deduced from its Application*

In the previous Remark we considered on the one hand the specific nature of the notion of the infinitesimal which is used in the differential calculus, and on the other the basis of its introduction into the calculus; both are abstract determinations and therefore in themselves also easy. The so-called *application*, however, presents greater difficulties, but also the more interesting side; the elements of this *concrete* side are to be the object of this Remark. The whole method of the differential calculus is complete in the proposition that $dx^n = nx^{n-1}dx$, or $\frac{f(x+i)-fx}{i} = P$, that is, is equal to the *coefficient* of the first term of the binomial $x + d$, or $x + i$, developed according to the powers of dx or i. There is no need to learn anything further: the development of the next forms, of the differential of a product, of an exponential magnitude and so on, follows mechanically; in little time, in half an hour perhaps—for with the finding of the differential the converse the finding of the original function from the differential, or integration, is also given—one can be in possession of the whole theory. What takes longer is simply the effort to understand, to make intelligible, how it is that, after having so easily accomplished the *first stage* of the task, the finding of the said differential, analytically, i.e. purely arithmetically, by the expansion of the function of the variable after this has received the form of a binomial by the addition of an increment; how it is that the *second stage* can be correct, namely the omission of all the terms except the first, of the series arising from the expansion. If all that were required were only this coefficient, then with its determination all that concerns the theory would, as we have said, be settled and done with in less than half an hour and the omission of the further terms of the series (with the determination of the first function, the determination of the second, third, etc., is also

accomplished) far from causing any difficulty, would not come into question since they are completely irrelevant.

We may begin by remarking that the method of the differential calculus shows on the face of it that it was not invented and constructed for its own sake. Not only was it not invented for its own sake as another mode of analytical procedure; on the contrary, the arbitrary omission of terms arising from the expansion of a function is absolutely contrary to all mathematical principles, it being arbitrary in the sense that the *whole* of this development is nevertheless assumed to belong *completely* to the matter in hand, this being regarded as the *difference* between the developed function of a variable (after this has been given the form of a binomial) and the original function. The need for such a mode of procedure and the lack of any internal justification at once suggest that the origin and foundation must lie elsewhere. It happens in other sciences too, that what is placed at the beginning of a science as its elements and from which the principles of the science are supposed to be derived is not self-evident, and that it is rather in the sequel that the *raison d'être* and proof of those elements is to be found. The course of events in the history of the differential calculus makes it plain that the matter had its origin mainly in the various so-called tangential methods, in what could be considered ingenious devices; it was only later that mathematicians reflected on the nature of the method after it had been extended to other objects, and reduced it to abstract formulae which they then also attempted to raise to the status of *principles*.

We have shown that the specific nature of the notion of the so-called infinitesimal is the *qualitative* nature of determinations of quantity which are related to each other primarily as quanta; to this was linked the empirical investigation aimed at demonstrating the presence of this specific nature in the existing descriptions and definitions of the infinitesimal in so far as this is taken as an infinitesimal difference and the like. This was done only in the interest of the abstract nature of the notion as such; the next question would be as to the nature of the transition from this to the mathematical formulation and application. To this end we must first pursue our examination of the theoretical side, the specific nature of the notion, which will not prove wholly unfruitful in itself; we must then consider the relation of the theoretical side to its application; and in both cases we must

demonstrate, so far as it is relevant here, that the general conclusions are at the same time adequate to the purpose of the differential calculus and to the way in which the calculus brings about its results.

First, it is to be remembered that the mathematical form of the determinateness of the notion under discussion has already been stated in passing. The specifically qualitative character of quantity is first indicated in the quantitative *relation* as such; but it was already asserted in anticipation when demonstrating the so-called kinds of reckoning (see the relative Remark), that it is the *relation of powers* (still to be dealt with in its proper place) in which number, through the equating of the moments of its Notion, unit and amount, is posited as returned into itself, thereby receiving into itself the moment of infinity, of being-for-self, i.e. of being self-determined. Thus, as we have already said, the express qualitative nature of quantity is essentially connected with the forms of powers, and since the specific interest of the differential calculus is to operate with qualitative forms of magnitude, its own peculiar subject matter must be the treatment of forms of powers, and the whole range of problems, and their solutions, show that the interest lies solely in the treatment of determinations of powers as such.

This foundation is important and at once puts in the forefront something definite in place of the merely formal categories of variable, continuous or infinite magnitudes or even of functions generally; yet it is still too general, for other operations also have to do with determinations of powers. The raising to a power, extraction of a root, treatment of exponential magnitudes and logarithms, series, and equations of higher orders, the interest and concern of all these is solely with relations which are based on powers. Undoubtedly, these together constitute a system of the treatment of powers; but which of the various relations in which determinations of powers can be put is the peculiar interest and subject matter of the differential calculus, this is to be ascertained from the calculus itself, i.e. from its so-called applications. These are, in fact, the core of the whole business, the actual procedure in the mathematical solution of a certain group of problems; this procedure was earlier than the theory or general part and was later called application only with reference to the subsequently created theory, the aim of which was to draw up the general

method of the procedure and, as well, to endow it with first principles, i.e. with a justification. We have shown in the preceding Remark the futility of the search for principles which would clarify the method as currently understood, principles which would really solve the contradiction revealed by the method instead of excusing it or covering it up merely by the insignificance of what is here to be omitted (but which really is required by mathematical procedure), or, by what amounts to the same thing, the possibility of infinite or arbitrary approximation and the like. If from the practical part of mathematics known as the differential calculus the general features of the method were to be abstracted in a manner different from that hitherto followed, then the said principles and the concern about them would also show themselves to be superfluous, just as they reveal themselves to be intrinsically false and permanently contradictory.

If we investigate this peculiarity by simply taking up what we find in this part of mathematics, we find as its subject matter:

(*a*) Equations in which any number of magnitudes (here we can simply confine ourselves to *two*) are combined into a qualitative whole in such a way that first, these equations have their determinateness in *empirical magnitudes* which are their fixed limits, and also in the kind of connection they have with these limits and with each other as is generally the case in an equation; but since there is only one equation for both magnitudes (similarly, relatively more equations for more magnitudes, but always fewer than the number of magnitudes), these equations belong to the class of indeterminate equations; and secondly, that one aspect of the determinateness of these magnitudes is that they are—or at least one of them is—present in the equation in a *higher power* than the first.

Before proceeding further, there are one or two things to be noticed about this. The first is that the magnitudes, as described under the first of the above two headings, have simply and solely the character of *variables* such as occur in the problems of *indeterminate* analysis. Their value is undetermined, but if one of them does receive a completely determined value, i.e. a numerical value, from outside, then the other too, is determined, so that one is a *function* of the other. Therefore, in relation to the specific quantitative determinateness here in question, the categories of variable magnitudes, functions and the like are, as

we have already said, merely *formal*, because they are still too general to contain that specific element on which the entire interest of the differential calculus is focused, or to permit of that element being explicated by analysis; they are in themselves simple, unimportant, easy determinations which are only made difficult by importing into them what they do not contain in order that this may then be derived from them—namely, the specific determination of the differential calculus. Then as regards the so-called *constant*, we can note that it is in the first place an indifferent empirical magnitude determining the variables only with respect to their empirical quantum as a limit of their minimum and maximum; but the nature of the connection between the constants and the variables is itself a significant factor in the nature of the particular function which these magnitudes are. Conversely, however, the constants themselves are also functions; in so far as a straight line, for example, has the meaning of being the *parameter* of a parabola, then this meaning is that it is the function $\frac{y^2}{x}$; and in the expansion of the binomial generally, the constant which is the coefficient of the first term of the development is the sum of the roots, the coefficient of the second is the sum of the products, in pairs, and so on; here, therefore, the constants are simply functions of the roots. Where, in the integral calculus, the constant is determined from the given formula, it is to that extent treated as a function of this. Further on we shall consider these coefficients in another character than that of functions, their meaning in the concrete object being the focus of the whole interest.

Now the difference between variables as considered in the differential calculus, and in their character as factors in indeterminate problems, must be seen to consist in what has been said, namely, that at least one of those variables (or even all of them), is found in a power higher than the first; and here again it is a matter of indifference whether they are all of the same higher power or are of unequal powers; their specific indeterminateness which they have here consists solely in this, that in *such a relation of powers they are functions of one another*. The alteration of variables is in this way *qualitatively* determined, and hence *continuous*, and this continuity, which again is itself only the purely formal category of an *identity*, of a determinateness which

is preserved and remains self-same in the alteration, has here its determinate meaning, solely, that is, in the power-relation, which does not have a quantum for its exponent and which forms the *non-quantitative*, permanent determinateness of the ratio of the variables. For this reason it should be noted, in criticism of another formalism, that the first power is only a power in relation to higher powers; *on its own*, x is merely any indeterminate quantum. Thus there is no point in differentiating for their own sakes the equations $y = ax + b$ (of the straight line), or $s = ct$ (of the plain uniform velocity); if from $y = ax$, or even $ax + b$, we obtain $a = \frac{dy}{dx}$, or from $s = ct$, $\frac{ds}{dt} = c$, then $a = \frac{y}{x}$ is equally the determination of the tangent, or $\frac{s}{t}$ that of velocity simply as such. The latter is given the form of $\frac{dy}{dx}$ in the context of what is said to be the development of the uniformly accelerated motion; but, as already remarked, the presence in the system of such a motion, of a moment of simple, merely uniform velocity, i.e. a velocity which is not determined by the higher power of one of the moments of the motion is itself an empty assumption based solely on the routine of the method. Since the method starts from the conception of the increment which the variable is supposed to acquire, then of course a variable which is only a function of the first power can also receive an increment; when now in order to find the differential we have to subtract the difference of the second equation thus produced from the given equation, the meaninglessness of the operation becomes apparent, for, as we have remarked, the equation for the so-called increments, both before and after the operation, is the same as for the variables themselves.

(*b*) What has been said determines the nature of the equation which is to be treated; we have now to indicate what is the interest on which the treatment of the equation is focused. This consideration can yield only known results, in a form found especially in Lagrange's version; but I have made the exposition completely elementary in order to eliminate the heterogeneous determinations associated with it. The basis of treatment of an equation of this kind shows itself to be this, that the power is taken as being *within itself* a relation or a *system of relations*. We

said above that power is number which has reached the stage where it *determines its own alteration*, where its moments of unit and amount are identical—as previously shown, *completely* identical first in the square, *formally* (which makes no difference here) in higher powers. Now power is *number* (magnitude as the more general term may be preferred, but it is *in itself* always number), and hence a *plurality*, and also is represented as a *sum*; it can therefore be directly analysed into an arbitrary amount of numbers which have no further determination relatively to one another or to their sum, other than that together they are equal to the sum. But the power can also be split into a *sum* of differences which are determined by the *form of the power*. If the power is taken as a sum, then its radical number, the root, is also taken as a sum, and arbitrarily after manifold divisions, which manifoldness, however, is the indifferent, empirically quantitative element. The sum which the root is supposed to be, when reduced to its simple determinateness, i.e. to its genuine universality, is the *binomial*; all further increase in the number of terms is a mere *repetition* of the same determination and therefore meaningless.[1] The sole point of importance here is the *qualitative determinateness* of the terms resulting from the *raising to a power* of the root taken as a sum, and this determinateness lies solely in the alteration which the potentiation is. These terms, then, are wholly *functions of potentiation and of the power*. Now this representation of number as a *sum* of a *plurality* of terms which are functions of potentiation, and the finding of the *form* of such functions and also this *sum* from the plurality of those terms, in so far as this must depend solely on that form, this constitutes, as we know, the special theory of *series*. But in this connexion it is essential to distinguish another object of interest, namely the *relation of the fundamental magnitude* itself (whose determinateness, since it is a complex, i.e. here an equation, includes *within itself* a power) *to the functions of its potentiation*. This relation, taken in complete

[1] It springs solely from the formalism of that *generality* to which analysis perforce lays claim when, instead of taking $(a + b)^n$ for the expansion of powers, it gives the expression the form of $(a + b + c + d \ldots)^n$ as happens too in many other cases; such a form is to be regarded as, so to speak, a mere affectation of a show of generality; the matter itself is exhausted in the binomial. It is through the expansion of the binomial that the *law* is found, and it is the law which is the genuine universality, not the external, mere *repetition* of the law which is all that is effected by this $a + b + c + d \ldots$

abstraction from the previously mentioned interest of the *sum*, will show itself to be the sole standpoint yielded by the practical aspect of the science.

But first, another determination must be added to what has been said, or rather, one which is implied in it must be removed. It was said that the variable into the determination of which power enters is regarded as within itself a sum, in fact a system of terms in so far as these are functions of the potentiation, and that thus the root, too, is regarded as a sum and in the simply determined form of a binomial: $x^n = (y + z)^n = (y + ny^{n-1} z + \ldots)$. This exposition started from the *sum* as such for the expansion of the power, i.e. for obtaining the functions of its potentiation; but what is concerned here is not a *sum* as such, or the *series* arising from it; what is to be taken up from the sum is only the *relation*. The *relation* as such of the magnitudes is, on the one hand, all that remains after abstraction is made from the *plus* of a sum as such, and on the other hand, all that is needed for finding the functions produced by the expansion of the power. But such relation is already determined by the fact that here the object is an equation, $y^m = ax^n$, and so already a *complex* of several (variable) magnitudes which contains a power determination of them. In this complex, each of these variables is posited simply as in *relation* to the others with the meaning, one could say, of a *plus* implicit in it—as a function of the other variables; their character, that of being functions of one another, gives them this determination of a *plus* which, however, for that same reason, is wholly *indeterminate*—not an increase or an increment, or anything of that nature. Yet even this abstract point of view we could leave out of account; we can quite simply stop at the point where the variables in the equation having received the form of functions of one another, such functions containing a relation of powers, the functions of *potentiation* are then also compared with one another—these second functions being determined simply and solely by the potentiation itself. To treat an equation of the powers of its variables as a relation of the functions developed by potentiation can, in the first place, be said to be just a *matter of choice* or a *possibility*; the *utility* of such a transformation has to be indicated by some further *purpose* or use; and the sole reason for the transformation was its utility. When we started above from the representation of these functions

of potentiation of a variable which is taken as a *sum complex within itself*, this served only partly to indicate the nature of such functions, but partly also to show the way in which they are found.

What we have here then is the ordinary analytical development which for the purpose of the differential calculus is operated in this way, that an increment *dx* or *i* is given to the variable and then the power of the binomial is developed by the terms of the series belonging to it. But the so-called increment is supposed to be not a quantum but only a *form*, the whole value of which is that it *assists* the development; it is admitted—most categorically by Euler and Lagrange and in the previously mentioned conception of limit—that what is wanted is only the resulting power determinations of the variables, the so-called *coefficients*, namely, of the increment and its powers, according to which the series is ordered and to which the different coefficients belong. On this we could perhaps remark that since an increment (which has no quantum) is assumed only for the sake of the development, it would be most appropriate to take 1 (the one) for that purpose, for in the development this always occurs only as a factor; the factor one, therefore, fulfils the purpose, namely, that the increment is not to involve any quantitative determinateness or alteration; on the other hand, *dx*, which is burdened with the false idea of a quantitative difference, and other symbols like *i* with the mere show—pointless here—of generality, always have the appearance and pretension of a quantum and its powers; which pretension then involves the trouble that they must nevertheless be removed and left out. In order to retain the form of a series expanded on the basis of powers, the designations of the exponents as indices could equally well be attached to the one. But in any case, abstraction must be made from the series and from the determination of the coefficients according to their place in the series; the relation between all of them is the same; the second function is derived from the first in exactly the same manner as this is from the original function, and for the function counted as second, the first derived function is itself original. But the essential point of interest is not the series but simply and solely the determination of the power resulting from the expansion in its relation to the variable which for the *power determination is immediate*. It should not therefore be defined as the *coefficient*

of the *first* term of the development, for it is *first* only in relation to the other terms following it in the series, and a power such as that of an increment, like the series itself, is here out of place; instead, the simple expression: derived function of a power, or as was said above: function of potentiation of a magnitude, would be preferable—the knowledge of the way in which the derivation is taken to be a development included within a power being presupposed.

Now if the strictly mathematical beginning in this part of analysis is nothing more than the finding of the function determined by the expansion of the power, the further question is what is to be done with the relation so obtained, where has it an application and use, or indeed, for what *purpose* are such functions sought. It is the finding of relations in *a concrete subject matter* which can be reduced to such a function that has given the differential calculus its great interest.

But as regards the applicableness of the relation, we need not wait for conclusions to be drawn from particular applications themselves, the answer follows directly and automatically from the nature of the matter which we have shown to consist in the form possessed by the moments of powers: namely, the expansion of the powers, which yields the functions of their potentiation, contains (ignoring any more precise determination) in the first place, simply the *reduction* of the magnitude to the next lower power. This operation is therefore *applicable* in the case of those objects in which there is also present such a difference of power determinations. Now if we reflect on the specific nature of space, we find that it contains the three dimensions which, in order to distinguish them from the abstract differences of height, length and breadth, we can call *concrete*—namely, line, surface and total space; and when they are taken in their simplest forms and with reference to self-determination and consequently to analytical dimensions, we have the straight line, plane surface and surface taken as a square, and the cube. The straight line has an empirical quantum, but with the plane there enters the qualitative element, the determination of power; further modifications, e.g. the fact that this also happens in the case of plane curves, we need not consider, for we are concerned primarily with the distinction in general. With this there arises, too, the *need to pass from a higher power to a lower, and vice versa*, when, for example, linear deter-

minations are to be derived from given equations of the plane, or vice versa. Further, the *motion* in which we have to consider the quantitative relation of the space traversed to the time elapsed, manifests itself in the different determinations of a motion which is simply uniform, or uniformly accelerated, or alternately uniformly accelerated and uniformly retarded, and thus a self-returning motion; since these different kinds of motion are expressed in accordance with the quantitative relation of their moments, of space and time, their equations contain different determinations of powers, and when it is necessary to determine one kind of motion, or a spatial magnitude to which one kind of motion is linked, from another kind of motion, the operation also involves the passage from one power-function to another, either higher or lower. These two examples may suffice for the purpose for which they are cited.

The appearance of arbitrariness presented by the differential calculus in its applications would be clarified simply by an awareness of the nature of the spheres in which its application is permissible and of the peculiar need for and condition of this application. But now the further point of interest within these spheres themselves is to know between what *parts* of the subject matter of the mathematical problem such a relation occurs as is posited peculiarly by the differential calculus. First, it must be observed that there are two kinds of relation. The operation of depotentiating an *equation* considered according to the derivative functions of its variables, yields a result which, *in itself*, is no longer truly an equation but a *relation*; this relation is the subject matter of the *differential calculus proper*. This also gives us, secondly, the relation of the higher power form (the original equation) itself to the lower (the derivative). This second relation we must ignore for the time being; it will prove to be the special subject matter of the *integral calculus*.

Let us start by considering the first relation; for the determination of its moment (to be taken from the application, in which lies the interest of the operation) we shall take the simplest example from curves determined by an equation of the second degree. As we know, the relation of the co-ordinates is given *directly* by the equation in a power form. From the fundamental determination follow the determinations of the other straight lines connected with the co-ordinates, tangent, subtangent, normal, and so on.

But the equations between these lines and the co-ordinate are *linear* equations; the wholes with respect to which these lines are determined as parts, are right-angled triangles formed by *straight* lines. The transition from the original equation which contains the power form, to said linear equations, involves now the above-mentioned transition from the original function (which is an *equation*), to the derived function (which is a *relation*, a relation, that is, between certain lines contained in the curve). The problem consists in finding the connection between the *relation* of these lines and the *equation* of the curve.

It is not without interest, as regards the historical element, to remark this much, that the first discoverers could only record their findings in a wholly empirical manner without being able to account for the operation, which remained a completely external affair. It will be sufficient here to refer to Barrow, to him who was Newton's teacher. In his *lect. Opt. et Geom.*, in which he treats problems of higher geometry according to the method of indivisibles, a method which, to begin with, is distinct from the characteristic feature of the differential calculus, he also puts on record[1] his procedure for determining tangents—'because his friends urged him to do so'. To form a proper idea of how this procedure is formulated simply as an *external rule*, in the same style as the 'rule of three', or better still the so-called 'test by casting out nines', one must read Barrow's own exposition. He draws the tiny lines afterwards known as the increments in the characteristic triangle of a curve and then gives the instruction, in the form of a mere *rule, to reject as superfluous* the terms which, as a result of the expansion of the equations, appear as powers of the said increments or as products (*etenim isti termini nihilum valebunt*); similarly, the terms which contain only magnitudes to be found in the original equation are to be rejected (the subsequent subtraction of the original equation from that formed with the increments); and finally, for the increments of the ordinate and abscissa, the ordinate itself and the subtangent respectively are to be substituted. The procedure, if one may say so, can hardly be set forth in a more schoolmasterlike manner; the latter substitution is the assumption of the proportionality of the increments of the ordinate and the abscissa with the ordinate and the subtangent, an assumption on which is based the determination of

[1] *Lect. X.*

the tangent in the ordinary differential method; in Barrow's rule this assumption appears in all its naïve nakedness. A simple way of determining the subtangent was found; the artifices of Roberval and Fermat have a similar character. The method for finding maximal and minimal values from which Fermat started rests on the same basis and the same procedure. It was a mathematical craze of those times to find so-called *methods*, i.e. rules of that kind and to make a secret of them—which was not only easy, but in one respect even necessary, for the same reason that it was easy—namely, because the inventors had found only an empirical, external rule, not a method, i.e. nothing derived from established principles. Leibniz accepted such so-called methods from his contemporaries and so did Newton who got them directly from his teacher; by generalizing their form and applicableness they opened up new paths for the sciences, but at the same time they also felt the need to wrest free the procedure from the shape of merely external rules and to try to procure for it the necessary justification.

If we analyse the method more closely, we find the genuine procedure to be as follows. Firstly the power forms (of the variables of course) contained in the equation are reduced to their first functions. But the *value* of the terms of the equation is thereby *altered*; there is now no longer an equation, but instead only a *relation* between the first function of the one variable and the first function of the other. Instead of $px = y^2$ we have $p : 2y$, or instead of $2ax - x^2 = y^2$, we have $a - x : y$, the relation which later came to be designated $\frac{dy}{dx}$. Now the equation represents a curve; but this relation, which is completely dependent on it and derived from it (above, according to a mere *rule*), is, on the contrary, a linear relation with which certain lines are in proportion: $p : 2y$ or $a - x : y$ are themselves relations of straight line of the curve, of the co-ordinates and parameters. *But with all this, nothing is as yet known.* The interest centres on finding that the *derived relation* applies to *other* lines connected with the curve, on finding the equality of two relations. And so there is, secondly, the question, which are the straight lines determined by the nature of the curve, standing in such a relation? But this is just what *was already known*: namely, that the relation so obtained is the relation of the ordinate to the subtangent. This the ancients

had found in an ingenious geometrical manner; what the moderns have discovered is the empirical procedure of so preparing the equation of the curve that it yields that first relation of which it *was already known* that it is equal to a relation containing the line (here the subtangent) which is to be determined. Now on the one hand, this preparation of the equation—the differentiation —has been methodically conceived and executed; but on the other hand the imaginary increments of the co-ordinates and an imaginary characteristic triangle formed by them and by an equally imaginary increment of the tangent, have been invented in order that the proportionality of the ratio found by lowering the degree of the equation to the ratio formed by the ordinate and subtangent, may be represented, not as something only empirically accepted as an already familiar fact, but as something demonstrated. However, in the said form of rules, the already familiar fact reveals itself absolutely and unmistakably as the sole occasion and respective justification of the assumption of the characteristic triangle and the said proportionality.

Now Lagrange rejected this pretence and took the genuinely scientific course. We have to thank his method for bringing into prominence the real point of interest for it consists in separating the two transitions necessary for the solution of the problem and treating and proving each of them separately. One part of this solution (for the more detailed statement of the process we shall confine ourselves to the example of the elementary problem of finding the subtangent), the theoretical or general part, namely, the finding of the *first function* from the given equation of the curve, is dealt with separately; the result is a *linear relation*, a relation therefore of straight lines occurring in the system determined by the curve. The other part of the solution now is the finding of those lines in the curve which stand in this relation. Now this is effected in a direct manner[1] i.e., without the characteristic triangle, which means that there is no assumption of infinitely small arcs, ordinates and abscissae, the last two being given the significance of dy and dx, that is, of being sides of that relation, and at the same time directly equating the infinitely small ordinate and abscissa with the ordinate and subtangent themselves. A line (and a point, too), is determined only in so far as it forms the side of a triangle and the determination of a point, too, falls only in

[1] *Théorie des Fonct. Anal.*, II. P. II Chap.

such triangle. This, it may be mentioned in passing, is the fundamental proposition of analytical geometry from which are derived the co-ordinates of that science, just as (it is the same standpoint) in mechanics it gives rise to the parallelogram of forces, for which very reason the many efforts to find a proof of this latter are quite unnecessary. The subtangent, now, is made to be the side of a triangle whose other sides are the ordinate and the tangent connected to it. The equation of the latter, as a straight line, is $p = aq$ (the determination does not require the additional term, $+ b$ which is added only on account of the fondness for generality);—the determination of the *ratio* $\frac{p}{q}$ falls within a, the coefficient of q which is the respective first function (derivative) of the equation, but may simply be considered only as $a = \frac{p}{q}$ being, as we have said, the essential determination of the straight line which is applied as tangent to the curve. But the first function (derivative) of the equation of the curve is equally the determination of a straight line; seeing then that the co-ordinate p of the first straight line and y, the co-ordinate of the curve, are assumed to be identical (so that the point at which the curve is touched by the first straight line assumed as tangent is also the starting point of the straight line determined by the first function of the curve), the problem is to show that this second straight line coincides with the first, i.e. is a tangent; or, algebraically expressed, that since $y = fx$ and $p = Fq$, and it is assumed that $y = p$ and hence that $fx = Fq$, therefore $f'x = F'q$. Now in order to show that the straight line applied as a tangent and the straight line determined by the first function of the equation coincide, and that therefore the latter is a tangent, Descartes has recourse to the *increment* i of the abscissa and to the increment of the ordinate determined by the expansion of the function. Thus here, too, the objectionable increment also makes its appearance; but its introduction for the purpose indicated and its rôle in the expansion of the function must be carefully distinguished from the previously mentioned employment of the increment in finding the differential equation and in the characteristic triangle. Its employment here is justified and necessary because it falls within the scope of geometry, the geometrical determination of a tangent as such implying that between it and the curve with which it has a point

in common, no other straight line can be drawn which also passes through the said point. For, as thus determined, the quality of tangent or not-tangent is reduced to a *quantitative difference*, that line being the tangent of which simply *greater smallness* is predicated with respect to the determination in point. This seemingly only relative smallness contains no empirical element whatever, i.e. nothing dependent on a quantum as such; in virtue of the nature of the formula it is explicitly qualitative if the difference of the moments on which the magnitude to be compared depends is a difference of powers. Since this difference becomes that of i and i^2 and i (which after all is meant to signify a number) is then to be conceived as a fraction, i^2 is therefore *in itself* and *explicitly* smaller than i, so that the very conception of an *arbitrary* magnitude in connection with i is here superfluous and in fact out of place. For the same reason the demonstration of the greater smallness has nothing to do with an infinitesimal, which thus need not be brought in here at all.

I must also mention the tangential method of Descartes, if only for its beauty and its fame—well-deserved but nowadays mostly forgotten; it has, moreover, a bearing on the nature of equations and this, again, calls for a further remark. Descartes expounds this independent method, in which the required linear determination is likewise found from the same derivative function, in his geometry which has proved to be so fruitful in other respects too[1]; in it he has taught the great basis of the nature of equations and their geometrical construction, and also of the application of analysis, thereby greatly widened in its scope, to geometry. With him the problem took the form of drawing straight lines perpendicularly to given points on a curve as a method for determining the subtangent, etc. One can understand the satisfaction he felt at his discovery, which concerned an object of general scientific interest at that time and which is so purely geometrical and therefore was greatly superior to the mere rules of his rivals, referred to above. His words are as follows: '*J'ose dire que c'est ceci le problème le plus utile et le plus général, non seulement que je sache, mais même que j'aie jamais désiré de savoir en géometrie.*' He bases his solution on the analytic equation of the right-angled triangle formed by the ordinate of the point on the curve to which the required straight line in the problem

[1] Liv. II (p 357 *sqq. Œuvres Compl.*, ed. Cousin, Tom. V).

is to be drawn perpendicularly, by this same straight line (the normal), and thirdly, by that part of the axis which is cut off by the ordinate and the normal (the subnormal). Now from the known equation of a curve, the value of either the ordinate or the abscissa is substituted in the said triangle, the result being an equation of the second degree (and Descartes shows how even curves whose equations contain higher powers reduce to this); in this equation, only *one* of the variables occurs, namely, as a square and in the first degree—a quadratic equation which at first appears as a so-called impure equation. Descartes now makes the reflection that if the assumed point on the curve is imagined to be a point of intersection of the curve and of a circle, then this circle will also cut the curve in another point and we shall then get for the unequal xs thus produced, two equations with the same constants and of the same form, or else only *one* equation with unequal values of x. But the equation only becomes *one* for the one triangle in which the hypotenuse is perpendicular to the curve or is the normal, the case being conceived of in this way, that the two points of intersection of the curve and the circle are made to coincide and the circle is thus made to touch the curve. But in that case it is also true that the x or y of the quadratic equation no longer have *unequal* roots. Now since in a quadratic equation with two equal roots the coefficient of the term containing the unknown in the first power is twice the single root, we obtain an equation which yields the required determinations. This procedure must be regarded as the brilliant device of a genuinely analytical mind, in comparison with which the dogmatically assumed proportionality of the subtangent and the ordinate with the postulated infinitely small, so-called increments, of the abscissa and ordinate drops into the background.

The final equation obtained in this way, in which the coefficient of the second term of the quadratic equation is equated with the double root or unknown, is the same as that obtained by the method of the differential calculus. The differentiation of $x^2 - ax - b = 0$ yields the new equation $2x - a = 0$; or $x^3 - px - q = 0$ gives $3x^2 - p = 0$. But it suggests itself here to remark that it is by no means self-evident that such a derivative equation is also correct. We have already pointed out that an equation with two variables (which, just because they are variables, do not lose their character of being unknown quantities) yields only a *proportion*;

and for the simple reason stated, namely, that when the functions of potentiation are substituted for the powers themselves, the value of both terms of the equation is altered and it is not yet known whether an equation still exists between them with their values thus altered. All that the equation $\frac{dy}{dx} = P$ expresses is that P is a *ratio* and no other real meaning can be ascribed to dy/dx. But even so, we still do not know of this ratio $= P$, to what other ratio it is equal; and it is only such equation or *proportionality* which gives a value and meaning to it. We have already mentioned that this meaning, which was called the application, was taken from another source, empirically; similarly, in the case of the equations here under discussion which have been obtained by differentiation, it is from another source that we must know whether they have equal roots in order that we may learn whether the equation thus obtained is still correct. But this fact is not expressly brought to notice in the textbooks; it is disposed of, certainly, when an equation with one unknown, reduced to zero, is straightway equated with y, with the result, of course, that differentiation yields a $\frac{dy}{dx}$, i.e. only a ratio. The functional calculus, it is true, is supposed to deal with functions of potentiation and the differential calculus with differentials; but it by no means follows from this alone that the magnitudes from which the differentials or functions of potentiation are taken, are themselves supposed to be *only* functions of *other* magnitudes. Besides, in the theoretical part, in the instruction to derive the differentials, i.e. the functions of potentiation, there is no indication that the magnitudes which are to be subjected to such treatment are themselves supposed to be functions of other magnitudes.

Further, with regard to the omission of the constant when differentiating, we may draw attention to the fact that the omission has here the meaning that the constant plays no part in the determination of the roots if these are equal, the determination being exhausted by the coefficient of the second term of the equation: as in the example quoted from Descartes where the constant is itself the square of the roots, which therefore can be determined from the constant as well as from the coefficients—seeing that, like the coefficients, the constant is simply a function of the roots of the equation. In the usual exposition, the omission

of the so-called constants (which are connected with the other terms only by *plus* and *minus*) results from the mere mechanism of the process of differentiation, in which to find the differential of a compound expression only the variables are given an increment, and the expression thereby formed is subtracted from the original expression. The meaning of the constants and of their omission, in what respect they are themselves functions and, as such, are or are not of service, are not discussed.

In connection with the omission of constants we may make a similar observation about the *names* of differentiation and integration as we did before about the expressions finite and infinite: that is, that the character of the operation in fact belies its name. To differentiate denotes that differences are posited, whereas the result of differentiating is, in fact, to reduce the dimensions of an equation, and to omit the constant is to remove from the equation an element in its determinateness. As we have remarked, the roots of the variables are made equal, and therefore their *difference* is *cancelled*. In integration, on the other hand, the constant must be added in again and although as a result the equation is integrated, it is so in the sense that the previously cancelled *difference* of the roots is restored, that is, what was posited as equal is differentiated again. The ordinary expression helps to obscure the essential nature of the matter and to set everything in a point of view which is not only subordinate but even alien to the main interest, the point of view, namely, of the infinitely small difference, the increment and the like, and also of the mere difference as such between the given and the derived function, without any indication of their specific, i.e. qualitative, difference.

Another important sphere in which the differential calculus is employed is *mechanics*. The *meanings* of the distinct power functions yielded by the elementary equations of its subject matter, *motion*, have already been mentioned in passing; at this point, I shall proceed to deal with them directly. The equation, i.e. the mathematical expression, for simply uniform motion, $c = \frac{s}{t}$, or $s = ct$, in which the spaces traversed are proportional to the times elapsed in accordance with an empirical unit c (the magnitude of the velocity), offers no meaning for differentiation: the coefficient c is already completely determined and known, and no further expansion of powers is possible. We have already

noticed how $s = at^2$, the equation of the motion of a falling body, is analysed; the first term of the analysis, $\frac{ds}{dt} = 2at$ is translated into language, and also into existence, in such a manner that it is supposed to be a factor in a *sum* (a conception we have long since abandoned), to be one part of the motion, which part moreover is attributed to the force of inertia, i.e. of a simply uniform motion, in such a manner that in *infinitely small* parts of time the motion is *uniform*, but in *finite* parts of time, i.e. in actually existent parts of time, it is non-uniform. Admittedly, $fs = 2at$; and the meaning of a and t themselves is known and so, too, the fact that the motion is determined as of uniform velocity; since $a = \frac{s}{t^2}$, $2at$ is equal simply to $\frac{2s}{t}$. But knowing this we are not a whit wiser; it is only the erroneous assumption that $2at$ is a part of the motion regarded as a *sum*, that gives the false appearance of a physical proposition. The factor itself, a, the empirical unit—a simple quantum—is attributed to gravity; but if the category of 'force of gravity' is to be employed then it ought rather to be said that the whole, $s = at^2$, is the effect, or, better, the law, of gravity. Similarly with the proposition derived from $\frac{ds}{dt} = 2at$, that *if* gravity ceased to act, the body, with the velocity reached at the *end* of its fall, would cover twice the distance it had traversed, in the same period of time as its fall. This also implies a metaphysics which is itself unsound: the *end* of the fall, or the *end* of a period of time in which the body has fallen, is itself still a period of time; if it were *not*, there would be assumed a state of *rest* and hence no velocity, for velocity can only be fixed in accordance with the space traversed *in a period* of time, not at its *end*. When, however, the differential calculus is applied without restriction in other departments of physics where there is no motion at all, as for example in the behaviour of light (apart from what is called its propagation in space) and in the application of quantitative determinations to colours, and the first function of a quadratic function here is also called a velocity, then this must be regarded as an even more illegitimate formalism of inventing an existence.

The motion represented by the equation $s = at^2$ we find, says Lagrange, empirically in falling bodies; the next simplest motion

would be that whose equation were $s = ct^3$, but no such motion is found in Nature; we do not know what significance the coefficient c could have. Now though this is indeed the case, there is nevertheless a motion whose equation is $s^3 = at^2$—Kepler's law of the motion of the bodies of the solar system; the significance here of the first derived function $\frac{2at}{3s^2}$ and the further direct treatment of this equation by differentiation, the development of the laws and determinations of that absolute motion *from this starting point*, must indeed present an interesting problem in which analysis would display a brilliance most worthy of itself.

Thus the application of the differential calculus to the elementary equations of motion does not of itself offer any *real* interest; the formal interest comes from the general mechanism of the calculus. But another significance is acquired by the analysis of motion in connection with the determination of its trajectory; if this is a curve and its equation contains higher powers, then transitions are required from rectilinear functions, as functions of potentiation, to the powers themselves; and since the former have to be obtained from the original equation of motion containing the factor of time, this factor being eliminated, the powers must at the same time be reduced to the lower functions of development from which the said linear equations can be obtained. This aspect leads to the interesting feature of the other part of the differential calculus.

The aim of the foregoing has been to make prominent and to establish the simple, specific nature of the differential calculus and to demonstrate it in some elementary examples. Its nature has been found to consist in this, that from an equation of power functions the coefficient of the term of the expansion, the so-called first function, is obtained, and the *relation* which this first function represents is demonstrated in moments of the concrete subject matter, these moments being themselves determined by the equation so obtained between the two relations. We shall also briefly consider the principle of the *integral calculus* to see what light is thrown on its specific, concrete nature by the application of the principle. The view of the integral calculus has been simplified and more correctly determined merely by the fact that it is no longer taken to be a *method of summation* in which it appeared essentially connected with the form of series; the

method was so named in contrast to differentiation where the increment counts as the essential element. The problem of this calculus is, in the first instance, like that of the differential calculus, theoretical or rather formal, but it is, as everyone knows, the converse of the latter. Here, the starting point is a function which is considered as *derived*, as the coefficient of the first term arising from the expansion of an equation as yet unknown, and the problem is to find the original power function from the derivative; what would be regarded in the natural order of the expansion as the original function is here derived, and the function previously regarded as derived is here the given, or simply original, function. Now the formal part of this operation seems to have been accomplished already in the differential calculus in which the transition and the relation of the original to the derived function in general has been established. Although in doing this it is necessary in many cases to have recourse to the *form of series* simply in order to obtain the function which is to be the starting point and also to effect the transition from it to the original function, it is important to remember that this form as such has nothing directly to do with the peculiar principle of integration.

The other part of the problem of the calculus appears in connection with its formal operation, namely the *application* of the latter. But this now is itself the *problem*: namely, to find the *meaning* in the above-mentioned sense, possessed by the original function of the given function (regarded as first) of a particular subject matter; it might seem that this doctrine, too, was in principle already finally settled in the differential calculus; but a further circumstance is involved which prevents the matter from being so simple. In the differential calculus, namely, it was found that the linear relation is obtained from the first function of the equation of a curve, so that it is also known that the integration of this relation gives the equation of the curve in the relation of abscissa and ordinate; or, if the equation for the area enclosed by the curve were given, then we should be supposed to know already from the differential calculus that the meaning of the first function of such equation would be that it represented the ordinate as a function of the abscissa, and therefore the equation of the curve.

The problem now is to determine which of the moments determining the subject matter is *given* in the equation itself; for

elementary mathematics) with the subject matter of the problem (in the given example, with the arc) this, too, being represented as infinitely small, and from this relation the magnitude required to be known can be found from the known magnitude of the parts originally taken. Thus, in connection with the rectification of curves, the three infinitely small elements mentioned are connected in the equation of the right-angled triangle, while for the quadrature of curves, seeing that area is taken arithmetically to be simply the product of lines, the ordinate and the infinitely small abscissa are connected in the form of a product. The transition from such so-called elements of the area, the arc, etc., to the magnitude of the total area or the whole arc itself, passes merely for the ascent from the infinite expression to the finite expression, or to the *sum* of the infinitely many elements of which the required magnitude is supposed to consist.

It is therefore merely superficial to say that the integral calculus is simply the converse, although in general the more difficult, problem of the differential calculus; the *real* interest of the integral calculus concerns almost exclusively the relation between the original and the derived function in the concrete subject matter.

Even in this part of the calculus, Lagrange did not smooth over the difficulties of its problems simply by making those direct assumptions. It will help to elucidate the nature of the matter in hand if here, too, we indicate the details of his method in one or two examples. The declared object of his method is, precisely, to provide an independent *proof* of the fact that between particular elements of a mathematical whole, for example, of a curve, there exists a relation of the original to the derived function. Now this proof cannot be effected in a direct manner because of the nature of the relation itself in this domain; in the mathematical object this relation connects terms which are *qualitatively* distinct, namely, curves with straight lines, linear dimensions and their functions with plane or surface dimensions and their functions, so that the required determination can only be taken as the mean between a *greater* and a *less*. Consequently, there spontaneously enters again the form of an increment with a *plus* and *minus* and the energetic '*développons*' is here in place; but we have already pointed out that here the increments have only an arithmetical, finite meaning. From the development of the condition that the required magnitude is greater than the one easily determinable

limit and smaller than the other, it is then deduced that, e.g. the function of the ordinate is the derived, first function of the function of the area.

Lagrange's exposition of the rectification of curves in which he starts from the principle of Archimedes is interesting because it provides an insight into the *translation* of the Archimedean method into the principle of modern analysis, thus enabling us to see into the inner, true meaning of the procedure which in the other method is carried out mechanically. The mode of procedure is necessarily analogous to the one just indicated. The principle of Archimedes, that the arc of a curve is greater than its chord and smaller than the sum of the two tangents drawn through the end points of the arc and contained between these points and the point of intersection of the tangents, gives no direct equation, but simply *postulates* an endless alternation between terms determined as too great or too small, the successive terms always being still too great or too small but within ever narrower limits of inaccuracy; its translation into the modern analytical form, however, takes the form of finding an expression which is *per se* a simple fundamental equation. Now whereas the formalism of the infinitesimal directly presents us with the equation $dz^2 = dx^2 + dy^2$, Lagrange's exposition, starting from the basis indicated, demonstrates that the length of the arc is the original function to a derived function whose characteristic term is itself a function coming from the relation of a derived function to the original function of the ordinate.

Because in Archimedes' method, as well as later in Kepler's treatment of stereometric objects, the idea of the infinitesimal occurs, this has often been cited as an authority for the employment of this idea in the differential calculus, although what is peculiar and distinctive in it has not been brought specifically to notice. The infinitesimal signifies, strictly, the negation of quantum as quantum, that is, of a so-called *finite* expression, of the completed determinateness possessed by quantum as such. Similarly, in the subsequent celebrated methods of Valerius[1] and Cavalieri, among others, which are based on the treatment of the *relations* of geometrical objects, the fundamental principle is that the *quantum* as such of the objects concerned, which are primarily considered

[1] Valerius, Lucas, died 1618 at Rome, called by Galileo the Archimedes of his time: *De Quadratura Parabolae Per Simplex Falsum.*

only in their constituent relations, is for this purpose to be left out of account, the objects thus being taken as *non-quantitative*. However, in these methods the affirmative aspect as such which is veiled by the merely negative determination fails to be recognized or brought to notice—that aspect namely which above presented itself abstractly, as the *qualitative* determinateness of quantity, and more precisely, as lying in the relation of powers; and also, since this relation itself embraces a number of more precisely determined relations such as that of a power and the function of its development; these also, in turn, are supposed to be based on and derived from the general and negative determination of the same infinitesimal. In the exposition of Lagrange just noticed, the specific affirmative aspect which is implied in Archimedes' method of developing the problem is brought to notice with the result that the procedure which is burdened with an unlimited progression is given its proper limit. The greatness of the modern invention *per se* and its capacity to solve previously intractable problems and to treat in a simple manner those previously soluble, is to be ascribed solely to the discovery of the relation of the original to the so-called derived functions and of those parts of a mathematical whole which stand in such a relation.

What has been said may suffice to signalize that distinctive relation of magnitudes which is the subject matter of the particular kind of calculus under discussion. It was possible to confine our exposition to simple problems and the methods of solving them; it would neither have been expedient as regards the determination of the Notion, which determination is here our sole concern, not would it have lain in the author's power to have reviewed the entire compass of the so-called application of the differential and integral calculus, and by reference of all the respective problems and their solutions to what we have demonstrated to be the principle of the calculus, to have carried out completely the induction that the application is based upon this principle. But sufficient evidence has been produced to show that just as each particular mode of calculation has as its subject matter a specific determinateness or relation of magnitude, such relation constituting addition, multiplication, the raising to powers and extraction of roots, and operations with logarithms and series, and so on, so too has the differential and integral calculus; the subject matter proper to this calculus might be most appropriately

named the relation between a power function and the function of its expansion or potentiation, because this is what is most readily suggested by an insight into the nature of the subject matter. Logarithms, circular functions and series are of course also employed in the calculus, especially for the purpose of making expressions more amenable for the operations necessary for deriving the original function from the functions of expansion; but they are only used in the same way that the other forms of calculation such as addition, etc., are also used in the calculus. The differential and integral calculus has, indeed, a more particular interest in common with the form of series namely, to determine those functions of expansion which in the series are called coefficients of the terms; but whereas the calculus is concerned only with the relation of the original function to the coefficient of the first term of its expansion, the series aims at exhibiting in the form of a sum, groups of the terms arranged according to powers which have these coefficients. The infinite which is associated with infinite series, the indeterminate expression of the negative of quantum in general, has nothing in common with the affirmative determination belonging to the infinite of this calculus. Similarly, the infinitesimal in the shape of the *increment*, by means of which the expansion is given the form of a series, is only an external means for the expansion, and the sole meaning of its so-called infinity is to have no other meaning beyond its significance as such means; the series, which in fact is not what is wanted, produces an excess, the elimination of which causes the unnecessary trouble. The method of Lagrange, who preferred to use the form of series again, is also burdened with this difficulty; although it is through his method, in what is called the *application*, that what is truly characteristic of the calculus is brought to notice, for, without forcing the forms of *dx*, *dy* and so on, *into the objects*, it is directly demonstrated to which part of the object the determinateness of the derived function (function of expansion) belongs; and thus it is evident that the matter in hand here is not the form of series.[1]

[1] In the critique quoted above (*Jahrb. für wissensch. Krit.*, II Bd. 1827, Nr. 155, 6 *sqq.*), are to be found interesting views of a profound scholar in this science, Herr Spehr; (Spehr, Friedrich Wilhelm, 1799–1833, a Brunswick mathematician: *Vollständiger Lehrbegriff der reinen Kombinationslehre*, 1824.) they are quoted from his *Neue Prinzipien des Fluentenkalkuls*, Brunswick, 1826, and concern a factor which has materially contributed to what is obscure and

Remark 3: *Further Forms Connected With the Qualitative Determinateness of Magnitude*

It has been shown that the infinitesimal of the differential calculus is, in its affirmative meaning, the *qualitative* determinateness of magnitude; and, more precisely, that it is present in the calculus not merely as a power determinateness in general but specifically as the relation of a power function to the power derived from the expansion of the function. But the qualitative determinateness is also present in another, so to speak, weaker form and this form, together with the use and the meaning of the infinitely small in this connection, are to be the subject matter of this Remark.

In making the foregoing our starting point we must, in this respect, first of all recollect that from the *analytical* side, the different power determinations appear in the first place as only formal and quite homogeneous, that they signify *numerical* magnitudes which as such do not possess that qualitative difference from each other. But in the application to spatial objects, the qualitative determinateness of the analytic relation is fully manifested as the transition from linear to planar determinations, from determinations of straight lines to those of curves, and so on. This application further involves that spatial objects, which by their nature are given in the form of *continuous* magnitudes, are

unscientific in the differential calculus and they agree with what we have said about the general character of the *theory* of this calculus. 'Purely arithmetical investigations,' he says, 'admittedly those which have a primary bearing on the differential calculus, have not been separated from the differential calculus proper, and in fact, as with Lagrange, have even been taken to be the calculus itself whilst this latter was regarded as only the application of them. These arithmetical investigations include the rules of differentiation, the derivation of Taylor's theorem, etc., and even the various methods of integration. But the case is quite the reverse, for it is precisely those applications which form the subject matter of the differential calculus proper, all those arithmetical developments and operations being *presupposed by the calculus* from analysis.' We have shown how, with Lagrange, it is just the separation of the so-called application from the procedure of the general part which starts from series, which serves to bring to notice the characteristic subject matter of the differential calculus. It is strange, however, that the author, who realizes that it is just these applications which form the subject matter of the differential calculus proper, should get involved in the formal metaphysics (adduced in that work) of *continuous magnitude*, *becoming*, *flow*, etc., and should want to add even fresh ballast to the old; these determinations are *formal*, in that they are only general categories which do not indicate just what is the *specific nature* of the subject matter, this having to be learned and abstracted from the concrete theory, that is, the applications.

taken to be *discrete*, the plane therefore as a multitude of lines, the line as a multitude of points, and so on. The sole interest of this procedure is to determine the points and the lines themselves into which the lines and planes respectively have been resolved, in order that from such determination, progress can be made analytically, that is, strictly speaking, arithmetically; these starting points for the required magnitudes are the *elements* from which the function and equation for the *concrete*, that is, the *continuous* magnitude, is to be derived. For the problems where the employment of this procedure is chiefly indicated, it is requisite that the element forming the starting point should be *self-determined*—in contrast to the *indirect* method which, on the contrary, can begin only with *limits* between which lies the self-determined element as the *goal* towards which the method advances. But then the result in both methods comes to the same thing if what can be found is only the law for progressively determining the required magnitude without the possibility of reaching the perfect, that is, so-called finite, determination demanded. To Kepler is ascribed the honour of first having thought of this reversal of the process and of having made the discrete the starting point. His explanation of how he understands the first proposition in Archimedes' cyclometry expresses this quite simply. Archimedes' first proposition, as we know, is that a circle is equal to a right-angled triangle having one of the sides enclosing the right angle equal to the diameter and the other to the circumference of the circle. Now Kepler takes the meaning of this proposition to be that the *circumference* of the circle has as many parts as it has *points*, that is, an infinite number, each of which can be regarded as the base of an isosceles triangle, and so on; he thus gives expression to the resolution of the continuous into the form of the discrete. The expression 'infinite' which occurs here is still far removed from the definition it is supposed to have in the differential calculus. When now a determinateness, a function, has been found for such discrete elements, they are supposed to be summed up, to be essentially elements of the continuous. But since a sum of points does not make a line, or a sum of lines a plane, the points are already directly taken as linear and the lines as planar. But because these linear elements are at the same time supposed not to be lines, which they would be if they were taken as quantum, they are represented as being

infinitely small. What is discrete can only be *externally* summed up, the moments of such sum retaining the meaning of the discrete one (unit); the analytic transition from these ones is made only to their *sum* and is not simultaneously the geometrical transition from the point to the line or from the line to the plane, and so on; therefore the element which is determined as point or line is at the same time also given the quality of being linear or planar respectively in order that the sum, as a sum of little lines, may become a line, or as a sum of little planes may become a plane.

It is the need to acquire this moment of qualitative transition and to have recourse for this purpose to the infinitely small, which must be regarded as the source of all the conceptions which, though they are meant to resolve this difficulty, constitute in themselves the greatest difficulty. Before one could dispense with this expedient, it would have to be possible to show that the analytic procedure itself which appears as a mere *summation*, in fact already contains a *multiplication*. But this involves a fresh assumption which forms the basis in this application of arithmetical relations to geometrical figures: the assumption, namely, that arithmetical multiplication is also for the geometrical determination a transition into a higher dimension—that the arithmetical multiplication of magnitudes spatially determined as *lines* also produces a plane from the linear determination; three times four linear feet gives twelve linear feet, but three linear feet times four linear feet gives twelve superficial feet and, in fact, square feet, since the unit in both factors as discrete quantities is the same. The multiplication of lines by lines at first sight appears meaningless in so far as multiplication concerns simply numbers, i.e. is an alteration of a subject matter that is perfectly homogeneous with what it passes over into, with the product, only the *magnitude* being altered. On the other hand, what was called multiplication of a line as such by a line—it has been called *ductus lineae in lineam*, like *plani in planum*, and is also *ductus puncti in lineam*—is an alteration not merely of magnitude, but of magnitude as a *qualitative* determination of spatial character, of a dimension; the transition of the line into a plane must be understood as the *self-externalization* of the line, and similarly the self-externalization of the point is a line, and of the plane a whole space [volume]. This is the same as the representation of the line

as the *motion* of a point, and so forth; but motion includes a determination of time and thus appears in this representation rather as merely a contingent, external alteration of state. The transition must be grasped from the standpoint of the Notion which was expressed as a self-externalization—the qualitative alteration which arithmetically is the multiplication of unit (point, etc.) by amount (line, etc.). We may here further remark that with the self-externalization of the plane, which would appear as a multiplication of the plane by a plane, there is seemingly a difference between the arithmetical and geometrical operations such that the self-externalization of the plane as *ductus plani in planum* would give arithmetically a multiplication of a two-dimensional factor by another such, and consequently a four-dimensional product which, however, is reduced by the geometrical determination to three. Although on the one hand number, because its principle is the one, yields the fixed determination for the external, quantitative element, yet equally, the result of operating with it is formal. Taken as a numerical determination 3×3 when it reproduces itself is $3 \times 3 \times 3 \times 3$; but this same magnitude as a plane, when it reproduces itself, is restricted to $3 \times 3 \times 3$, because space, represented as an expansion outwards from the point, from the merely abstract limit, has its true limit as a *concrete* determinateness beyond the line in the third dimension. The difference referred to could prove itself effective as regards free motion in which one side, the spatial side has a geometrical significance (in Kepler's law, $s^3 : t^2$) and the other, the temporal side, is an arithmetical determination.

It will now be evident, without further comment, how the qualitative element here considered differs from the subject of the previous Remark. There, the qualitative element lay in the determinateness of power; here, like the infinitely small, it is only the factor as arithmetically related to the product, or as the point to the line or the line to the plane, and so on. Now the qualitative transition which has to be made from the discrete (into which continuous magnitude is imagined to be resolved), to the continuous, is effected as a process of summation.

But that the alleged pure summation does in fact include a multiplication and therefore the transition from linear to planar dimensions, this comes to view most simply in the way in which, for example, it is shown that the area of a trapezium is equal to

the product of the sum of the two opposite parallel lines and half the height. This height is represented as being merely the *amount* of a multitude of *discrete* magnitudes which must be summed up. These magnitudes are lines which lie parallel between the said limiting parallels; there are infinitely many of them for they are supposed to constitute the plane, and yet are lines which therefore, in order to possess the character of a plane, must at the same time be posited with negation. In order to escape the difficulty that a sum of lines is supposed to give a plane, the lines are directly assumed to be planes but also as *infinitely narrow*, for they have their determination solely in the linear quality of the parallel limits of the trapezium. As parallel and bounded by the other pair of rectilinear sides of the trapezium, these lines can be represented as the terms of an arithmetical progression, having a simply uniform difference which does not, however, require to be determined, and whose first and last terms are these two parallel lines; as we know, the sum of such a series is the *product* of the parallels and half the *amount* or *number* of terms. This last quantum is called *amount* or *number* simply and solely with reference to the conception of infinitely many lines; it is simply the specific magnitude of something which is *continuous*—the height. It is clear that what is called a sum is at the same time a *ductus lineae in lineam*, a *multiplication* of lines by lines, and so according to the above determination the result is something having the quality of a plane. In the simplest case of any rectangle AB, each of the two factors is a simple magnitude; but even in the further, still elementary example of the trapezium, only one of the factors is simple as half of the height. The other, on the contrary, is determined by a progression; it is also linear but its specific magnitude is more complex; and since it can be expressed only by a series, the problem of summing it is called analytical, i.e. arithmetical; but the geometrical moment in it is multiplication, the qualitative element of the transition from the dimension of line to that of plane; the one factor was taken to be *discrete* only for the arithmetical determination of the other; by itself it is, like the other, the magnitude of a line.

But the method of representing planes as sums of lines is also often employed when multiplication as such is not used to produce the result. This happens when the problem is to indicate the magnitude in the equation not as a quantum but as a proportion.

It is, for example, a familiar way of showing that the area of a circle bears the same proportion to the area of an ellipse, the major axis of which is the diameter of the circle, as the major axis does to the minor axis, each of these areas being taken as the *sum* of the relative *ordinates*; each ordinate of the ellipse is to the corresponding ordinate of the circle as is the minor to the major axis; therefore, it is concluded, the *sums* of the ordinates, i.e. the *areas*, are also in the same proportion. Those who want to avoid here the representation of an area as a sum of lines resort to the usual, quite unnecessary expedient of making the ordinates into *trapezia* of infinitely small breadth; since the equation is only a proportion, only one of the two linear elements of the plane comes into the comparison. The other, the abscissa axis, is assumed as equal in ellipse and circle, as a factor therefore arithmetically equal to 1, and consequently the proportion depends solely on the relation of the one determining moment. The two dimensions are necessary to the *representation* of a plane, but the quantitative determination required to be indicated in this proportion affects only the *one* moment; to be swayed by the representation of a plane, or to help it out by adding the idea of *sum* to this *one* moment, is really to fail to recognize the essential mathematical element here involved.

The foregoing exposition also contains the criterion for Cavalieri's method of indivisibles referred to above which equally is justified by it and does not need to be helped out by the infinitely small. These indivisibles are lines when he is considering a plane, and squares or plane circles when he is considering a pyramid or a cone, etc. The base line or basic plane which is assumed as determined, he calls the *regula*; it is the constant, and with reference to a series it is its first or last term. The indivisibles are regarded as parallel with the *regula* and therefore as having the same determination as this with respect to the figure. Now Cavalieri's general fundamental proposition is 'that all figures, both plane and solid, are *proportionate* to all their indivisibles, these being compared with each other collectively and, if there is a common proportion in the figures, distributively'.[1] For this purpose, he compares in figures of the *same* base and height, the proportions between lines drawn parallel to the base and *equidistant* from it; all such lines in a figure have

[1] *Exerc. Geometr.*, VI: in the later work: *Exerc.*, I, p. 6.

one and the same determination and constitute its whole content. In this way, too, Cavalieri proves for example the elementary proposition that parallelograms of equal height are proportional to their bases; any two lines drawn equidistant from and parallel to the base in both figures are in the same proportion as the base lines and so therefore are the whole figures. The lines do not in fact constitute the whole content of the figure as *continuous*, but only the content in so far as it is to be arithmetically *determined*; it is the line which is the element of the content and through it alone must be grasped the specific nature of the figure.

This leads us to reflect on the difference which exists with respect to that feature into which the determinateness of a figure falls; this is either the *height*, as here, or an *external limit*. Where this determinateness is an external limit, it is admitted that the *continuity* of the figure, so to speak, *follows upon* the equality or the proportion of the limit; for example, the equality of figures which *coincide* follows from the fact that their boundaries coincide. But in parallelograms of equal height and base, only the latter determinateness is an external limit; the height (not the parallelism as such), on which is based the second main determination of the figures, their *proportion*, introduces a second principle of determination additional to the external limits. Euclid's proof of the equality of parallelograms having the same height and base reduces them to triangles, to *continuous* figures *limited externally*; in Cavalieri's proof, primarily in that of the proportionality of parallelograms, the limit is simply a *quantitative determinateness as such*, which is explicated in every pair of lines drawn at the same distance from each other in both figures. These lines, which are equal or in an equal ratio with the base, taken *collectively* give figures standing in the same ratio. The conception of an *aggregate* of lines is incompatible with the continuity of the figure; the essential determinateness is completely exhausted by a consideration of the lines alone. Cavalieri frequently answers the objection that the conception of indivisibles involves the comparison of lines or planes as if they were infinite in *amount*;[1] he makes the correct distinction that he does not compare their *amount*, which we do not know (and which is, as we have remarked, merely an empty idea assumed in support of the theory), but only the *magnitude*, i.e. the the quantitative determinateness as such which

[1] *Geom.*, Lib. II. Prop. I. Schol.

is equal to the space occupied by these lines; because this space is enclosed within limits, its magnitude too is enclosed within the same limits; *the continuous figure is nothing other than the indivisibles themselves*, he says; if it were something apart from them it would not be comparable; but it would be absurd to say that bounded continuous figures were not comparable with each other.

It is evident that Cavalieri means to distinguish what belongs to the *outer existence* of the continuous figure from what constitutes its *determinateness*; it is the latter alone to which we must attend when comparing the continuous or constructing theorems about it. The categories he employs in this connection, namely that the continuous is *composed* or *consists* of indivisibles, and the like, are of course inadequate since they demand at the same time the intuition of the continuous or, as already said, its outer existence; instead of saying that 'the continuous is nothing other than the indivisibles themselves', it would be more correct and also directly self-explanatory to say that the quantitative determinateness of the continuous is none other than that of the indivisibles themselves. Calvalieri does not support the erroneous conclusion that there are greater and lesser infinites which is drawn *by the schools* from the idea that the indivisibles constitute the continuous; further on[1] he gives expression to his quite definite awareness that his method of proof by no means forces on him the idea that the continuous is composed of indivisibles; continuous figures follow only the *proportion* of the indivisibles. He says he has taken the aggregate of indivisibles not as an apparent infinity for the sake of an *infinite* number of lines or planes, but in so far as they possess a specific kind of limitedness. But then, in order to remove this stumbling block, he does not spare himself the trouble of proving the principal proportions of his geometry (in the seventh book specially added for this purpose) in a way which remains free from any admixture of infinity. This method reduces the proofs to the usual form of the *coincidence* of figures above mentioned, i.e. as remarked, to the conception of determinateness as an external spatial limit.

About this form of coincidence, we may add that it is, on the whole, a so to speak childish aid for sense perception. In the elementary theorems about triangles, two traingles are represented side by side; of their six component parts, three are

[1] *Geom.*, Lib. VII. Praef.

assumed equal to the corresponding three of the other triangle and it is then shown that such triangles are congruent, i.e. that the remaining three parts of each triangle are also equal to those of the other triangle—because by virtue of the equality of the first three parts, the triangles *coincide*. If we take the matter more abstractly, then it is just because of this equality of each pair of corresponding parts in both triangles that there is only *one* triangle before us; in this it is assumed that three parts are *already determined* and from this follows the *determinateness* of the three remaining parts. In this way, the determinateness is exhibited as completed in the three parts; hence for the determinateness as such, the three remaining parts are a superfluity, the superfluity of sensuous existence, i.e. of the intuition of continuity. Expressed in this form, the qualitative determinateness stands out in its distinction from what is given in intuition, from the whole as continuous within itself; the form of coincidence does not bring this distinction to notice.

With parallel lines and with parallelograms there enters, as we have observed, another factor, partly the equality of the angles only and partly the height of the figures, from which latter their external limits, the sides of the parallelograms, are distinct. This gives rise to uncertainty whether in these figures, besides the determinateness of one side, the base, which is an external limit, we are to take for the other determinateness, the other external limit, namely the other side of the parallelogram, or else the height. In the case of two such figures having the same base and height, one of them being rectangular and the other having very acute angles (the opposite angles therefore being very obtuse), the latter triangle can easily look greater than the former, when its long side is taken as the determinant and, as in Cavalieri's method, the *planes* are compared according to the *aggregate* of parallel lines intersecting them; the *longer* side can be regarded as a potentiality of *more* lines than is given by the vertical side of the rectangle. Such a conception, however, is no argument against Cavalieri's method; the *aggregate* of parallel lines imagined in the two parallelograms, for the purpose of comparison, also presupposes the equidistance of the lines from each other or from the base; from which it follows that the height, and not the other side of the parallelogram, is the other determining moment. But the case is different again when the comparison is between two

parallelograms having the same height and base but not lying in the same plane and making different angles with a third plane; here, the parallel sections which arise, when the third plane is imagined as cutting through the parallelograms and moving parallel to itself are no longer equidistant from each other and the two planes are unequal. Cavalieri is very careful to draw attention to this distinction which he defines as the difference between a *transitus rectus* and a *transitus obliquus* of the indivisibles and thus prevents a superficial misunderstanding which could arise on this point.[1] I remember that an objection to indivisibles by Tacquet,[2] an acute geometer who was also working at this time on new methods, concerns this very point: it is referred to in the above-mentioned work of Barrow[3] who also uses the method of indivisibles, although with him it is tainted with the assumption (which he passed on to his pupil Newton and other contemporary mathematicians including Leibniz, too), that a curvilinear triangle, like the so-called characteristic triangle, may be equated with a rectilinear triangle if both are infinitely, that is, *very* small. The difficulty raised by Tacquet likewise concerns the question which line, in the calculation of conical and spherical surfaces, should be taken as the basis of determination in the method based on the employment of the discrete. Tacquet's objection to the method of indivisibles is that in the calculation of the *surface* of a right-angled cone, this atomistic method represents the triangle of the cone to be composed of straight *lines* parallel to the base and perpendicular to the axis, which are at the same time *radii of the circles* of which the *surface* of the cone consists. Now if this surface is defined as a sum of the circumferences and this sum is determined from the number of their radii, i.e. from the length of the axis or the height of the cone, then, says Tacquet, such a result clashes with the truth formerly taught and demonstrated by Archimedes. Now Barrow counters this by showing that to determine the surface it is not the axis but the *side* of the triangle of the cone which must be taken as the line the revolution of which generates the surface; consequently, it is this line and not the axis which must be assumed as the specific magnitude for the aggregate of the circumferences.

[1] Both in *Exercit.*, I *n*, XII *sqq.* and in *Geometr.*, I. II.

[2] Tacquet, Andr., 1611–1660, Professor in the Jesuit College at Antwerp: *Cylindricorum Et Annularium*, Libri V, 1651–9.

[3] *Lect. Geom.*, II, p. 21.

Objections and uncertainties of this kind have their origin solely in the indefinite idea employed of the *infinite* aggregate of points of which the line (or of lines of which the plane) is supposed to consist; this idea obscures the essential determinateness of the magnitude of the lines or planes.

The intention of these Remarks has been to bring to notice the *affirmative* meanings which, in the various applications of the infinitely small in mathematics, remain so to speak in the background, and to lift them out of the nebulosity in which that category, merely negatively held, has concealed them. In infinite series, as in Archimedes' cyclometry, the meaning of the infinite is nothing more than this: that the law determining the series being known, but the so-called *finite*—i.e. arithmetical—expression not being given, the reduction of the arc to the straight line cannot be effected; this incommensurability is their qualitative difference. The qualitative difference between the discrete and the continuous generally, equally contains a negative determination which makes them appear as incommensurable and introduces the infinite in the sense that the continuous which is to be taken as discrete, is no longer to possess, as continuous, any quantum. The continuous which is to be taken arithmetically as a *product*, is therefore posited as in its own self discrete, i.e. it is analysed into the elements which are its factors and in these lies its quantitative determinateness; just because they are these factors or elements, they are of a lower dimension, and as powers are concerned, are of a lower power than the magnitude of which they are the elements or factors. This difference appears arithmetically as a purely quantitative one, that of the root and power, or whatever degree of powers it may be; however, if the expression is to be taken only quantitatively, for example, $a : a^2$ or $d.a^2 = 2a : a^2 = 2 : a$, or for the law of descent of a falling body, $t : a^2$, then it yields the meaningless ratios of $1 : a$, $2 : a$, $1 : at$; in supersession of their merely quantitative aspect, the sides would have to be held apart by their different qualitative significance, as $s = at^2$, the magnitude in this way being expressed as a quality, as a function of the magnitude of another quality. Here then, we are faced merely with a quantitative determinateness; there is no difficulty in operating with this in accordance with its own manner and no objection can be offered if the magnitude of one line is multiplied by the magnitude of another;

but the multiplication of these same magnitudes at the same time results in the qualitative alteration of the transition from line into plane; and to that extent a negative determination comes into play. It is this that occasions the difficulty, a difficulty which is resolved by an insight into its peculiarity and into the simple nature of the matter; but the introduction of the infinite which is meant to remove the difficulty only serves to aggravate it and prevent its solution.

CHAPTER 3

THE QUANTITATIVE RELATION OR QUANTITATIVE RATIO

The infinity of quantum has been determined to the stage where it is the negative beyond of quantum, which beyond, however, is contained within the quantum itself. This beyond is the qualitative moment as such. The infinite quantum as the unity of both moments, of the quantitative and qualitative determinateness, is in the first instance a *ratio*.

In the ratio, quantum no longer has merely an indifferent determinateness but is qualitatively determined as simply related to its beyond. It continues itself into its beyond; this, in the first place, is simply *another* quantum. But they are essentially related to each other not as external quanta, but the determinateness of each consists in this relation to the other. In this their otherness they have thus returned into themselves; what each is, it is in the other; the other constitutes the determinateness of each. The flight of quantum away from and beyond itself has now therefore this meaning, that it changed not merely into an other, or into its abstract other, into its negative beyond, but that in this other it reached its determinateness, finding *itself* in its beyond, which is another quantum. The *quality* of quantum, the specific nature of its Notion, is its externality as such, and in ratio the quantum is now *posited* as having its determinateness in its externality, in another quantum, and as being in its beyond what it is.

They are quanta which stand to each other in the relation just described. This *relation* is itself also a magnitude; the quantum is not only *in* a ratio, but it is itself *posited as a ratio*; there is only a *single* quantum and this has the said qualitative determinateness *within itself*. As such ratio, it is a self-enclosed totality and indifferent to limit, and it expresses this by containing within itself the externality of its determining and by being in this externality related only to itself. It is therefore, in its own self infinite.

Ratio as such is:

1. *direct* ratio. In this, the qualitative moment does not yet emerge explicitly as such; its mode is still only that of quantum, namely, to be posited as having its determinateness in its very externality. The quantitative ratio is in itself the contradiction of externality and self-relation, of the affirmative being of quanta and their negation; next, it is sublated
2. in the *indirect* or *inverse* ratio, in which is posited the negation of one of the quanta with the alteration of the other, and the alterableness of the direct relation itself;
3. in the *ratio of powers*, however, the unity which in its difference is self-related, vindicates itself as a simple self-production of the quantum; this qualitative moment itself, when finally posited in a simple determination and as identical with the quantum, becomes *measure.*

About the nature of the following ratios, much has been anticipated in the preceding Remarks concerning the infinite of quantity, i.e., the qualitative moment in it; it only remains therefore to expound the abstract Notion of these ratios.

A. THE DIRECT RATIO

1. In the ratio which, as immediate, is *direct*, the determinateness of either quantum lies reciprocally in the determinateness of the other. There is only *one* determinateness or limit of both and this is itself a quantum, namely, the *exponent* of the ratio.

2. The exponent is any quantum; but it is *self-related* in its own externality and a qualitatively determined quantum only in so far as it has within itself its own difference, its beyond and its otherness. This difference of quantum present within it is, however, the difference of unit and amount, unit being a being which is determined as for-itself, and amount the indifferent fluctuation of the determinateness, the external indifference of quantum. Unit and amount were at first *moments* of quantum; now, in the ratio, in the realisation so far of quantum, each of its moments appears as a quantum on its own, and as a determination of its existence—as a limiting of the otherwise merely external, indifferent determinateness of quantity.

The exponent is this difference as a simple determinateness, i.e. it has immediately within it the significance of both determinations. It is *first* of all a quantum. As such it is amount; if the one side of the ratio which is taken as unit is expressed numerically as one—and it counts only as such—then the other, the amount, is the quantum of the exponent itself. *Secondly*, it is simple determinateness as the qualitative moment of the sides of the ratio. When the quantum of one side is determined, the other, too, is determined by the exponent and it is quite immaterial how the first is determined; it no longer has any significance as a determinate quantum on its own, but can equally well be any other quantum without altering the value of the ratio, which depends solely on the exponent. The one which is taken as unit always remains unit however great it becomes, and the other, no matter how great it, too, becomes in consequence, must remain the *same* amount of that unit.

3. The two therefore constitute strictly only *one* quantum; the one has relatively to the other only the value of unit, not of amount, the other only that of amount; consequently, *according to the specific nature of their Notion*, they themselves are *not complete quanta*. But this incompleteness is a negation in them and a negation, not as regards their alterableness generally in virtue of which one of them (and it can be either of the two) can assume any possible magnitude, but as regards the determination that when one is altered the other is increased or diminished by the same amount; this means, as has been shown, that the quantum of only one of them, the unit, is altered, while the other side, the amount, remains the same quantum *of units*; but the former too still *counts* only as a unit no matter how it is altered as quantum. Thus each side is only one of the two moments of quantum and the independence which belongs to the peculiar character of quantum is in principle *negated*; in this qualitative relationship they are to be *posited* as *negative* relatively to each other.

The exponent ought to be the complete quantum, since the determination of *both* sides coincides in it; but in fact, even as quotient it has the value only of amount or of unit. There is nothing to determine which side of the ratio must be taken as unit or which as amount; if the quantum B is measured in terms of quantum A as unit, then the quotient C is the amount of such units; but if A is itself taken as amount, the quotient C is the

unit which to the amount A is required for the quantum B. This quotient therefore is, as exponent, not posited as what it ought to be—the determinant of the ratio, or the ratio's qualitative unity. It is only posited as this in so far as it has the value of being the *unity of both moments*, of unit and amount. True, these sides are present as quanta as they should be in the explicated quantum, in the ratio; but at the same time they have only the value proper to them as sides of the ratio, namely to be *incomplete* quanta and to count only as one of those qualitative moments. They must therefore be posited with this their negation and thus there arises a more developed form of the ratio, one which corresponds more to its character, a ratio in which the exponent has the significance of the product of the sides. As thus determined, it is the *inverse* ratio.

B. INVERSE RATIO

1. The ratio as now before us is the *sublated* direct relation. At first, the ratio was *immediate* and therefore not yet truly determinate; the determinateness it now possesses is such that the exponent counts as a product, as a unity of unit and amount. As we have already seen, the exponent as immediate could equally well be taken either as unit or as amount; it was then also only a simple quantum and therefore, by choice, an amount; one side was the unit, to be taken as a numerical one, of which the other side is a fixed amount and, at the same time, the exponent; the quality of this latter, therefore, was simply that this quantum is taken as fixed, or rather that this fixed quantum has the meaning only of quantum.

Now in the inverse ratio the exponent as quantum is likewise immediate and is any quantum assumed as fixed. But this quantum is not a *fixed amount* to the *one* of the other quantum in the *ratio*; this ratio, which previously was fixed, is now on the contrary posited as alterable; if in place of the unit on one side of the ratio another quantum is taken, then the other side is no longer the *same amount* of units of the first side. In the direct ratio this unit is only the common element of both sides; as such, it continues itself into the other side, into the amount; and the amount itself taken by itself, or the exponent, is indifferent to the unit.

But now the ratio is so determined that the amount as such is

altered relatively to the other side of the ratio, to the unit; when another quantum is taken for the unit, the quantum of amount also is altered. Consequently, although the exponent is also only an immediate quantum arbitrarily assumed as fixed, it is not preserved as such in the side of the ratio, but this side, and with it the direct ratio of the sides, is alterable. In the ratio, then, as now before us, the exponent as the determining quantum is posited as negative towards itself as a quantum of the ratio, and hence as qualitative, as a limit—with the result that the qualitative moment is manifested independently and in distinct contrast to the quantitative moment. In the direct ratio, the *alteration* of the two sides is only the one alteration of the quantum which is taken as unit, the common element of both sides; by as many times as the one side is increased or diminished, so also is the other: the ratio itself is not affected by this alteration which is external to it. In the indirect ratio on the other hand, although the alteration, in keeping with the indifferent quantitative moment is also arbitrary, it is *confined within the ratio*, and this arbitrary quantitative fluctuation, too, is limited by the negative determinateness of the exponent as by a limit.

2. We have now to consider more closely this qualitative nature of the inverse ratio, more particularly in its realisation, and to unravel the entanglement of the affirmative moment with the negative which is contained in it. The indirect or inverse ratio is quantum posited as a qualitative quantum, i.e., displaying itself as self-determining, as a limit of itself within itself. As such, the quantum is, *first*, an immediate magnitude as a *simple* determinateness, the *whole* as a quantum simply affirmatively present. But secondly, this immediate *determinateness* is also a *limit*; for that purpose the quantum is differentiated into two quanta which in the first instance are mutually related as others; but the quantum as their qualitative and, moreover, complete determinateness is the unity of unit and amount, a product of which the two quanta are the factors. Thus on the one hand the exponent of their ratio is in them identical with itself and is their affirmative moment which constitutes them quanta; on the other hand the exponent, as the negation posited in them, is the *unity* in them, so that although each is in the first place simply an immediate, limited quantum, it is at the same time limited in such a manner that it is only *in principle* [*an sich*] identical with its other. Thirdly,

the exponent as the simple determinateness is the negative unity of this differentiation of itself into two quanta and is the limit of their reciprocal limiting.

In conformity with these determinations, each of the two moments has its *limit* within the exponent, and since this is their specified unity each is the negative of the other; one of them becomes as many times smaller as the other becomes greater, the magnitude of each depending on its containing the magnitude which the other lacks. Each in this way continues itself *negatively* into the other; to the extent that each is amount, the amount of the other is cancelled, and each is what it is only through the negation or limit posited in it by the other. In this manner each also *contains* the other and is measured by it, for each is supposed to be only that quantum which the other is not; the magnitude of the other is an indispensable factor in the value of each and is therefore inseparable from it.

This continuity of each in the other constitutes the moment of *unity* through which they are in ratio—the moment of the *one* determinateness, of the simple limit which is the exponent. This unity, the whole, constitutes the *in-itself*, the *principle* of each, from which their *actual* magnitude is distinct; in accordance with the latter, each only *is* to the extent that it takes from the other a part of their common in-itself, the whole. But it can take from the other only as much as will make its own self equal to this in-itself; it has its maximum in the exponent which in accordance with the stated second determination is the limit of their reciprocal limiting. And since each is a moment of the ratio only in so far as it limits the other and is simultaneously limited by it, it loses this its determination in making itself equal to its in-itself; for in so doing not only does the *other* magnitude become zero, but it vanishes itself, since what it is supposed to be is not a mere quantum as such, but only quantum as such moment of a ratio. Thus each side is the contradiction between its determination as the in-itself, i.e. as unity of the whole, which is the exponent, and its determination as moment of the ratio; this contradiction is *infinity* again in a fresh, peculiar form.

The exponent is a *limit* of the sides of its ratio within which they increase and decrease relatively to each other; but they cannot become equal to the exponent because of the latter's affirmative determinateness as quantum. As thus the limit of their

reciprocal limiting, the exponent, is (α) their beyond, to which they *infinitely* approximate but which they cannot reach. This infinity in which they approach their beyond is the spurious infinity of the infinite progress; it is itself finite, is bounded by its opposite, by the finitude of each side and of the exponent itself and is consequently only *approximation*. But (β) the spurious infinity is here also *posited* as what it is *in truth*, namely, as only the *negative moment* in general, in accordance with which the exponent is the *simple limit* of the differentiated quanta of the ratio as their in-itself; their finitude, as their simple alterableness, is related to this in-itself which, however, remains absolutely distinct from them as their negation. This infinity, then, to which these quanta can only approximate is likewise *affirmatively* present and actual—the simple quantum of the exponent. In it is reached the beyond with which the sides of the ratio are burdened; it is *implicitly* [*an sich*] the unity of both, and so implicitly the other side of each; for each has only as much value as the other has not, the whole determinateness of each thus resides in the other, and this their in-itself as an affirmative infinity is simply the exponent.

3. The outcome of this, however, is the transition of the inverse ratio into a different determination from that which it had at first. This consisted in an immediate quantum being also related to another quantum in such a way that its increase is proportional to the decrease of the other, that it is what it is through a negative relationship with the other; also, a third magnitude is the common limit of this their fluctuating increase. This fluctuation here is their distinctive character—in contrast to the qualitative moment as a *fixed* limit; they have the character of *variable* magnitudes, for which the said fixed limit is an infinite beyond.

But the determinations which have emerged and which we have to summarize are not only that this infinite beyond is at the same time some present finite quantum or other, but that its fixity—which constitutes it such infinite beyond relatively to the quantitative moment, and which is the qualitative moment of being only as abstract self-relation—has developed as a mediation of itself with itself in its other, in the finite terms of the ratio. The general result can be indicated by saying that the whole, as exponent, is the limit of the reciprocal limiting of both terms and is therefore posited as *negation of the negation*, hence as infinity, as an *affirmative* relation to itself. More specifically the

exponent, simply as product, is *implicitly* the unity of unit and amount, but as each term is only one of these two moments, the exponent also includes them within itself and in them is *implicitly* related to itself. But in the inverse ratio, the difference has developed into the *externality* of quantitative being, and the qualitative moment is not merely the fixity of the exponent, or merely the immediate inclusion in it of the two moments of unit and amount, but is the identification of the exponent with itself in its *self-external otherness*. It is this determination which stands out as result in these moments as explicated. The exponent, namely, is found to be the in-itself which is realized in the simple alterableness of its moments as quanta; the indifference of their magnitudes in their alteration is displayed as an infinite progress, the basis of which is that in their indifference their determinateness is to have their value in the value of the other. Hence (α), in accordance with the affirmative aspect of the quantum, each is *implicitly* the whole of the exponent. Similarly (β), the quanta have for their negative moment, for their reciprocal limiting, the magnitude of the exponent; their limit is that of the exponent. The fact that they no longer have any other immanent limit, a fixed immediacy, finds expression in the infinite progress of their determinate being and of their limitation, in the negation of every particular value. This negation is, accordingly, the *negation* of the self-externality of the exponent which is displayed in the moments of the ratio; and the exponent, which is itself a simple quantum and is also differentiated into quanta, is, therefore, posited as preserving itself and uniting with itself in the negation of the indifferent existence of the quanta, thus being the determinant of its self-external otherness.

The ratio is now specified as the *ratio of powers*.

C. THE RATIO OF POWERS

1. The quantum which, in its otherness, is identical with itself and which determines the beyond of itself, has reached the stage of being-for-self. As a qualitative totality—for it posits itself as *developed*—it has for its moments the determinations of the Notion of number, unit and amount; in the inverse ratio, the latter is still a plurality determined not by the unit itself as such, but from elsewhere, by a Third; but now it is posited as determined

only by the unit. This is the case in the ratio of powers where the unit, which in its own self is amount, is also amount relatively to itself as unit. The otherness, the amount of units, is the *unit* itself. The power is a plurality of units each of which is this same plurality. The quantum as an indifferent determinateness undergoes alteration; but in so far as this alteration is a raising to a power, this its otherness is limited purely by itself. Thus in the power, quantum is posited as returned into itself; it is at once its own self and also its otherness.

The *exponent* of this ratio is no longer an immediate quantum as it is in the direct ratio and also in the inverse ratio. In the ratio of powers it is of a wholly *qualitative* nature—this *simple* determinateness that the amount is the unit itself, that the quantum in its otherness is *identical* with itself. In this is also contained the *quantitative* aspect of its nature, namely, that the limit or negation is not present simply affirmatively as an immediacy, but that the determinate being [of the quantum] is posited as continued into its otherness; for the truth of quality is just this, to be quantity, immediate determinateness as sublated.

2. The ratio of powers appears at first to be an external alteration to which any quantum can be subjected; but it has a closer connection with the *Notion* of quantum: namely, that in the determinate being into which it has developed in the ratio of powers, quantum has reached its Notion and has completely realized it. This ratio is the display of what quantum is *in itself* and it expresses that determinateness or *quality* of quantum which is its distinctive feature. Quantum is the *indifferent* determinateness, i.e., *posited* as *sublated*, determinateness as a limit which is equally no limit, which continues itself into its otherness and so remains identical with itself therein. In the ratio of powers this quality of quantum is *posited*; quantum itself determines its otherness, its going beyond itself into another quantum.

Comparing the progressive realization of quantum in the preceding ratios, we see that the quality of quantum as the posited difference of itself from itself is simply this: to be a ratio. As a direct ratio it is at first only the simple or unmediated form of such posited difference, so that its self-relation which it has as exponent, in contrast to its differences, counts only as the fixity of an amount of the unit. In the inverse ratio, the quantum is negatively determined as relating itself to itself—to itself as a

negation of itself in which, however, it has its value; as an affirmative relation to itself it is an exponent which, as quantum, is only *in principle* the determinant of its moments. But in the ratio of powers, quantum is present in the difference as *its own difference from itself*. The *externality* of the determinateness is the quality of quantum and this externality is now posited in conformity with the Notion of quantum, as the latter's own self-determining, as its relation to its own self, as its *quality*.

3. But with the *positing* of quantum in conformity with its Notion, it has undergone transition into another determination; or, as we may also express it, its *determination* is now also a *determinateness*, what quantum is *in principle* it is now also *in reality*. It is *quantum* in so far as the externality or indifference of its determining (as it is said, it is that which can be increased or decreased) counts and is posited only *simply* or *immediately*; it has become the other of itself, namely, quality—in so far as this externality is now *posited* as mediated by quantum itself, and thus as moment of itself—so that *in this very externality quantum is self-related*, is *being* as quality.

At first, then, quantity as such appears in opposition to quality; but quantity is itself *a* quality, a purely self-related determinateness distinct from the determinateness of its other, from quality as such. But quantity is not only *a* quality; it is the truth of quality itself, the latter having exhibited its own transition into quantity. Quantity, on the other hand, is in its truth the externality which is no longer indifferent but has returned into itself. It is thus quality itself in such a manner that apart from this determination there would no longer be any quality as such. The *positing* of the totality requires the *double* transition, not only of the one determinateness into its other, but equally the transition of this other, its return, into the first. The first transition yields the identity of both, but at first only *in itself* or in principle; quality is contained in quantity, but this is still a one-sided determinateness. That the converse is equally true, namely, that quantity is contained in quality and is equally only a sublated determinateness, this results from the second transition—the return into the first determinateness. This observation on the necessity of the *double* transition is of great importance throughout the whole compass of scientific method.

Quantum is now no longer an indifferent or external determina-

tion but as such is sublated and is quality, and is that by virtue of which something is what it is; this is the truth of quantum, to be Measure.

Remark

In the Remarks above on the quantitative infinite, it was shown that this infinite and also the difficulties associated with it have their origin in the *qualitative* moment which makes its appearance in the sphere of quantity, and also how the qualitative moment of the ratio of powers in particular is the source of various developments and complexities. It was shown that the chief obstacle to a grasp of the Notion of this infinite is the stopping short at its merely negative determination as the negation of quantum, instead of advancing to the simple affirmative determination which is the qualitative moment. The only further remark to be made here concerns the intrusion of quantitative forms into the pure qualitative forms of thought in philosophy. It is the relationship of powers in particular which has been applied recently to the determinations of the Notion. The Notion in its immediacy was called the *first power* or *potence*; in its otherness or difference, in the determinate being of its moments, the *second power*; and in its return into itself or as a totality, the *third power*. It is at once evident that *power* as used thus is a category which essentially belongs to quantum—these *powers* do not bear the meaning of the *potentia*, the δύναμις of Aristotle. Thus, the relationship of powers expresses determinateness in the form or difference which has reached its truth, but difference as it is in the *particular Notion* of quantum, not as it is in the Notion as such. In quantum, the negativity which belongs to the nature of the Notion is still far from being posited in the determination proper to the Notion; differences which are proper to quantum are superficial determinations for the Notion itself and are still far from being determined as they are in the Notion. It was in the infancy of philosophic thinking that numbers were used, as by Pythagoras, to designate universal, essential distinctions—and first and second power, and so on are in this respect not a whit better than numbers. This was a preliminary stage to comprehension in the element of pure thought; it was not until after Pythagoras that thought determinations themselves were discovered, i.e., became *on their own account* objects for consciousness. But to retrogress from such determina-

tions to those of number is the action of a thinking which feels its own incapacity, a thinking which, in opposition to current philosophical culture which is accustomed to thought determinations, now also makes itself ridiculous by pretending that this impotence is something new, superior, and an advance.

There is as little to be said against the expression *power* when it is used only as a *symbol*, as there is against the use of numbers or any other kind of symbols for Notions—but also there is just as much to be said against them as against all symbolism whatever in which pure determinations of the Notion or of philosophy are supposed to be represented. Philosophy needs no such help either from the world of sense or from the products of the imagination, or from subordinate spheres in its own peculiar province, for the determinations of such spheres are unfitted for higher spheres and for the whole. This unfitness is manifest whenever categories of the finite are applied to the infinite; the current determinations of force, or substantiality, cause and effect, and so on, are likewise only symbols for expressing, for example, vital or spiritual relationships, i.e. they are untrue determinations for such relationships; and still more so are the powers of quantum and degrees of powers, both for such and for speculative relationships generally. If numbers, powers, the mathematical infinite, and suchlike are to be used not as symbols but as forms for philosophical determinations and hence themselves as philosophical forms, then it would be necessary first of all to demonstrate their philosophical meaning, i.e. the specific nature of their Notion. If this is done, then they themselves are superfluous designations; the determinateness of the Notion specifies its own self and its specification alone is the correct and fitting designation. The use of those forms is, therefore, nothing more than a convenient means of evading the task of grasping the determinations of the Notion, of specifying and of justifying them.

Section Three: Measure

Abstractly expressed, in measure quality and quantity are united. *Being* as such is an immediate identity of the determinateness with itself. This immediacy of the determinateness has sublated itself. *Quantity* is being which has returned into itself in such a manner that it is a simple self-identity as indifference to the determinateness. But this indifference is only the externality of having the determinateness not in its own self but in an other. Thirdly, we now have self-related externality; as self-related it is also a *sublated* externality and has within itself the difference from itself—the difference which, as an externality is the *quantitative*, and as taken back into itself is the *qualitative*, moment.

In transcendental idealism the categories of quantity and quality are followed, after the insertion of relation, by *modality*, which may therefore be mentioned here. This category has there the meaning of being the relation of the object to thought. According to that idealism thought generally is essentially external to the thing-in-itself. In so far as the other categories have only the transcendental character of belonging to consciousness, but to the *objective* element of it, so modality as the category of relation to the subject, to this extent contains relatively the determination of *reflection*-into-self; i.e. the objectivity which belongs to the other categories is lacking in the categories of modality; these, according to Kant, do not in the least add to the concept as a determination of the object but only express the relation to the faculty of cognition.[1] The categories which Kant groups under modality—namely, possibility, actuality and necessity—will occur later in their proper place; Kant did not apply the infinitely inportant form of triplicity—with him it manifested itself at first only as a formal spark of light—to the genera of his categories (quantity, quality, etc.), but only to their species which, too, alone he called categories. Consequently he was unable to hit on the third to quality and quantity.

With Spinoza, the *mode* is likewise the third after substance and attribute; he explains it to be the *affections* of substance, or that element which is in an other through which it is comprehended. According to this concept, this third is only externality as such;

[1] *Critique of Pure Reason*, 2nd ed., pp. 99, 266.

as has already been mentioned, with Spinoza generally, the rigid nature of substance lacks the return into itself.

The observation here made extends generally to those systems of pantheism which have been partially developed by thought. The first is being, the one, substance, the infinite, essence; in contrast to this abstraction the second, namely, all determinateness in general, what is only finite, accidental, perishable, non-essential, etc. can equally abstractly be grouped together; and this is what usually happens as the next step in quite formal thinking. But the connection of this second with the first is so evident that one cannot avoid grasping it as also in a unity with the latter; thus with Spinoza, the attribute is the whole substance, but is apprehended by the intellect which is itself a limitation or mode; but in this way the mode, the non-substantial generally, which can only be grasped through an other, constitutes the other extreme to substance, the third generally. Indian pantheism, too, in its monstrous fantasies has in an abstract way received this development which runs like a moderating thread through its extravagances; a point of some interest in the development is that Brahma, the one of abstract thought, progresses through the shape of Vishnu, particularly in the form of Krishna, to a third form, that of Siva. The determination of this third is the mode, alteration, coming-to-be and ceasing-to-be—the field of externality in general. This Indian trinity has misled to a comparison with the Christian and it is true that in them a common element of the nature of the Notion can be recognized; but it is essential to gain a more precise consciousness of the difference between them; for not only is this difference infinite, but it is the true, the genuine infinite which constitutes it. This third principle is, according to its determination, the dispersal of the unity of substance into its opposite, ***not the return of the unity to itself***—not spirit but rather the non-spiritual. In the true trinity there is not only unity but union, the conclusion of the syllogism is a unity possessing ***content*** and ***actuality***, a unity which in its wholly concrete determination is spirit. This principle of the mode and of alteration does not, it is true, altogether exclude the unity; in Spinozism, for example, it is precisely the mode as such which is untrue; substance alone is true and to it everything must be brought back. But this is only to submerge all content in the void, in a merely formal unity lacking all content. Thus Siva,

too, is again the great whole, not distinct from Brahma, but Brahma himself. In other words, the difference and the determinateness only vanish again but are not preserved, are not sublated, and the unity does not become a concrete unity, neither is the disunity reconciled. The supreme goal for man placed in the sphere of coming-to-be and ceasing-to-be, of modality generally, is submergence in unconsciousness, unity with Brahma, annihilation; the Buddhist Nirvana, Nibbana etc., is the same.

Now although the mode as such is abstract externality, indifference to qualitative and quantitative determinations, and in essence the external and unessential elements are not supposed to count, it is still, on the other hand, admitted in many cases that everything depends on the kind and manner of the mode; such an admission means that the mode itself is declared to belong essentially to the substantial nature of a thing, a very indefinite connection but one which at least implies that this external element is not so abstractly an externality.

Here the mode has the specific meaning of measure. Spinoza's mode, like the Indian principle of change, is the measureless. The Greek awareness, itself still indeterminate, that everything has a measure—even Parmenides, after abstract being, introduced necessity as the ancient limit by which all things are bounded—is the beginning of a much higher conception than that contained in substance and in the difference of the mode from substance.

Measure in its more developed, more reflected form is necessity; fate, Nemesis, was restricted in general to the specific nature of measure, namely, that what is presumptuous, what makes itself too great, too high, is reduced to the other extreme of being brought to nothing, so that the mean of measure, mediocrity is restored. 'The absolute, God, is the *measure* of all things' is not more intensely pantheistic than the definition: 'The absolute, God, is *being*,' but it is infinitely truer. Measure, it is true, is an external kind and manner of determinateness, a more or less, but at the same time it is equally reflected into itself, a determinateness which is not indifferent and external but intrinsic; it is thus *the concrete truth of being*. That is why mankind has revered measure as something inviolable and sacred.

The Idea of *essence*, namely, to be self-identical in the immediacy of its determined being, is already immanent in measure; so that the immediacy is thus reduced by this self-identity to

something mediated, which equally is mediated only through this externality, but is a mediation *with itself*—that is, reflection, the determinations of which *are*, but in this being are nothing more than moments of their negative unity. In measure, the qualitative moment is quantitative; the determinateness or difference is indifferent and so is no difference, is sublated. This nature of quantity as a return-into-self in which it is qualitative constitutes that being-in-and-for-itself which is essence. But measure is only *in itself* or in its Notion essence; this *Notion* of measure is not yet posited. Measure, still as such, is itself the *immediate* [*seiende*] unity of quality and quantity; its moments are determinately present as a quality, and quanta thereof; these moments are at first inseparable only in principle [*an sich*], but do not yet have the significance of this reflected determination. The development of measure contains the differentiation of these moments, but at the same time their *relation*, so that the identity which they are *in themselves* becomes their relation to each other, i.e. is *posited*. The significance of this development is the realization of measure in which it posits itself as in relation with itself, and hence as a moment. Through this mediation it is determined as sublated; its immediacy and that of its moments vanishes; they are reflected. Measure, having thus *realized* its own Notion, has passed into essence.

At first, measure is only an *immediate* unity of quality and quantity, so that: (1), we have a quantum with a qualitative significance, a *measure*. The progressive determining of this consists in *explicating* what is only *implicit* in it, namely, the difference of its moments, of its qualitatively and quantitatively determined being. These moments further develop themselves into wholes of measure which as such are *self-subsistent*. These are essentially in relationship with each other, and so measure becomes (2), a *ratio* of specific quanta having the form of self-subsistent measures. But their self-subsistence also rests essentially on quantitative relation and quantitative difference; and so their self-subsistence becomes a transition of each into the other, with the result that measure perishes in the *measureless*. But this beyond of measure is the negativity of measure only in principle; this results (3), in the positing of the *indifference* of the determinations of measure, and the positing of real measure—real through the negativity contained in the indifference—as an *inverse ratio of measures*

which, as self-subsistent qualities, are essentially based only on their quantity and on their negative relation to one another, thereby demonstrating themselves to be only moments of their truly self-subsistent unity which is their reflection-into-self and the positing thereof, *essence.*

The development of measure which has been attempted in the following chapters is extremely difficult. Starting from immediate, external measure it should, on the one hand, go on to develop the abstract determination of the *quantitative* aspects of natural objects (a mathematics of nature), and on the other hand, to indicate the connection between this determination of measure and the *qualities* of natural objects, at least in general; for the specific proof, derived from the Notion of the concrete object, of the *connection* between its qualitative and quantitative aspects, belongs to the special science of the concrete. Examples of this kind concerning the law of falling bodies and free, celestial motion will be found in the *Encyclop. of the Phil. Sciences*, 3rd ed., Sections 267 and 270, Remark. In this connection the general observation may be made that the different forms in which measure is realized belong also to different spheres of natural reality. The complete, abstract indifference of developed measure, i.e. the *laws* of measure, can only be manifested in the sphere of *mechanics* in which the concrete bodily factor is itself only *abstract* matter; the qualitative differences of such matter are essentially quantitatively determined; space and time are the purest forms of externality, and the multitude of matters, masses, intensity of *weight*, are similarly external determinations which have their characteristic determinateness in the quantitative element. On the other hand, such quantitative determinateness of abstract matter is deranged simply by the plurality of conflicting qualities in the inorganic sphere and still more even in the organic world. But here there is involved not merely a conflict of qualities, for measure here is subordinated to higher relationships and the *immanent development* of measure tends to be reduced to the simple form of immediate measure. The limbs of the animal organism have a measure which, as a simple quantum, stands in a ratio to the other quanta of the other limbs; the proportions of the human body are the fixed ratio of such quanta. Natural science is still far from possessing an insight into the connection between such quantities and the organic functions on which they

wholly depend. But the readiest example of the reduction of an immanent measure to a merely externally determined magnitude is *motion*. In the celestial bodies it is free motion, a motion which is determined solely by the Notion and whose quantitative elements therefore equally depend solely on the Notion (see above); but such free motion is reduced by the living creature to arbitrary or mechanically regular, i.e. a wholly abstract, formal motion.

And in the realm of spirit there is still less to be found a characteristic, free development of measure. It is quite evident, for example, that a republican constitution like that of Athens, or an aristocratic constitution tempered by democracy, is suitable only for States of a certain size, and that in a developed civil society the numbers of individuals belonging to different occupations stand in a certain ratio to one another; but all this yields neither laws of measure nor characteristic forms of it. In the spiritual sphere as such there occur differences of *intensity* of character, *strength* of imagination, sensations, general ideas, and so on; but the determination does not go beyond the indefiniteness of *strength* or *weakness*. How insipid and completely empty the so-called laws turn out to be which have been laid down about the relation of strength and weakness of sensations, general ideas, and so on, comes home to one on reading the psychologies which occupy themselves with such laws.

CHAPTER I

SPECIFIC QUANTITY

Qualitative quantity in the *first* place an immediate, *specific quantum. Secondly*, this quantum as relating itself to another becomes a quantitative specifying, a sublating of the indifferent quantum. This measure is so far a *rule* and contains the two moments of measure distinguished; namely, the intrinsic quantitative determinateness and the external quantum. In this distinction, however, these two sides become qualities and the rule becomes a relation between them; consequently measure exhibits itself *thirdly*, as a relation of qualities. These at first have a single measure, but this is further specified within itself into distinct measures.

A. THE SPECIFIC QUANTUM

1. Measure is the simple relation of the quantum to itself, its own determinateness within itself; the quantum is thus qualitative. At first, as an immediate measure it is an immediate quantum, hence just some specific quantum or other; equally immediate is the quality belonging to it, some specific quality or other. The quantum as this no longer indifferent limit but as a self-related externality, is thus itself quality, and although distinguished from it does not transcend it, neither does the quality transcend the quantum. It is thus the determinateness which has returned into simple identity with itself, one with the specific determinate being, just as this latter is one with its quantum.

If it is desired to make a proposition out of the determination in question, it can be expressed thus: all that exists has a measure. Everything that exists has a magnitude and this magnitude belongs to the nature of the something itself; it constitutes its specific nature and its being-within-self. Something is not indifferent to this magnitude, so that if this were altered it would continue to be what it is; on the contrary, an alteration of the magnitude would alter the quality of the something. Quantum, as measure, has ceased to be a limit which is no limit; it is now the

determination of the thing, which is destroyed if it is increased or diminished beyond this quantum.

A measure taken as a standard in the usual meaning of the word is a quantum which is arbitrarily assumed as the *intrinsically determinate* unit relatively to an external amount. Such a unit can, it is true, also be in fact an intrinsically determinate unit, like a foot and suchlike original measures; but in so far as it is also used as a standard for other things it is in regard to them only an external measure, not their original measure. Thus the diameter of the earth or the length of a pendulum may be taken, each on its own account, as a specific quantum; but the selection of a particular fraction of the earth's diameter or of the length of the pendulum, as well as the degree of latitude under which the latter is to be taken for use as a standard, is a matter of choice. But for other things such a standard is still more something external. These have further specified the general specific quantum in a particular way and have thereby become particular things. It is therefore foolish to speak of a *natural* standard of things. Moreover, a universal standard ought only to serve for *external* comparison; in this most superficial sense in which it is taken as a *universal measure* it is a matter of complete indifference what is used for this purpose. It ought not to be a fundamental measure in the sense that it forms a scale on which the natural measures of particular things could be represented and from which, by means of a rule, they could be grasped as specifications of a universal measure, i.e. of the measure of their universal body. Without this meaning, however, an absolute measure is interesting and significant only as a *common element*, and as such is a universal not in itself but only by agreement.

This immediate measure is a simple quantitative determination as, for example, the size of organic beings, of their limbs etc. But everything that exists has a size which makes it what it is, and in general enables it to have an external reality. As a quantum it is an indifferent magnitude open to external determination and capable of increase and decrease. But as a measure it is also distinguished from itself as a quantum, as such an indifferent determination, and is a limitation of that indifferent fluctuation about a limit.

Since the quantitative determinateness of anything is thus twofold —namely, it is that to which the quality is tied and also that which can be varied without affecting the quality—it follows that the

destruction of anything which has a measure takes place through the alteration of its quantum. On the one hand this destruction appears as *unexpected*, in so far as the quantum can be changed without altering the measure and the quality of the thing; but on the other hand, it is made into something quite easy to understand through the idea of *gradualness*. The reason why such ready use is made of this category to render conceivable or to *explain* the disappearance of a quality or of something, is that it seems to make it possible almost to watch the disappearing with one's eyes, because quantum is posited as the external limit which is by its nature alterable, and so *alteration* (of quantum only) requires no explanation. But in fact nothing is explained thereby; the alteration is at the same time essentially the transition of one quality into another, or the more abstract transition of an existence into a negation of the existence; this implies another determination than that of gradualness which is only a decrease or an increase and is a one-sided holding fast to quantity.

2. The sudden conversion into a change of quality of a change which was apparently merely quantitative had already attracted the attention of the ancients who illustrated in popular examples the contradictions arising from ignorance of this fact; they are familiar under the names of 'the bald' and 'the heap'. These *elenchi* are, according to Aristotle's explanation, ways in which one is compelled to say the opposite of what one had previously asserted. The question was asked: does the pulling out of a single hair from the head or from a horse's tail produce baldness, or does a heap cease to be a heap if a grain is removed? An answer in the negative can be given without hesitation since such a removal constitutes only a quantitative difference, a difference moreover which is itself quite insignificant; thus a hair, a grain, is removed and this is repeated, only one of them being removed each time in accordance with the answer given. At last the qualitative change is revealed; the head or the tail is bald, the heap has disappeared. In giving the said answer, what was forgotten was not only the repetition, but the fact that the individually insignificant quantities (like the individually insignificant disbursements from a fortune) *add up* and the total constitutes the qualitative whole, so that finally this whole has vanished; the head is bald, the purse is empty.

The dilemma, the contradiction which results therefrom, is not

a sophism in the usual sense of the word; for such contradiction is not a sham or a deception. The real mistake is committed by the assumed Other (i.e. our ordinary consciousness), the mistake, namely, of assuming a quantity to be only an indifferent limit, i.e. of assuming that it is just a quantity in the specific sense of quantity. This assumption is refuted by the truth to which it is brought —to wit, that quantity is a moment of measure and is connected with quality. What is refuted is the error of one-sidedly holding fast to the abstract determinateness of quantum. Therefore these examples, too, are not a pointless or pedantic joke but have their own correctness; they are the product of a mentality which is interested in the phenomena which occur in thinking.

Quantum, when it is taken as an indifferent limit, is the aspect of an existence which leaves it open to unsuspected attack and destruction. It is the cunning of the Notion to seize on this aspect of a reality where its quality does not seem to come into play; and such is its cunning that the aggrandizement of a State or of a fortune, etc., which leads finally to disaster for the State or for the owner, even appears at first to be their good fortune.

3. Measure in its immediacy is an ordinary quality with a specific magnitude attaching to it. Now that aspect of the quantum according to which it is an indifferent limit which can be exceeded without altering the quality, is also distinguished from its other aspect according to which it is qualitative and specific. Both are quantitative determinations of one and the same thing; but because of the initial immediacy of measure, this distinction is also to be taken as immediate, and therefore both aspects also have a distinct existence. The existence of measure, then, which is *intrinsically* determinate magnitude, is in its behaviour towards the existence of the alterable, external aspect, a sublating of its indifference, a *specifying* of measure.

B. SPECIFYING MEASURE

This is first a rule, a measure which is external with reference to mere quantum. Secondly it is a specific quantity which determines the external quantum, and thirdly *both sides*, as *qualities* of a specific quantitative determinateness, are related to one another as *one* measure.

(*a*) *The Rule*

The rule or standard, which has already been mentioned, is in the first place an intrinsically determinate magnitude which is a unit with reference to a quantum having a particular existence in a something other than the something of the rule; this other something is ***measured*** by the rule, i.e. is determined as an amount of the said unit. This ***comparison*** is an ***external*** act, the unit itself being an arbitrary magnitude which in turn can equally be treated as an amount (the foot as an amount of inches). But measure is not only an external rule; as a specifying measure its nature is to be related in its own self to an other which is a quantum.

(*b*) *Specifying Measure*

Measure is a specific determining of the ***external***, i.e. indifferent magnitude which is now posited by some other existence in general in the measurable something; true, this latter is itself a quantum but, as distinguished from it, it is the qualitative side determining the merely indifferent, external quantum. The measurable something has in it this side of being-for-other to which the indifferent increasing and decreasing is proper. This immanent measuring standard is a quality of the something to which is opposed the same quality in another something; but in the latter something the quality is at first only relative, the quantum having no significance as a measure in relation to the other something which is determined as the standard of measuring.

Something, in so far as it is a measure within itself, has the magnitude of its quality altered from outside itself; it does not accept this externally imposed alteration as an arithmetical amount: its measure reacts against it, behaves towards the amount as an intensive quantum and assimilates it in a characteristic way; it alters the externally imposed alteration, makes this quantum into a different one and through this specifying shows itself to be *self*-determined in this externality. This ***specifically assimilated*** amount is itself a quantum which is also dependent on the other, or is for it only an ***external*** amount. Consequently the specified amount is also alterable, but is not therefore a quantum as such but the external quantum specified in a constant manner. The determinate being of measure is thus a ***ratio***, the specific element of which is in general the ***exponent*** of this ratio.

When considering ***intensive*** and ***extensive*** quantum we found

that it is the *same* quantum which is present, once in the form of intensity and again in the form of extension. In this difference the quantum lying at the base suffers no alteration, the difference being only an outer form. In the specifying measure, on the contrary, the quantum is taken in the first instance in its immediate magnitude, but in the second instance it is taken through the exponent of the ratio in another amount.

The exponent which constitutes the specific element can at first seem to be a fixed quantum, as a quotient of the ratio between the external and the qualitatively determined quantum. But as such, it would be nothing but an external quantum; the exponent here must be understood as nothing else but the qualitative moment itself which specifies the quantum as such. As we have already seen, the strictly immanent qualitative form of the quantum is solely its determination as a *power*. It must be such a determination which constitutes the ratio and which here, as the intrinsic determination of the quantum, confronts the quantum as externally constituted. The principle of the quantum is the numerical *one* which constitutes its intrinsic determinedness, and the mode of relation of the numerical one is external; and the alteration which is specified only by the nature of the immediate quantum as such, consists by itself in the addition of such a numerical one and then another and so on. If in this way the alteration of the external quantum is an arithmetical progression, the specifying reaction of the qualitative nature of measure produces another series which is related to the first, increases and decreases with it, but not in a ratio determined by a numerical exponent but in a number of incommensurable ratios, according to a determination of powers.

Remark

To cite an example, *temperature* is a *quality* in which these two sides of external and specified quantum are distinguished. As a quantum it is an external temperature, and that too, of a body as a general medium, and it is assumed that the alteration of the temperature proceeds on the scale of an arithmetical progression, increasing or decreasing uniformly. On the other hand, the particular bodies in the medium differ in the way they absorb the temperature, for through their immanent measure they determine it as received from outside themselves and the change of temper-

ature in any one of them does not correspond in a direct ratio with that of the medium or of the other bodies among themselves. Different bodies compared at one and the same temperature give the numerical ratios of their specific heats, of their thermal capacities. But the thermal capacities of bodies vary in different temperatures and associated with this is a change in the specific shape. Thus a particular specification is manifested in the increase or decrease of temperature. The ratio of the temperature, taken as external, to the temperature of a specific body which is at the same time dependent on the former temperature, has no fixed exponent; the increase or decrease of this heat does not proceed uniformly with the increase or decrease of the external heat. Here a temperature is assumed which is purely external and whose changes are merely external or purely quantitative. But the temperature is itself the temperature of the air or some other specific temperature. The ratio, therefore, if looked at more closely, would strictly speaking have to be taken not as the ratio of a merely quantitative quantum to a qualitative one, but as the ratio of two specific quanta. In fact, the determining of the specifying ratio has now advanced to the stage where the moments of measure not only consist of a quantitative side and a side qualifying the quantum, both being sides of one and the same quality, but are related to each other as two qualities which are in themselves measures.

(*c*) *Relation of the two Sides as Qualities*

1. The qualitative, intrinsically determinate side of the quantum exists only as a relation to the externally quantitative side; as a specifying of the latter it is a sublating of its externality through which quantum as such is. This qualitative side thus has a quantum for its presupposition and its starting point. But this quantum is also qualitatively distinguished from the quality itself; this difference between them is now to be posited in the *immediacy* of being as such, in which determination measure still is. The two sides are thus qualitatively related and each is on its own account a qualitative determinate being; and the one quantum which at first was only a formal quantum indeterminate in itself, is the quantum of a something and of its quality, and also—now that the connection between them is determined as a measure—the specific magnitude of these qualities. These qualities are related

to each other according to their determination as measures—which determination is their exponent. But they are already implicitly related to each other in the *being-for-self* of measure; the quantum in its dual character is both external and specific so that each of the distinct quantities possesses this twofold determination and is at the same time inseparably linked with the other; it is in this way alone that the qualities are determined. They are therefore not only simply determinate beings existing for each other but they are posited as inseparable and the specific magnitude connected with them is a qualitative unity—a single determination of measure in which, in accordance with their Notion, they are implicitly bound up with each other. Measure is thus the *immanent* quantitative relationship of *two* qualities to each other.

2. In measure there enters the essential determination of *variable magnitude*, for measure is quantum as sublated, and therefore no longer what, as quantum, it is supposed to be, but quantum and something else; this something else is the qualitative element and, as we have seen, is nothing else than its relation of powers. In immediate measure this alteration is not yet posited; it is only an arbitrary, single quantum to which a quality is bound. In the specifying of measure (the preceding determination), which is an alteration of the merely external quantum by the qualitative element, there is posited the distinction between the two specific magnitudes and hence generally the plurality of measures in a common, external quantum. It is in this distinguishedness of the quantum from itself that it first shows itself to be a real [*daseiendes*] measure; for it now appears as a determinate being which is both one and the same (e.g. the constant temperature of the medium), and also quantitatively varied (in the different temperatures of the bodies present in the medium). This distinguishedness of the quantum in the different qualities (the different bodies) gives a further form of measure, that in which the two sides are mutually related as qualitatively determinate quanta. This can be called *realized measure.*

Magnitude, simply as magnitude, is alterable, for its determinateness is a limit which is at the same time no limit, so that the alteration concerns only a particular quantum, the place of which is taken by another. But the genuine alteration is that of the quantum as such; when understood in this way, we have the interesting determination of the variable magnitude in higher

mathematics; here we must not stop short at the merely formal determination of *variability* in general, neither must we introduce any determination other than the simple determination of the Notion according to which the *other of quantum is only quality*. The genuine determination, therefore, of real variable magnitude is that it is magnitude qualitatively determined, that is, as has been sufficiently demonstrated, determined by a ratio of powers. In this variable magnitude the fact is *posited* that what counts is not quantum as such but quantum determined in accordance with its other, i.e. qualitatively determined.

The two sides thus related have, in keeping with their abstract aspect as qualities generally, some particular significance—for example, space and time. Taken at first simply as specific magnitudes in their ratio as measures, one of them is an amount which increases and decreases in an external arithmetical progression, the other is an amount which is specifically determined by the first, which for it is unit. Now if each were only just a particular quality, that element of difference would be lacking which, with regard to their character as quantities, would indicate which of them was to be taken as merely externally quantitative and which as varying according to the specifying of its magnitude. If, for example, they are related as root and square, it is immaterial which is regarded as increasing or decreasing merely externally in arithmetical progression, and which, on the other hand, as specifically determining the other quantum.

But the difference between the qualities is not undefined, for as moments of *measure* the specifying of measure must be present in them. The next determinateness of the qualities themselves is that one is *extensive*—is in its own self externality—the other is *intensive*, the being-within-self or negative relatively to the other. Accordingly, the former of the quantitative moments is amount, and the latter is unit; in the simple direct ratio the former is to be taken as the dividend, the latter as divisor, and in the specifying ratio the former as the power or the becoming-other, the latter as the root. In so far as we still *count* here, i.e. still reflect on the external quantum (which is thus the quite contingent specific magnitude, the empirical amount), the alteration, too, being likewise taken as an external, arithmetical progression, then this falls on the side of the unit, of the intensive quality; the external, extensive side, on the other hand, is to be represented as altering in

the specified series. But the direct ratio $\left(\text{like velocity as such, } \frac{s}{t}\right)$ is here reduced to the merely formal determination which has no existence except as an intellectual abstraction; and even though in the ratio of root and square (as in $s = at^2$) the root is to be taken as an empirical quantum varying in an arithmetical progression, and the other side is to be taken as specified, yet the higher realization of the qualifying of the quantitative, a realization more in harmony with the Notion, is that in which both sides are related to each other in higher determinations of powers (as is the case in $s^3 = at^2$).

Remark

The exposition here of the connection between the qualitative nature of something and its quantitative determination has its application in the already indicated example of motion. First of all, in *velocity* as the direct ratio of space traversed and time elapsed, the magnitude of time is taken as denominator while that of space is taken as numerator. If velocity as such is only a ratio of the space and time in a motion, it is immaterial which of the two moments is to be considered as amount or as unit. Space, however, like weight in specific gravity, is an external, real whole as such—hence amount—whereas time, like volume, is the ideal, negative factor, the side of unity. But here there essentially belongs the more important ratio, that which holds between the magnitudes of space and time in *free motion*; at first, in the still conditioned motion of a falling body where the time factor is determined as a root and the space factor as a square, or in the absolutely free motion of the celestial bodies where the period of revolution is lower by one power than the distance from the sun, the former being a square and the latter a cube. Fundamental relationships of this kind rest on the nature of the interrelated qualities of space and time and on the kind of relation in which they stand, either as a mechanical motion, i.e. as an unfree motion which is not determined by the Notion of the moments of space and time, or as the descent of a falling body, i.e. as a conditionally free motion, or as the absolutely free celestial motion. These kinds of motion, no less than their laws, rest on the development of the Notion of their moments, of space and time, since these qualities as such (space and time) prove to be in themselves, i.e. in their Notion,

inseparable and their quantitative relationship is the being-for-self of measure, is only *one* measure-determination.

In regard to the absolute relations of measure, it is well to bear in mind that the mathematics of nature, if it is to be worthy of the name of science, must be essentially the science of measures—a science for which it is true much has been done empirically, but little as yet from a strictly scientific, that is, philosophical point of view. Mathematical principles of natural philosophy—as Newton called his work—if they are to fulfil this description in a profounder sense than that accorded to them by Newton and by the entire Baconian species of philosophy and science, must contain things of quite a different character in order to bring light into these still obscure regions which are, however, worthy in the highest degree of consideration. It is a great service to ascertain the empirical numbers of nature, e.g. the distances of the planets from one another; but it is an infinitely greater service when the empirical quanta are made to disappear and they are raised into a universal form of determinations of quantity so that they become moments of a *law* or of measure—immortal services which Galileo for the descent of falling bodies and Kepler for the motion of the celestial bodies have achieved. The laws they discovered they have proved in this sense, that they have shown the whole compass of the particulars of observation to correspond to them. But yet a still higher *proof* is required for these laws; nothing else, that is, than that their quantitative relations be known from the qualities or specific Notions of time and space that are correlated. Of this kind of proof there is still no trace in the said mathematical principles of natural philosophy, neither is there any in the subsequent works of this kind. It has already been remarked in connection with the show of mathematical proofs of certain relationships in nature, a show based on the misuse of the infinitely small, that it is absurd to try to demonstrate such proofs on a strictly mathematical basis, i.e. neither empirically nor from the standpoint of the Notion. These proofs *presuppose* their theorems, those very laws, from experience; what they succeed in doing is to reduce them to abstract expressions and convenient formulae. Undoubtedly the time will come when, with a clearer understanding of what mathematics can accomplish and has accomplished, the entire, real merit of Newton as against Kepler—the sham scaffolding of proofs being discarded—will clearly be seen

to be restricted to the said transformation of Kepler's formula[1] and to the elementary analytical treatment accorded to it.

C. BEING-FOR-SELF IN MEASURE

1. In the form of specified measure just considered, the quantitative element of both sides is qualitatively determined (both in the ratio of powers); hence they are moments of *one* measure-determinateness of qualitative nature. At the same time, however, the two sides are so far posited only as immediate, *merely different* qualities, which do not themselves stand in the same relationship as their quantitative determinatenesses; that is, they cannot be said to have no meaning or existence *outside* that relationship, as is the case in the determinateness of quantity as a ratio of powers. The qualitative element thus masks itself, specifying not itself but the quantitative determinateness; only in the latter is it *posited*, remaining on its own account an *immediate* quality as such which, beside the fact that it explicitly differentiates the magnitude and beside its relation to its other, still has an independent determinate being of its own. Thus space and time, apart from that specification contained in their quantitative determinateness in the descent of a falling body, or in the absolutely free motion, count as space in general and time in general, space having an enduring existence of its own apart from and without time, and time flowing on its own independently of space.

This immediacy of the qualitative element as against its specific measure relation is, however, just as much bound up with a quantitative immediacy and with the indifference of this *quantitative* aspect in it towards this its relation; the immediate quality has also a merely *immediate quantum*. Consequently, the specific measure has also a side which is, to begin with, subject to external alteration in the sense of a merely arithmetical progression unaffected by the specific measure and in which falls the external, hence only empirical, determinateness of magnitude. Quality and quantum as thus also appearing outside the specific measure are at the same time correlated with it; immediacy is a moment

[1] See *Encyclopaedia of the Philosophical Sciences*, Remark on Section 270, on the transformation of Kepler's $\frac{S^3}{T^2}$ into Newton's $\frac{S^2 \times S}{T^2}$, the fraction $\frac{S}{T^2}$ being called the law of gravity.

of those sides which themselves belong to measure. Thus the immediate qualities also belong to measure, are likewise in relation and stand in a quantitative relationship which, as outside the specified determination, the ratio of powers, is itself only a direct ratio and an immediate measure. This conclusion and its import is to be indicated in more detail.

2. Even though the immediately determined quantum as such is, in virtue of its being a moment of measure, established as in itself determinable by the Notion, it is still with reference to the specific measure an externally given quantum. But the immediacy which is thus posited is the negation of the qualitative determination of measure; this has already been demonstrated in respect of the sides of this determination of measure which for that reason appeared as independent qualities. Such negation and the return to the immediate determination of quantity lies in the qualitatively determined relation, for the relation of distinct terms as such implies their correlation as *one* determinateness which latter, in distinction from the determination of the relation, is here in the sphere of quantity a quantum. As a negation of the distinct qualitatively determined sides, this exponent is a being-for-self or absolute determinedness; but it is such being-for-self only *in principle* [*an sich*]; as a determinate being it is a simple immediate quantum which is the quotient or exponent of a direct ratio between the sides of the measure, but in general is the unit appearing as empirical, in the quantitative side of measure. In the motion of falling bodies the spaces traversed are proportional to the squares of the elapsed times, $s = at^2$. This is the specifically determined relation of space and time, a ratio of powers. The other, the direct ratio, would concern space and time as mutually indifferent qualities; it is supposed to be the ratio of the space traversed to the *first* unit of time. The same co-efficient a remains in all the following units of time—the *unit* being an ordinary quantum, while the *amount* is determined by the specifying measure. This unit at the same time counts as the exponent of that direct ratio which belongs to the imaginary, spurious velocity i.e. the merely formal velocity which is not specifically determined by the Notion. Such a velocity does not exist here, any more than the one previously mentioned which is supposed to accrue to the falling body at the *end* of a unit of time. That velocity was ascribed to the *first* unit of time in the motion of a falling body; but this

so-called unit of time is itself only an assumed unit and has as such atomic point no real being. The beginning of the motion—the alleged smallness of it could make no difference—is straightway a magnitude and one specified by the law of descent of a falling body. The said empirical quantum is attributed to the force of gravity, on the supposition that this force itself has no connection with the actual specification, with the ratio of powers characteristic of a determination of measure. The *immediate* moment, that in the motion of a falling body the amount of some fifteen spatial units, taken as feet, is traversed in a unit—the so-called *first* unit—of time, a second, is an immediate measure, like the measurements of the limbs of the human body, the distances and diameters of the planets and so forth. The determination of such a measure falls elsewhere than in the qualitative measure determination itself, here in the law of descent of a falling body; however, the concrete sciences have so far failed to throw any light on the basis of determination of such numbers, which are only immediate and consequently are the empirical embodiment of a measure. Here we are concerned only with the determinateness specified by the Notion, namely, that the said empirical co-efficient constitutes the moment of *being-for-self* in the measure-determination, and that too only in so far as this moment is *unexplicated* [*an sich*], and hence an immediacy. The second moment is the *developed* side of this being-for-self, the specific measure determinateness of the sides. In the motion of falling, a motion which is still half-conditioned and half-free, gravity, according to this second moment, is to be regarded as a force of nature, so that the relationship expressed by the law of descent of a falling body is determined by the nature of space and time, and consequently the said specification, namely, the ratio of powers, falls within gravity; the above-mentioned simple direct ratio expresses only a mechanical relationship between space and time in the [merely] formal velocity which is externally produced and determined.

3. Measure has now acquired the character of a specified quantitative relation which, as qualitative, has in it the ordinary external quantum; but this is not a quantum in general, but essentially a determinant of the relation as such; hence in the sense of an exponent, and by virtue of the immediacy of its determination, of a fixed exponent, namely that of the already mentioned direct ratio between the same qualities whose quantita-

tive relationship is at the same time specifically determined by the ratio. This direct ratio is, so to speak, anticipated and assumed as given in the example used of a measure, namely the law of descent of a falling body; but still, as we remarked, it does not *exist* in this motion. The fact, however, that the two sides of measure are themselves measures, the one immediate and external, and the other immanently specified, both being contained within the unity of measure itself, means that measure is now further determined, is *realised*. As this unity, measure contains the relation in which the magnitudes are determined and posited as differently specified by the nature of the qualities; its determinateness is accordingly wholly immanent and self-subsistent, and has at the same time collapsed into the being-for-self of an immediate quantum, the exponent of a direct ratio. The self-determination of the relation is thus *negated*, for in this its other it has its final, explicit determinateness; and conversely, the immediate measure which is supposed to be in its own self qualitative, possesses in truth such qualitative determinateness only in the other side of the relation. This negative unity is a *real being-for-self*, the category of a *something* as a unity of qualities which are related as measures—a completely *self-subsistent* something. The two sides which have presented themselves as distinct relations also immediately possess a twofold existence; or, to put it more explicitly, a self-subsistent whole of this kind, just because it is a real being-for-self, is at the same time a repulsion into *distinct self-subsistent* somethings whose qualitative nature and subsistence (materiality) lies in their measure determinateness.

CHAPTER 2

REAL MEASURE

Measure is now determined as a correlation of measures which constitute the quality of distinct self-subsistent somethings—or *things*. The relations of measure just considered concern abstract qualities like space and time; those now about to be considered are exemplified in specific gravity and later on in chemical properties, i.e. in determinations characteristic of *material* existence. Space and time are also moments of such measures but their relationship no longer depends simply on their own nature because they are now subordinated to further determinations. Among the determining moments in sound, for example, there is the *time* in which a number of vibrations occur, and the *spatial* element of length and thickness of the vibrating body; but the magnitudes of those ideal moments are determined externally. Space and time are no longer in a relation of powers but in the ordinary direct relation, harmony being reduced to a quite external simplicity of numbers whose relations can be grasped with the utmost ease and hence afford a satisfaction falling entirely within the element of sensation since there is an absence for spirit of figurate conception, fantasy, abstract thought, and the like. In that the sides which now constitute the measure relation are themselves measures, but at the same time real somethings, their measures are, in the first place, immediate measures and as regards their relations, direct relations. It is the inter-relationship of such relations that is now to be considered in its progressive determination.

Measure, as now real measure, is first, a self-subsistent measure of a material thing which is related to others and in this relation specifies them and with them their self-subsistent materiality. This specification as an external relating to a plurality of others in general, produces other relations and hence other measures; and the specific self-subsistence does not continue as a single direct relation but passes over into a specific determinateness which is a series of measures. Secondly, the direct relations thus produced are in themselves determinate and exclusive measures

(elective affinities); but because the difference between them is also only quantitative, the progressive determination of the relations presents itself as in part a merely externally quantitative development which, however, is also interrupted by qualitative relationships and forms a *nodal line* of specific self-subsistent measure relations. Thirdly, however, in this development measure gives place to the *measureless* as such, and more specifically to the *infinity* of measure. In this, the self-exclusive and self-subsistent measures are one with each other, and the self-subsistent measure enters into a negative relation with itself.

A. THE RELATION OF SELF-SUBSISTENT MEASURES

Measures now are no longer merely immediate but self-subsistent, because they have become in themselves relations of measures which are themselves specified; and thus in this being-for-self are physical somethings, in the first instance, material things. The whole, which is a relation of such measures is, however, (*a*) first, itself *immediate;* thus the two sides which are determined as such self-subsistent measures exist apart in particular things and their *combination* is effected *externally*; (*b*) but what the self-subsistent material things are *qualitatively*, they are only in virtue of their *quantitative* determination as measures; hence through this same quantitative connection with others they are determined as *differently* specified in regard to them (so-called *affinity*), namely, as *members of a series* of such quantitative relationships; (*c*) at the same time, this *indifferent*, manifold interrelationship finishes by converting itself into *exclusive* being-for-self—so-called *elective affinity*.

(*a*) *Combination of Two Measures*

Something is immanently determined as a measure relation of quanta which also possess qualities and the something is the connection of these qualities. One of them is the *being-within-self* or *inwardness* of the something by virtue of which it is a real being-for-self, a material thing (such as, taken intensively, weight, or in its extensive aspect, the multiplicity of *material* parts); the other quality is the *externality* of this inwardness (the abstract, ideal element of space). These qualities are quantitatively determined and their correlation constitutes the qualitative nature of

the material something—e.g. the ratio of weight to volume: specific gravity. The volume, the ideal aspect, is to be taken as unit, but the intensive aspect, which manifests quantitatively and in comparison with the former as an extensive magnitude, as a plurality of independent ones, is to be taken as amount. The purely qualitative relation of the two specific magnitudes, that is, as a ratio of powers, has vanished, because with the self-subsistence of the material thing immediacy has returned and in this the specific magnitude is an ordinary quantum whose relation to the other side is likewise determined as the ordinary exponent of a direct ratio.

This exponent is the specific quantum of the something, but it is an immediate quantum and this is determined—and with it the specific nature of such something—only in the *comparison* with other exponents of such ratios. The exponent constitutes the specific intrinsic determinedness, the inner characteristic measure of something; but because this its measure rests on a quantum, it too is only an external, indifferent determinateness, with the consequence that the something, in spite of its inner determination as a measure, is alterable. The other to which it can, as alterable, enter into relation is not a material plurality, quantum in general—this it can withstand through its specific intrinsic determinedness —but it is a quantum which is at the same time also an exponent of such a specific ratio. Two things with different internal measures stand in relation and enter into combination—such as two metals of different specific gravities (the combination in question would not be, for example, of a metal and water); but what other kind of homogeneity is required to make possible such a combination will not be considered here. Now on the one hand, each of the two measures, just because it is a measure, preserves itself in the alteration which it ought to suffer through the externality of the quantum, but on the other hand this self-preservation is itself a negative relation towards this quantum, a specification of it; and since the quantum is the exponent of the measure relation, the self-preservation is an alteration of the measure itself and moreover a reciprocal specification.

As purely quantitatively determined, the compound would be a mere addition of the two magnitudes of the one quality and the two magnitudes of the other, e.g., the sum of the two weights and of the two volumes in the case of a compound of two material

substances of different specific gravities; thus not only the weight of the mixture would remain equal to the said sum, but also the space occupied by the mixture would be equal to the sum of the said spaces. But this is true only of the weight of the mixture, which is equal to the sum of the weights before the combination; addition applies to that quality which, as a real being-for-self, has acquired a fixed determinate being with a permanent immediate quantum—the weight of the material thing, or, what counts as the same from the quantitative point of view, the number or amount of material parts. The exponents however are subject to alteration since they are the expression of the qualitative aspect of the compound, of its being-for-self in the form of measure relations; and since the quantum as such suffers contingent external alteration by an increase which is summed, the being-for-self at the same time displays itself as negating this externality. Because this immanent determining of the quantitative element cannot, as we have seen, be manifested in the weight, it displays itself in the other quality which is the ideal side of the relation. From the point of view of sense perception it may appear remarkable that the mixing of two specifically different material substances should be followed by an alteration—usually a diminution—in the sum of the two volumes, for it is space itself which constitutes the subsistence of matter in its external separated existence. But this subsistence in face of the negativity immanent in the being-for-self lacks intrinsic being, it is subject to alteration. In this manner space is posited as what it truly is, an ideal being [*das Ideelle*].

Not only, then, is one of the qualitative sides posited as alterable but measure itself—and so too the qualitative nature of the something based on it—has shown that it is unstable in its own self and, like the quantum as such, has its determinateness in other measure relations.

(*b*) *Measure as a Series of Measure Relations*

(1) If two things forming a compound body owed their respective specific natures only to a simple qualitative determination, they would only destroy each other when combined. But a thing which is an immanent measure relation is self-subsistent; it is therefore also capable of combining with another such thing. But in being reduced to an element of this unity, it preserves itself through

the persistence of its indifferent, quantitative character and at the same time functions as a specifying moment of a new measure relation. Its quality is masked in the quantitative element and is thus also indifferent towards the other measure, continuing itself in it and in the newly formed measure. The exponent of the new measure is itself only some quantum or other, an external determinateness, and its indifference finds expression in the fact that the specifically determined thing effects, in association with other such measures, precisely similar neutralizations of the reciprocal measure relations; it is in only one measure relation formed by itself and another specifically determined thing that its specific peculiarity is not expressed.

(2) This combination with a number of others which are likewise measures within themselves, yields different ratios which therefore have different exponents. The self-subsistent measure has the exponent of its intrinsic determinedness only in the comparison with others; its neutrality with the others, however, consititutes its real comparison with them; it is its comparison with them through its own self. But the exponents of these ratios differ and the qualitative exponent of the self-subsistent measure is thus represented as the *series* of these *different amounts* of which it is the *unit*, i.e. as a series whose members are in a specific relationship to others. The qualitative exponent, as one immediate quantum, expresses only one relation. The distinctive character of the self-subsistent measure finds its true expression in the characteristic series of exponents which it, taken as unit, forms with other such self-subsistent measures; for one of these measures when brought into relation with the rest of them and taken as unit forms another series. Now it is the interrelationship of the members of such a series that constitutes the qualitative aspect of the self-subsistent measure.

At first, it seems that a self-subsistent measure which forms a series of exponents with a series of such measures, is distinguished from another measure—outside this series—with which it is *compared*, by the fact that this other measure forms another series of exponents with the members of the first series. But in this way these two measures would *not be comparable*, because each is thus regarded as unit with respect to its exponents, and between the two series arising from this relation there is no *specified difference*. The two measures which, as self-subsistent, are supposed to be compared are at first contra-distinguished only as quanta; in order

to determine their relation, an independent unit common to them both is required. This specific unit is to be sought, as has been indicated, only in that feature which embodies the specific determinate being of the measures to be compared, i.e. in the ratio which the exponents of the ratios of the members of the series have to each other. This ratio of the exponents themselves is, however, such independent and actually specific unit only in so far as the members of the series together have it as a *constant* ratio to both measures; in that way it can be *their common unit.* It is this alone, therefore, which makes it possible to compare the two self-subsistent measures which were assumed to be not reciprocally neutralising measures, but indifferent to each other. Each of them taken separately and apart from the comparison is the unit of the ratio it forms with the opposite members, which are the amounts relatively to that unit and which thus represent the series of exponents. But conversely, this series is the unit for the two measures which when compared with each other are related as quanta; as such they are themselves different amounts of the unit just indicated.

But further, those measures which together with the two, or rather indefinitely many self-subsistent measures of the first series—measures which are compared only with each other—yield a series of exponents of the ratios between the members of that series, are similarly in themselves self-subsistent measures, each being a specific something with its own intrinsic measure ratio. Each of these then is similarly to be taken as unit so that they have a series of exponents in the two, or rather indefinitely many members of the first series which are compared merely among themselves, and these exponents are the numbers resulting from comparison among themselves of the measures just named; and conversely, the comparative numbers of the measures which are now also to be taken singly as self-subsistent are the series of exponents for the members of the first series. In this way, both sides are series in which each number is firstly simply a unit to the opposite series in which it has its self-determined character as a series of exponents; secondly, each number is itself one of the exponents for each member of the opposite series; and thirdly, it is a comparative number for the other numbers of its series and as such amount, which belongs to it also as an exponent, it has its own specifically determined unit in the opposite series.

3. In this form of relationship there is a return to the particular kind of way in which quantum is posited as self-determined, i.e., as degree: namely, it is simple or unitary, but it has its quantitative determinateness in a quantum existing outside it, which is a circle of quanta. In measure, however, this external aspect is not merely a quantum and a circle of quanta, but a series of numerical ratios and it is in the entirety of these that the self-determinedness of measure lies. As is the case in the being-for-self of quantum as degree, the nature of the self-subsistent measure is converted into this externality of itself. Its self-relation is in the first place an *immediate* relation and therefore its indifference to an other consists only in the quantum. It is into this externality therefore that its qualitative side falls and its *relationship to its other* becomes that quantitative mode of relationship which constitutes the specific determination of this self-subsistent measure, a mode which is determined just as much by the other as by the measure itself; and this other is a series of quanta and the measure is reciprocally a quantum. But this relation in which two specific measures specify themselves in a third something, the exponent, also implies that the one has not passed into the other; that therefore there is not only *one* negation, but that *both* are posited as negative in the relation, and since in this each preserves itself as indifferent towards the other *its negation* is also in turn *negated*. This their qualitative unity is thus a self-subsistent *exclusive* unit. It is only as exclusive that the exponents, which are primarily comparative numbers of the members of the series, have in them their genuinely specific determinateness relatively to one another, and their difference thus acquires a qualitative nature. But this difference has a quantitative basis: first, the self-subsistent measure is related to a *plurality* of its qualitatively other side only because in this relationship it is, at the same time, indifferent; secondly, the neutral relationship, in virtue of its quantitative aspect, is now not simply an alteration but is posited as a negation of the negation and as an exclusive unit. Consequently, the *affinity* of a self-subsistent measure with the measures of the other side is no longer an indifferent relationship but an *elective affinity*.

(*c*) *Elective Affinity*

The expression *elective* affinity used here and the terms *neutrality* and *affinity* employed in the preceding paragraphs, refer to the

chemical relationship. For a chemical substance has its specific determinateness essentially in its relation to its other and exists only as this difference from it. Furthermore, this specific relation is bound up with quantity and is at the same time the relation, not only to a single other but to a series of specifically different others opposed to it. The combinations with this series are based on a so-called affinity with *every* member of the series; but along with this indifference each member is at the same time exclusive towards the others and it is this correlation of opposed determinations which we have still to consider. It is, however, not only in the sphere of chemistry that the specific relation is represented in a circle of combinations; the individual note, too, only has meaning in relationship and combination with another note and with a series of others. The harmony or disharmony in such a circle of combinations constitutes its qualitative nature which is at the same time based on quantitative ratios; these form a series of exponents and are the ratios of the two specific ratios which each of the combined notes is in its own self. The individual note is the key of a system, but again it is equally an individual member in the system of every other key. The harmonies are exclusive elective affinities whose characteristic quality is equally dissolved again in the externality of a merely quantitative progression. What it is, however, that constitutes the principle of a measure for those affinities which (whether chemical or musical or what else) are elective affinities between and in opposition to the others, will be the subject of a further Remark in connection with chemical affinity; but this higher question is very closely bound up with the specific nature of the strictly qualitative aspect and belongs to the particular concrete natural sciences.

Inasmuch as the member of a series has its qualitative unity in its relation to all the members of an opposite series, whose members however are distinguished only by the quantum required for neutralizing them with the member of the first series, the more specific determinateness in this multiple affinity is likewise only quantitative. In elective affinity as an exclusive, qualitative correlation, the relationship is rid of this quantitative difference. The next determination which offers itself is this; that in accordance with the difference of the amounts, that is, of *extensive* magnitude, of the substances of the one series required for the neutralization of a substance in the other series, the elective

affinity of the latter substance would also be directed towards the substances of the first series with all of which it has an affinity. The exclusion which would thereby be established in the form of a *firmer* holding together against other possibilities of combination, would appear, thus transformed, in a proportionately greater *intensity* in virtue of the previously demonstrated identity of the forms of extensive and intensive magnitude, the quantitative determinateness being one and the same in both forms. However, this sudden conversion of the one-sided form of extensive magnitude into its other, intensive form, makes no difference to the nature of the fundamental determination, which is one and the same quantum; consequently, no real exclusion would, in fact, result therefrom and there could take place equally well either only one combination, or a combination of an indefinite number of substances, provided that the portions of them entering into the combination corresponded to the required quantum in accordance with the ratios existing between them.

The combination which we have also called neutralization, however, is not only the form of intensity; the exponent is essentially a measure determination and therefore exclusive. In this aspect of exclusive relations, numbers have lost their continuity with one another and their fluid combinatory nature; the relationship is one of *more* or *less*, which acquires a negative character and the *preference* which one exponent has over another does not remain confined to the quantitative determinateness. But equally, too, there co-exists this other aspect which again makes it a matter of indifference that a substance receives the neutralizing quantum from several substances of the opposed series and from each according to its specific ratio relatively to the others; the exclusive, negative relation is thus at the same time adversely affected by the quantitative aspect. The effect of this is an actual conversion of an indifferent, merely quantitative relationship into a qualitative one, and conversely, a transition from a specifically determined relation into a merely external one—a series of relations which are sometimes of a merely quantitative nature and sometimes are specific relations and measures.

Remark: *Berthollet on Chemical Affinity and Berzelius's Theory of it*

Chemical substances are the most characteristic examples of those

measures which, as moments of a measure, are characterized solely by their relationship to other such measure moments. Acids and alkalis or bases generally, appear to be intrinsically determinate things just as they are; but the fact is that they are incomplete elements of bodies, constituents which strictly speaking do not exist for themselves but only as a tendency to get rid of their isolatedness by combining with another constituent. Further, the difference in virtue of which they are *self-subsistent*, does not consist in this immediate quality but in the peculiar quantitative mode of the relationship. This, namely, is not restricted to the chemical opposition of acid and alkali or base in general, but is a specific *measure of saturation* and consists in the specific determinateness of the quantity of the substances which neutralize each other. This specific quantity required for saturation constitutes the qualitative nature of a substance; it makes it what it is on its own account and the number which expresses this is essentially one of several exponents for an opposed unit. A substance of this kind has a so-called affinity with another. If this connection remained of a purely qualitative nature, then, as in the case of the magnetic poles or positive and negative electricity, the one determinateness would be only the negative of the other and both sides would not at the same time show themselves to be indifferent towards each other. But because the connection has also a quantitative side, each of these substances is capable of neutralizing itself with *more* than one and is not restricted only to the one to which it is opposed. It is not only an acid and an alkali or base which are in relation, but acids and alkalis or bases which are related to one another. They are specifically distinguished from each other primarily according to whether one acid, for example, requires a greater amount of an alkali for its saturation than another acid does. But the independent, self-determined character of the substances is displayed in the exclusiveness of the relation between the affinities, one having a preference over another in that one acid can by itself enter into combination with any alkali, and conversely. Thus the cardinal difference between two acids consists in one of them having a closer affinity to a base than the other, i.e., in a so-called elective affinity.

The law of the chemical affinities of acids and alkalis has been discovered and it states that if two neutral solutions are mixed resulting in dissociation followed by two new compounds, these

products, too, are neutral. From this it follows that the amounts of two alkaline bases required for the saturation of an acid must be in the same ratio to saturate another acid; and in general, when for one alkali, taken as unit, the series of numerical ratios has been determined in which the various acids saturate it, then this series is the same for any other alkali, though the different alkalis must be taken in different amounts relatively to one another—amounts which again on their part form a similar fixed series of exponents for each of the opposite acids since they are related to any one acid in the same ratio as to any other. Fischer[1] was the first to extract these series in their simplicity from the works of Richter,[2] and Berthollet.[3] Since that was written, our knowledge of the numerical ratios of mixtures of chemical elements has been greatly expanded in every direction and to consider it here would be a digression, all the more so because this empirical expansion which is in part only hypothetical remains confined within the same group of concepts. We may however add a few remarks on the categories employed here and also on the views about chemical elective affinity itself and its relation to the quantitative aspect as well as the attempt to base this on specific physical qualities.

It is well known that Berthollet modified the general conception of elective affinity by the concept of the activity of a *chemical mass*. This modification does not affect the quantitative ratios of the chemical laws of saturation themselves, but its effect on the qualitative moment of exclusive elective affinity as such is not only to weaken it but rather to eliminate it, and this is a point that must not be overlooked. If two acids act on an alkali and the one which has a greater affinity for it is also present in the requisite amount for saturating it, then according to the concept of elective affinity this is the only saturation which occurs; the other acid remains quite inactive and is excluded from the neutral combination. According to the concept of the activity of a *chemical mass*, on the other hand, each of the two acids is active in a proportion which is composed of the amounts of the acids present and their

[1] Fischer, Ernst Gottfried, 1754–1831. Professor of Physics at Berlin, Member of the Academy.

[2] Richter, Jeremiah Benjamin, 1762–1807. Bergassessor in Berlin. See his observations to the translation of Berthollet's treatise on *The Laws of Affinity in Chemistry*, p. 232.

[3] Berthollet, Claude Louis, Count, 1748–1822: *Statique Chimique*, I Part p. 134 *sqq*. Professor at the Polytechnic School in Paris.

saturation capacity or so-called affinity. Berthollet's investigations have indicated in greater detail the circumstances in which the activity of the chemical mass is nullified and one acid (the one with a stronger affinity) appears to expel and to *exclude* the action of the other acid (with a weaker affinity) that is, appears to be active in the sense of elective affinity. He has shown that this exclusion takes place in certain *circumstances*, such as strength of cohesion, or the insolubility in water of the salts formed, but the qualitative *nature* of the agents as such does not come into play; and the action of these circumstances can in turn be nullified by other circumstances, for example, by temperature. With the removal of these obstacles the chemical mass enters unimpeded into activity and what appeared as a purely qualitative exclusion, as an elective affinity, proves to depend only on external modifications.

It is Berzelius[1] principally who should be heard further on this subject although in his *Textbook of Chemistry* he does not put forward anything original or more specific on the matter. The views of Berthollet are taken up and repeated literally, only decked out in the metaphysics peculiar to an uncritical reflection, the categories of which are therefore all that offer themselves for a more detailed consideration. The theory goes beyond the limits of experience and, on the one hand, invents sensuous images such as are not given in experience and on the other hand, applies categories of thought, in both cases making itself a subject for logical criticism. We propose therefore to hear what he has to say about the theory in the *Textbook* itself. Now there we read 'that one *must imagine* that in a uniformly mixed liquid, each *atom* of the dissolved substance is *surrounded* by an *equal number* of *atoms* of the solvent; and if several substances are dissolved together they must *share between them* the *interstices* between the atoms of the solvent, so that with a uniform mixture of the liquid there is produced a *symmetry in the arrangement* of the atoms such that *all the atoms* of the individual substances are *uniformly arranged in relation to the atoms* of the other bodies; it could therefore be said that the solution is characterized by *symmetry in the arrangement* of the atoms, and the combination by the fixed proportions.'[2] This is then illustrated by an example of compounds formed when

[1] Berzelius, Johann Jakob, Baron von, 1779–1848: *Textbook of Chemistry*, 3 volumes, 1808–1828. From 1807 Professor of Chemistry at Stockholm.

[2] Vol. III, Section 1, Wöhler's translation, pp. 82 *sqq*.

sulphuric acid is added to a solution of copper chloride; but the example certainly does not demonstrate that *atoms* exist; neither does it show that a number of atoms of the dissolved substances *surround* atoms of the fluid, nor that free atoms of the two acids *arrange themselves around* the atoms which remain combined (with the copper oxide), nor that there exists a *symmetry* in their *position and arrangement*, nor that interstices between the atoms exist—and least of all that the dissolved substances *share among themselves* the *interstices* between the atoms of the solvent. This would mean that the atoms of the dissolved substances take up their positions where the solvent is *not*—for the interstices between the atoms of the solvent are spaces *empty* of it—and hence that the dissolved substances are *not* present in the solvent, but rather are *outside* it, even if the solvent is arranged around them or they around it, and it is also certain therefore that they are not dissolved by it. One fails therefore to see why it is *necessary* to form such conceptions which are not empirically demonstrated, are in essence directly self-contradictory and are not corroborated in any other way. Corroboration could be provided only by a consideration of these conceptions themselves, i.e. by metaphysics, which is logic; but this cannot confirm them any more than experience can—on the contrary! For the rest, Berzelius admits what was also said above, that Berthollet's propositions are not opposed to the theory of fixed proportions, although he does add that they are also not opposed to the views of the corpuscular theory, i.e. the ideas mentioned above of atoms, of the filling of the interstices of the solvent by the atoms of the solid substances, and so on; this latter baseless metaphysics however, has essentially nothing to do with the proportions of saturation as such.

Hence, what finds specific expression in the laws of saturation concerns only the amount of units themselves quantitative (not atoms), of a substance with which the quantitative *unit* (equally not an atom) of another substance chemically distinct from the first, is neutralised; the difference between them consists solely in these different proportions. When Berzelius, then, notwithstanding the fact that his theory of proportions is wholly and solely a determination of *amounts*, nevertheless also speaks of *degrees* of affinity[1] in explaining Berthollet's *chemical mass* as the sum of the *degree of affinity*, from the given quantity of the active

[1] For example, on p. 86.

substance—although Berthollet is more consistent for he uses the expression *capacité de saturation*—then he himself lapses into the form of *intensive* magnitude; but it is this form which is the characteristic feature of the so-called *dynamic* philosophy which earlier on he calls 'the speculative philosophy of certain German schools',[1] and emphatically rejects in favour of the excellent 'corpuscular theory'. He there states of this dynamic philosophy that it assumes that the elements *interpenetrate* one another in their chemical combination, and that neutralization consists in this *mutual interpenetration*; this means nothing else than that the chemically different particles, which are interrelated as a plurality, collapse into the simplicity of an *intensive* magnitude, a fact which also finds expression as a diminution of volume. In the corpuscular theory, on the other hand, even the atoms which are chemically *combined* are supposed to be preserved in the interstices, i.e. *outside* one another (juxtaposition); in such a relationship which is one of merely extensive magnitude, of a perpetuated *amount*, a *degree* of affinity has no meaning. In the same place it is stated that for the dynamic view the phenomena of specific proportions came as something quite unforeseen; but this would be only an external, historical circumstance, apart from the fact that Richter's stoechiometric series in Fischer's compilation of them were already known to Berthollet and are quoted in the first edition of this *Logic* which proves the nullity of the categories on which the old, like the would-be new, corpuscular theory is based. But Berzelius is in error when he judges that if 'the dynamic view' had prevailed, the phenomena of specific proportions would have remained 'for ever' unknown—meaning that this view is incompatible with the determinateness of proportions. This is, in any case, only a determinateness of quantity, no matter whether in an extensive or an intensive form; so that even Berzelius, much as he adheres to the first of these forms, that of aggregate or amount [*Menge*], himself makes use of the conception of degrees of affinity.

Since in this way affinity is reduced to a quantitative difference, it is sublated as *elective* affinity; but the *exclusive* factor which occurs in it is ascribed to *circumstances*, i.e. to determinations which appear as something external to the affinity, to cohesion, insolubility of the compounds formed, and so on. A partial

[1] P. 29, *loc. cit.*

comparison may be made between this conception and the manner of considering the effect of gravity on a moving pendulum. Through gravity the pendulum necessarily passes into a state of rest; but this intrinsic effect of gravity itself is treated as a merely concomitant circumstance of the external resistance of the air, the thread and so on, and it is ascribed solely to *friction* instead of to gravity. Here it makes no difference to the nature of the *qualitative* element present in elective affinity whether this is manifested in the form of these circumstances taken as its conditions, and is so interpreted. With the qualitative aspect as such there begins a new order, the specifying of which is no longer only a matter of quantitative difference.

Now although the *chemical* affinity in a series of quantitative ratios has thus been accurately distinguished from *elective* affinity which occurs as a qualitative determinateness whose behaviour in no way coincides with the order of that series, this distinction in turn gets utterly confused by the way in which electrical action has recently been coupled with chemical action; and the hope that this supposedly profounder principle would throw light on the most important relation, that of measure, has met with complete disappointment. We need not here consider more closely this theory in which the phenomena of electricity and chemistry are completely *identified*, since it concerns the physical nature of substances and not merely their measure relations and it calls for mention only in so far as the distinctive character of the determinations of measure is confused by it. The theory as such must be dubbed shallow, for shallowness consists in omitting the difference between distinct terms and then treating them as identical. As for affinity, chemical processes being thus identified with electrical, and also with the phenomena of fire and light, this is reduced 'to neutralisation of opposite electricities'. It is almost comical to find the identification of electricity and chemical action expounded in the following manner: 'it is true that electrical phenomena explain the action of bodies at a greater or lesser distance, their *attraction* before combination (i.e. a behaviour which is not yet chemical) and the *fire*(?) caused by this combination, but they throw no light on the cause of the combination which persists with such strength *after* the opposite electrical condition has been destroyed;'[1] that is to say, the theory tells us

[1] P. 63, *loc. cit.*

that electricity is the cause of the chemical action, but about the specifically chemical nature of the chemical process electricity tells us nothing. Chemical difference as such being thus reduced to the opposition of positive and negative electricity, the different affinities of the agents on either side are determined as the order of two series of electro-positive and electro-negative substances. In identifying electricity and chemical action it is overlooked that the former generally (and its neutralization) is *transient* and remains *external* to the quality of substances, whereas chemical action, especially in the process of neutralization, embraces and alters the entire qualitative nature of substances. Equally transient within electricity is its opposition of positive and negative; this is so unstable that it is dependent on the most trivial outer circumstances and cannot be compared with the definiteness and fixity of the opposition between acids, for example, and metals, and so on. The alterations which can be produced by chemical action under extremely powerful influences, e.g., of a raised temperature are not comparable with the superficial nature of the opposition in electricity. Also the further distinction *within the series* of each of the two sides, between a more or less positive-electrical or more or less negative-electrical disposition is quite uncertain and entirely unconfirmed. But it is on the basis of the 'electrical dispositions' of these series of substances that 'the electro-chemical system is to be set up, which above all would be best fitted to provide an idea of chemistry'[1]: these series are now quoted; but what their nature really is, is indicated in the remark 'that this is *approximately* the order of these substances, but so little investigation has been made into this matter, that as yet *nothing really certain* can be ascertained about this relative order.'[2] Both the numerical ratios of the series of affinities (first made by Richter) as well as the extremely interesting reduction by Berzelius of the combinations of two substances to the simplicity of a few quantitative ratios, are absolutely independent of that hypothetical electro-chemical hotch-potch. If the experimental method has been the correct guiding star in the theory of proportions and its universal expansion since Richter, then the mixing of these great discoveries with the so-called corpuscular theory, a desert lying away from the path of experience, forms all the greater contrast with it; only this beginning, the abandoning of the principle of

[1] Berzelius, *op. cit.*, p. 84 *sq.* [2] P. 67.

experience, could be the reason for taking up again and developing that idea introduced earlier, especially by Ritter,[1] namely, the setting up of fixed classifications of electro-positive and electro-negative substances, which classifications were also supposed to have a chemical significance.

Even if the opposition of electro-positive and electro-negative substances were more in keeping with the facts than it is, then the nullity of this assumed basis of chemical affinity soon shows itself even experimentally and this again leads to further inconsistencies. It is admitted that two so-called electro-negative substances such as sulphur and oxygen combine in a much more intimate way than, e.g. oxygen and copper, although the latter is electro-positive.[2] Here therefore, the basis for the affinity founded on the general opposition of positive and negative electricity must give place to a mere *more* or *less* within one and the same series of electrical quality. From this it is now inferred that the *degree of affinity* of the substances depends therefore not only on their specific *unipolarity* (with what hypothesis this determination is connected is irrelevant here; it is significant here only for the 'either' of the positive and the 'or' of the negative); the degree of affinity must be derived mainly from the *intensity* of their *polarity* generally. At this point then, the consideration of affinity passes on to the relationship of *elective* affinity which is our chief concern; let us see then what the result now is for this subject. It is at once admitted that the *degree* of this polarity, if this does not exist merely in our imagination, does not seem to be a *constant quality* but to depend very much on temperature;[3] so that, after all, the result turns out to be not only that every chemical action is therefore at bottom an electrical phenomenon, but also that what seems to be an effect of so-called elective affinity is brought about *only* by an electrical polarity which in certain substances is present in greater strength than in others. The conclusion then after all this meandering in hypothetical conceptions, is that we are left with the category of *greater intensity*, which is the same *formal* determination as elective affinity in general; and since the latter is made to depend on a greater intensity of electrical polarity, it is not a whit nearer to being put on a physical basis than it was

[1] Ritter, Johann Wilhelm, 1776–1810: *Das Elektrische System der Körper*, 3 volumes, 1805–1806. Member of the Academy of Munich.

[2] P. 73, *loc. cit.* [3] *loc. cit.*, p. 73

before. But even what is here supposed to be determined as a greater specific intensity is subsequently reduced to the modifications demonstrated by Berthollet which have already been cited.

The merit and fame which Berzelius has earned by his theory of proportions, which has been extended to all chemical relations, ought not as such to be made a reason for not setting forth the weaknesses of this theory; but a more particular reason for doing so must be the circumstance that such merit in one aspect of a science, as with Newton, tends to become an *authority* for a baseless structure of spurious categories which is attached to it and that it is just this kind of metaphysics which is proclaimed and echoed too with the greatest pretension.

Apart from the forms of measure relation connected with chemical and elective affinity, others too can be considered with respect to quantities which are specified into a system. Chemical substances form a system of relations with respect to saturation; saturation itself rests on the specific proportion in which the reciprocal amounts of two substances, each of which has a particular material existence, combine with each other. But there are also measure relations the moments of which are inseparable and cannot be displayed in a separate and distinct existence of their own. These are what we called earlier on, *immediate self-subsistent* measures and which are displayed in the *specific gravities* of substances. They are a ratio within the substances of weight to volume; the exponent of the ratio, which is the expression of the difference between one specific gravity and another, is a definite quantum only as a result of *comparison.* This is a relationship external to the substances in an external reflection, and is not founded on the one substance's own qualitative behaviour towards another contrasted substance. The problem would be to recognize the exponents of the ratios of the series of specific gravities as a *system* based on a *rule* which would specify a merely arithmetical plurality into a series of harmonic nodes. The same demand would apply to our knowledge of the series of chemical affinities already mentioned. But the accomplishment of this task still lies a long way ahead, as far ahead as the problem of grasping the numbers of the relative distances of the planets from the sun as elements in a system of measure.

Although at first specific gravities do not seem to have any qualitative relationship to one another, yet they likewise enter

into a qualitative relation. When substances are chemically combined, or even form only amalgams or synsomates, there occurs also a *neutralization* of the specific gravities. We mentioned earlier on the fact that the volume of a mixture, even a mixture of substances remaining really indifferent chemically to each other, is not of the same magnitude as the sum of the volumes of the substances before mixing. There is a reciprocal modification in the mixture of the quantum of specific gravity with which the substances enter into the relation and in this way they indicate their qualitative behaviour towards each other. Here the quantum of specific gravity is expressed not merely as a fixed *comparative number*, but as a *numerical ratio* which can be varied; and the exponents of the mixtures give series of measures the specifying principle of which is other than the numerical ratios of the specific gravities of the substances in combination. The exponents of these ratios are not exclusive determinations of measure; their progress is continuous but it contains an immanent specifying law which is distinct from the formally progressive ratios in which the amounts are combined and makes the former progress incommensurable with the latter.

B. NODAL LINE OF MEASURE-RELATIONS

The last determination of the measure relation was that being specific it is *exclusive*; the neutrality is exclusive because it is a *negative* unity of the distinct moments. For the relation of this *self-subsistent* unity, of the elective affinity to the other neutralities, no further principle of specification has offered itself; the specification resides only in the quantitative determination of affinity in general, according to which the amounts which neutralize themselves are specific and therefore stand opposed to other relative elective affinities of their moments. But further, because the fundamental determination is quantitative, the *exclusive* elective affinity also continues itself into the opposed neutralities; and this continuity is not only an external relation of the different ratios of the neutralities in the form of a comparison, but the neutrality is, as such, *separable into the moments* which united to produce it, since it is as self-subsistent somethings that these enter into relation indifferently with one or the other of the opposite series, although combining in different, specifically

determined amounts. This measure, based on such a relation, is thus infected with its own indifference; it is in its own self something external and alterable in its relation to itself.

The relation *to itself* of the measure relation is distinct from its externality and alterableness which represent its quantitative aspect. As related to itself in contrast to these, it is an affirmatively present [*seiende*], qualitative foundation—a permanent, material substrate which, as also the continuity of the measure *with itself* in its externality, must contain in its quality the principle of the specification of this externality referred to above.

Now the exclusive measure as thus more precisely determined is external to itself in its being-for-self and hence repels itself from itself, positing itself both as another measure relation and also as another, merely quantitative, relation; it is determined as in itself a specifying unity which produces measure relations within itself. These relations differ from the affinities of the kind above-mentioned in which a self-subsistent measure relates itself to self-subsistent measures of a different quality and to a series of such. They take place in *one and the same* substrate within the same moments of the neutrality; the self-repelling measure develops other, merely quantitatively different relations which likewise form affinities and measures, alternating with those which remain only *quantitatively* different. They form in this way a nodal line of measures on a scale of more and less.

Here we have a measure relation, a self-subsistent reality which is qualitatively distinguished from others. Such a being-for-self, because it is at the same time essentially a relation of quanta, is open to externality and to quantitative alteration; it has a range within which it remains indifferent to this alteration and does not change its quality. But there enters a point in this quantitative alteration at which the quality is changed and the quantum shows itself as specifying, so that the altered quantitative relation is converted into a measure, and thus into a new quality, a new something. The relation which has taken the place of the first is determined by this, partly according to the qualitative identity of the moments which are in affinity, and partly according to the quantitative continuity. But because the difference falls into this quantitative aspect, the relation between the new something and its predecessor is one of indifference; their difference is the external one of quantum. The new something has therefore not

emerged from or developed out of its predecessor but directly from itself, that is, from the inner specifying unity which has not yet entered into existence. The new quality or new something is subjected to the same progressive alteration, and so on to infinity.

Since the progress from one quality [to another] is in an uninterrupted continuity of the quantity, the ratios which approach a specifying point are, quantitatively considered, only distinguished by a more and a less. From this side, the alteration is *gradual.* But the gradualness concerns merely the external side of the alteration, not its qualitative aspect; the preceding quantitative relation which is infinitely near the following one is still a different qualitative existence. On the qualitative side, therefore, the gradual, merely quantitative progress which is not in itself a limit, is absolutely interrupted; the new quality in its merely quantitative relationship is, relatively to the vanishing quality, an indifferent, indeterminate other, and the transition is therefore a *leap*; both are posited as completely external to each other. People fondly try to make an alteration *comprehensible* by means of the gradualness of the transition; but the truth is that gradualness is an alteration which is merely indifferent, the opposite of qualitative change. In gradualness too, the connection of the two realities, whether these are taken to be states or self-subsistent things, is eliminated; gradualness necessarily implies that neither of the two is the limit of the other but that each is completely external to the other. In this way there is eliminated the very factor which is necessary for an *understanding* of change, although little enough is required for that purpose.

Remark: *Examples of Such Nodal Lines*; *the Maxim*, '*Nature Does Not Make Leaps*'

The system of natural numbers already shows a nodal line of qualitative moments which emerge in a merely external succession. It is on the one hand a merely quantitative progress and regress, a perpetual adding or subtracting, so that each number has the same *arithmetical* relation to the one before it and after it, as these have to their predecessors and successors, and so on. But the numbers so formed also have a *specific* relation to other numbers preceding and following them, being either an integral multiple of one of them or else a power or a root. In the musical scale which is built

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up on quantitative differences, a quantum gives rise to an harmonious relation without its own relation to those on either side of it in the scale differing from the relation between these again and their predecessors and successors. While successive notes seem to be at an ever-increasing distance from the keynote, or numbers in succeeding each other arithmetically seem only to become other numbers, the fact is that there suddenly emerges a *return*, a surprising accord, of which no hint was given by the quality of what immediately preceded it, but which appears as an *actio in distans*, as a connection with something far removed. There is a sudden interruption of the succession of merely indifferent relations which do not alter the preceding specific reality or do not even form any such, and although the succession is continued quantitatively in the same manner, a specific relation breaks in *per saltum*.

Such qualitative nodes and leaps occur in chemical combinations when the mixture proportions are progressively altered; at certain points in the scale of mixtures, two substances form products exhibiting particular qualities. These products are distinguished from one another not merely by a more or less, and they are not already present, or only perhaps in a weaker degree, in the proportions close to the nodal proportions, but are bound up with these nodes themselves. For example, different oxides of nitrogen and nitric acids having essentially different qualities are formed only when oxygen and nitrogen are combined in certain specific proportions, and no such specific compounds are formed by the intermediate proportions. Metal oxides, e.g. the lead oxides, are formed at certain quantitative points of oxidation and are distinguished by colours and other qualities. They do not pass gradually into one another; the proportions lying in between these nodes do not produce a neutral or a specific substance. Without having passed through the intervening stages, a specific compound appears which is based on a measure relation and possesses characteristic qualities. Again, water when its temperature is altered does not merely get more or less hot but passes through from the liquid into either the solid or gaseous states; these states do not appear gradually; on the contrary, each new state appears as a leap, suddenly interrupting and checking the gradual succession of temperature changes at these points. Every birth and death, far from being a progressive gradualness, is an

interruption of it and is the leap from a quantitative into a qualitative alteration.

It is said, *natura non facit saltum*; and ordinary thinking when it has to grasp a coming-to-be or a ceasing-to-be, fancies it has done so by representing it as a *gradual* emergence or disappearance. But we have seen that the alterations of being in general are not only the transition of one magnitude into another, but a transition from quality into quantity and *vice versa*, a becoming-other which is an interruption of gradualness and the production of something qualitatively different from the reality which preceded it. Water, in cooling, does not gradually harden as if it thickened like porridge, gradually solidifying until it reached the consistency of ice; it suddenly solidifies, all at once. It can remain quite fluid even at freezing point if it is standing undisturbed, and then a slight shock will bring it into the solid state.

In thinking about the gradualness of the coming-to-be of something, it is ordinarily assumed that what comes to be is already sensibly or *actually in existence*; it is not yet perceptible only because of its smallness. Similarly with the gradual disappearance of something, the *non-being* or the other which takes its place is likewise assumed to be *really there*, but not yet observable, and *there*, too, not in the sense of being implicitly or ideally contained in the first something, but *really there*, only not observable. In this way, coming-to-be and ceasing-to-be lose all meaning; that is to say, the form of the in-itself, the inner being of something before it actually exists, is transformed into a smallness of an outer existence, and the essential difference, that of the Notion, is converted into an external difference of mere magnitude. The attempt to explain coming-to-be or ceasing-to-be on the basis of the gradualness of the alteration is tedious like any tautology; what comes to be or ceases to be is assumed as already complete and in existence beforehand and the alteration is turned into a mere change of an external difference, with the result that the explanation is in fact a mere tautology. The intellectual difficulty attendant on such an attempted explanation comes from the qualitative transition from something into its other in general, and then into its opposite; but the *identity* and the *alteration* are misrepresented as the indifferent, external determinations of the quantitative sphere.

In the moral sphere, in so far as it is considered under the

categories of being, there occurs the same transition from quantity into quality and different qualities appear to be based on a difference of magnitude. It is through a more and less that the measure of frivolity or thoughtlessness is exceeded and something quite different comes about, namely crime, and thus right becomes wrong and virtue vice. Thus states, too, acquire through their quantitative difference, other things being assumed equal, a distinct qualitative character. With the expansion of the state and an increased number of citizens, the laws and the constitution acquire a different significance. The state has its own measure of magnitude and when this is exceeded this mere change of size renders it liable to instability and disruption under that same constitution which was its good fortune and its strength before its expansion.

C. THE MEASURELESS

The exclusive measure, even in its realised being-for-self, remains burdened with the moment of quantitative determinate being and is therefore open to movement up and down a scale of fluctuating ratios. Something, or a quality, based on such a ratio is impelled beyond itself into the *measureless* and is destroyed by the mere alteration of its magnitude. Magnitude is that side of determinate being through which it can be caught up in a seemingly harmless entanglement which can destroy it.

The abstract measureless is the quantum as such which lacks an inner significance and is only an indifferent determinateness which does not alter the measure. Measure in the nodal line of measures is at the same time posited as specifying and the abstract measureless raises itself into a qualitative determinateness; the new measure relation into which the original one passes is, with respect to this, measureless, but in its own self it is equally a quality on its own account. Thus there is posited the alternation of specific existences with one another and of these equally with relations remaining merely quantitative—and so on *ad infinitum*. What therefore is present in this transition is both the negation of the specific relations and the negation of the quantitative progress itself—the *infinite* which is *for itself*. The *qualitative* infinite, as simply a determinate being, was the eruption of the infinite in the finite as an *immediate transition* and vanishing of

the latter in its beyond. The *quantitative* infinite on the other hand is, simply by virtue of its determinateness, the *continuity* of the quantum, a continuity of it into its be*yond*. The qualitative finite *becomes* the infinite; the quantitative finite is in its own self its beyond and *points beyond itself*. But this infinity of the specification of measure *posits* both the qualitative and the quantitative as *sublating* themselves in each other, and hence posits their first, immediate *unity*, which is measure as such, as returned into itself and therefore as itself *posited*. The transition of the qualitative, of one specific existence, into another, is such that all that occurs is an alteration of the specific magnitude of a ratio. Hence the alteration of the qualitative itself into the qualitative is posited as an external and indifferent change, as a coming together with itself; moreover, the quantitative, in being converted into the qualitative, into that which is determined in and for itself, sublates itself. This unity which thus continues itself into itself in its alternating measures is the truly persisting, self-subsistent material substance or thing.

What therefore is present here is (α) one and the same thing which is posited as the perennial substrate of its differentiations. This severance of being from its determinateness begins already in quantum as such; under the category of *magnitude*, a thing is indifferent to its affirmative determinateness. In measure, the thing itself is already in itself the unity of its qualitative and quantitative moments, the two moments which constitute the element of difference within the general sphere of being and of which one is the beyond of the other; in this way the perennial substrate has directly in its own self the determination of affirmative infinity. (β) This self-sameness of the substrate is *posited* in the fact that the qualitative self-subsistent measures into which the specifying unity is dispersed consist only of quantitative differences, so that the substrate continues itself into this differentiation of itself; (γ) in the infinite progress of the nodal series there is posited the continuation of the qualitative moment into the quantitative progress as into an indifferent alteration, but equally too, there is posited the negation of the qualitative moment contained therein and hence of the merely quantitative externality too. The quantitative reference beyond itself to an other which is itself quantitative perishes in the emergence of a measure relation, of a quality; and the qualitative transition is sublated in the very

fact that the new quality is itself only a quantitative relation. This transition of the qualitative and the quantitative into each other proceeds on the basis of their unity, and the meaning of this process is only to *show* or to *posit* the *determinate being* of such a substrate underlying the process, a substrate which is their unity.

In the series of self-subsistent measure relations the one-sided members of the series are immediately qualitative somethings (specific gravities or chemical substances, bases, alkalis, or acids for example), and then their neutralizations (by which must also be understood here the compounds of substances of different specific gravity) are self-subsistent and even exclusive measure relations, self-determined and mutually indifferent totalities of determinate being. Now such relations are determined only as nodal points of one and the same substrate. Consequently, the measures and the self-subsistent things posited with them are reduced to *states*. The alteration is only change of a *state*, and the *subject* of the transition is posited as remaining the *same* in the process.

Surveying the progressive determinations which measure has passed through we can summarise them as follows. Measure is, in the first instance, only the immediate unity of quality and quantity as an ordinary quantum which is, however, specific. As thus a specific quantity which is related not to another but to itself, it is essentially a *ratio*. It therefore also contains its moments as sublated and undivided within itself; as is always the case in a Notion, the difference in the ratio is present in such a manner that each of its moments is itself a unity of quality and quantity. The difference therefore is real and yields a number of measure relations which, as formal totalities in themselves, are self-subsistent. The two series formed by the sides of these ratios are the same constant arrangement for each individual member which, as belonging to one side, enters into relationship with all the members of the opposite series. This unity as a mere *arrangement* is still quite external, and although it shows itself to be an immanent specifying unity of a self-subsistent measure distinguished from its specifications, it is not yet the free Notion which alone gives its differences an immanent determination: it is as yet only a substrate, a material, and for its differentiation into totalities, i.e., into differences embodying the nature of the unchanged substrate, it is dependent solely on the external,

quantitative determination which shows itself at the same time as a difference of quality. In this unity of the substrate with itself the measure determination is sublated and its quality is an external state determined by the quantum. This process is equally the progressive determination of measure in its realization and also the reduction of measure to the status of a moment.

CHAPTER 3

THE BECOMING OF ESSENCE

A. ABSOLUTE INDIFFERENCE

Being is the abstract equivalence [*Gleichgültigkeit*]—for which, since it is to be thought of by itself as *being*, the expression indifference [*Indifferenz*] has been employed—in which there is supposed to be as yet no determinateness of any kind; pure quantity is indifference as open to all determinations provided that these are external to it and that quantity has no immanent connection with them; but the indifference which can be called absolute is the indifference which, *through the negation* of every determinateness of being, i.e., of quality, quantity, and their at first immediate unity, measure, is a process of *self-mediation* resulting in a simple unity. Any determinateness it still possesses is only a *state*, i.e. something qualitative and external which has the indifference for a *substrate*.

But what has thus been determined as qualitative and external is only a vanishing determinateness; quality as thus external to being is the opposite of itself and as such is only the sublation of itself. In this manner, the determinateness is still only posited in the substrate as an empty differentiation. But it is just this empty differentiation which is indifference itself as a result; and indifference is thus concrete, a mediation-with-self through the negation of every determination of being. As this mediation it contains negation and relation, and what was called state is its immanent, self-related differentiation; it is precisely externality and its vanishing which make the unity of being into indifference and they are therefore *within* this indifference, which therewith ceases to be only a substrate and *in its own self* only abstract.

B. INDIFFERENCE AS INVERSE RATIO OF ITS FACTORS

We have now to see how this determination of indifference is posited within the indifference itself and how the latter is therewith posited as being for itself.

1. The reduction of measure relations which at first ranked as self-subsistent measures, establishes their common substrate;

this is their continuation into one another and hence the indivisible self-subsistent measure which is *wholly* present in its differentiations. For this difference, there are present the determinations of quality and quantity which the measure contains, and everything turns solely on how these are posited in it. This, however, is determined by the fact that the substrate is in the first place a result and only *in principle* [*an sich*] a mediation; but because this mediation is not yet posited as such *in the measure itself*, this latter is in the first place a substrate and with respect to determinateness, *indifference*.

Consequently, at first it is essentially the merely quantitative external difference which is present in it; there are two distinct quanta of one and the same substrate which in this way would be their *sum* and therefore would itself be determined as quantum. But the indifference is this fixed measure, the implicit, absolute limit, only in *relation* to those differences in such a manner that it would not be in its own self a quantum or opposed in any way, either as a sum or even as an exponent, to other quanta whether sums or indifferences. It is only the abstract determinateness which falls into the indifference; the two quanta, in order that they may be posited in it as moments, are variable, indifferent, greater or smaller relatively to one another. Bounded however by the fixed limit of their sum, they are related to each other not externally but negatively, and this now is the qualitative determination of their relationship. They are consequently *inversely proportional*. This relation is distinguished from the earlier formal inverted relation or inverse ratio by the fact that here the whole is a real substrate and each of the two sides is posited as having to be itself *in principle* [*an sich*] this whole.

According to the stated qualitative determinateness, the difference is present, further, as *two qualities*, one of which is sublated by the other; but because both are held in a unity which they together constitute, neither is separable from the other. The substrate itself, as an indifference, is likewise in itself the unity of both qualities; therefore each side of the relation, too, contains both sides within itself and is distinguished from the other side only by a more of the one quality and a less of the other, and *vice versa*. The one quality is through its quantum only *preponderant* in the one side, and so, too, the other quality in the other side.

Consequently each side is in its own self an inverted relation. As formal, this relation recurs in the two distinct sides. These sides thus continue themselves into each other also in respect of their qualitative determinations; each quality is self-related in the other and is present in each of the two sides, only in a different quantum. Their quantitative difference is that indifference in accordance with which they continue themselves into each other and this continuation as the self-sameness of the qualities is in each of the two unities. The sides, however, each as the whole of the two determinations and hence containing the indifference itself, are thus at the same time posited as self-subsistent relatively to each other.

2. As this indifference, being is now the specification of measure no longer in its immediacy, but measure as developed in the manner just indicated; it is *indifference* first as *in itself*, the whole of the determinations of being which are resolved into this unity and secondly, as a *determinate being*, as a totality of the posited realization, in which the moments themselves are implicitly the totality of the indifference, borne by this latter as their unity. But because the unity is only an *indifference*, and hence is held fast only as an *implicit* unity, and the moments are not yet explicitly *self-determined*, i.e. are not yet determined as sublating themselves into a unity *within themselves* and *through one another*, what is here present is simply the *indifference* of the unity itself *towards itself* as a developed determinateness.

This self-subsistent measure as thus indivisible is now to be considered in more detail. It is immanent in all its determinations and in them remains in unity with itself and unaffected by them; but (α) as remaining *implicitly* the totality, it possesses the determinatenesses which are sublated in it, only as groundlessly *emerging* in it. The *implicit* being of the indifference and this *real* being of the latter are unconnected; the determinatenesses show themselves in the indifference in an immediate manner and the indifference is wholly present in each of them. Consequently, the difference between them is posited in the first place as sublated, therefore as only *quantitative*, but not as the self-repulsion of the indifference, and the indifference is not posited as self-determining but as being determinate and determined only *externally*. (β) The two moments are in an inverted quantitative relation—a to and fro in the scale of magnitude; but this fluctuation is determined

not by the indifference, which is just the indifference of this fluctuation, but is determined herewith only externally. The principle of determination resides not in the indifference, but in something lying outside it. The absolute, as indifference, has in this aspect the second defect of the *quantitative* form, namely that the determinateness of the difference is not determined by the absolute itself; just as it has the first defect in the fact that the differences simply *emerge* in it, that is to say, the absolute's positing is immediate in character, is not the mediation of the absolute with itself. (γ) The quantitative determinateness of the moments which are now *sides* of the relation constitutes this mode of their *subsistence*; through this indifference their determinate being is freed from the transition of the qualitative sphere. But in distinction from this their real being, they have an implicit subsistence in the fact that they are *in themselves* the indifference itself, each being itself the unity of the two *qualities* into which the qualitative moment splits itself. The difference between the two sides is restricted to this, that one quality is posited in the one side with a more and in the other with a less, and the other quality similarly, but conversely. Hence each side is in its own self the totality of the indifference. Each of the two qualities taken singly on its own account also remains the same sum which the indifference is; it continues itself out of the one side into the other and is not bounded by the quantitative limit which is thereby posited in it. In consequence of this the determinations come into immediate opposition and this develops itself into a contradiction which we have now to consider.

3. Namely, each quality enters *within* each side into relation to the other, and does so in such a manner that, as has been determined, this relation too is to be only a quantitative difference. If the two qualities are self-subsistent—taken, say, as if they were sensuous things independent of each other—then the whole determinateness of indifference falls asunder; their unity and totality would be empty names. But they are at the same time expressly determined as comprised in a single unity, as inseparable, each as having meaning and reality only in this one qualitative relation to the other. But now, *because their quantitativity is simply and solely of this qualitative nature, each reaches only as far as the other.* If the qualities are regarded simply as distinct quanta, then the one would reach beyond the other and would have in its *more*

an indifferent determinate being which the other would not have. But in their qualitative connectedness, each is only in so far as the other is. From this it follows that they are in *equilibrium*; that by as much as the one increases or decreases, the other likewise would increase or decrease and in the same proportion.

Therefore on the basis of their *qualitative* connection there can be no question of a *quantitative* difference or of a *more* of the one quality. The more by which *one* of the correlated moments would exceed the *other* would only be a baseless determination, or else this *more would only be the other itself again*; but in this their equality, both would have vanished, for their determinate being was supposed to be based solely on the inequality of their quantum. Each of these hypothetical factors vanishes, whether it is supposed to be *beyond* or *equal to* the other. This vanishing appears to quantitative conception as a disturbance of the equilibrium so that the one factor becomes greater than the other; thus there is posited the sublating of the quality of the other and its lack of any support. One factor becomes preponderant as the other diminishes with an accelerated velocity and is overpowered by the first, which therefore constitutes itself the sole self-subsistent quality; but this being so, there are no longer two specific moments and factors but only the one whole.

This unity thus posited as the totality of the process of determining, in which it is itself determined as indifference, is a contradiction in every respect; it therefore has to be *posited* as sublating this its contradictory nature and acquiring the character of a self-determined, self-subsistent being which has for its result and truth not the unity which is merely indifferent, but that immanently negative and absolute unity which is called essence.

Remark: Centripetal and Centrifugal Force

The relationship of a whole which is supposed to have its determinateness in the quantitative difference of two factors determined qualitatively against each other, is applied to the elliptical motion of the celestial bodies. This example exhibits primarily only two qualities in inverse relation to each other, not two sides, each of which is itself the unity of both and their inverse relation. The fact, the inverse relation, rests on a firm empirical foundation, but the theoretical explanation of it involves a consequence which is overlooked, namely the destruction of the basic fact; or if, as is

proper, the fact is retained it escapes notice that the theory proves to be meaningless in face of the fact. The ignoring of this consequence allows the fact and the theory conflicting with it to exist calmly side by side. The simple fact is that in the elliptical motion of the celestial bodies their velocity is accelerated as they approach perihelion and retarded as they approach aphelion. The quantitative side of this fact has been accurately ascertained by the untiring diligence of observation, and further, it has been reduced to its simple law and formula. Hence all that can properly be required of a theory has been accomplished; but to reflective understanding this did not appear sufficient. For the purpose of a so-called explanation of the phenomenon and its law a centripetal and a centrifugal force are assumed as qualitative moments of the curvilinear motion. Qualitatively, their difference consists in the opposed direction of the moments, and quantitatively, the moments being determined as unequal, in the fact that as the one increases the other is supposed to decrease, and *vice versa*; then further, that their relationship is reversed again so that after a period during which the centripetal force has been increasing and the centrifugal force decreasing, a point is reached where the former decreases and the latter increases.[1] But this way of representing the matter is contradicted by the essentially qualitative relation between their respective determinatenesses which makes their separation from each other completely out of the question. Each has meaning only with reference to the other; consequently, in so far as the one had an excess over the other, to that extent it would have no relation to it and the other would no longer exist. If it is assumed that one of them is at one time greater than the other, being *related* to it as the greater to the smaller, then what was said above applies, namely, that the greater would acquire absolute preponderance and the other would vanish; the other would be posited as a vanishing moment without any support; and nothing would be altered by supposing the vanishing to take place only gradually any more than by supposing that by as much as the vanishing moment decreases in magnitude the other increases, for the latter, too, is destroyed with the former, since it is what it is, only in so far as the other is. It requires but little

[1] In the first edition the following sentence is added here: 'I have elucidated this subject in an earlier dissertation and have demonstrated the futility of this distinction and the explanations based on it.'

consideration to see that if, for example, as is alleged, the body's centripetal force increases as it approaches perihelion, while the centrifugal force is supposed to decrease proportionately, the latter *would no longer be able* to tear the body away from the former and to set it again at a distance from its central body; on the contrary, for once the former has gained the preponderance, the other is overpowered and the body is carried towards its central body with accelerated velocity. Just as conversely, if the centrifugal force gains the upper hand when infinitely near to aphelion, it is equally contradictory that now, in the aphelion itself, it should be overpowered by the weaker force. Further, it is evident that it would be an *alien* force which effected this reversal; and this means that this alternation of accelerated and retarded velocity of the motion *cannot be ascertained* or, as it is said, *explained* from the assumed determination of the factors although these have been assumed for the express purpose of explaining this difference. The conclusion which follows from the vanishing of one or the other direction and hence of the elliptical motion altogether, is ignored and concealed because of the undeniable fact that this motion does go on and pass from the accelerated into the retarded velocity. The assumed transformation of the weakness of the centripetal force in aphelion into a preponderant strength over the centrifugal force, and conversely in perihelion, implies first the conclusion arrived at above, namely, that each side of the inverse relation is in its own self the whole inverse relation; for the side of the motion from aphelion to perihelion (when the centripetal force is supposed to be preponderant) is still supposed to contain the centrifugal force which, however, diminishes as the other increases, while on the side of the retarded motion the centrifugal force is assumed to be present in that same inverse relation in an ever-increasing preponderance over the centripetal force. Consequently, on neither side has either force vanished but only become increasingly smaller up to the point of its transformation into a preponderance over the other. All that recurs then on either side is the defect characteristic of this inverse relation, namely, either each force is credited with an independent self-subsistence, the pair being merely *externally* associated in a motion (as in the parallelogram of forces), in which case the unity of the Notion, the nature of the thing itself, is left out of account; or else, since each is qualitatively related to the other by the Notion, neither

can attain an indifferent independent subsistence in face of the other, a subsistence supposedly imparted to it by a *more*. The form of intensity, the so-called dynamic factor, does not help, because this too has its determinateness in quantum and consequently can express only as much force (which is the measure of its existence) as is opposed to it by the opposite force. Secondly, this sudden change round from a preponderance into its opposite implies the alternation of the qualitative determination of positive and negative; an increase in the one means an equivalent loss in the other. In the theory, the inseparable qualitative connectedness of this qualitative opposition is distorted into a *succession in time*; but it fails thereby to give an *explanation* of this alternation and more particularly of this distortion itself. The semblance of unity which is still implied in the increasing of one side with an equivalent decreasing of the other vanishes here completely; what is alleged is a merely *external* succession of the sides which only contradicts what is necessarily implied in their qualitative connectedness, namely that with the preponderance of one side the other must disappear.

The same relationship has been applied to the forces of attraction and repulsion for the purpose of explaining the different *densities* of bodies; and the inverse ratio of sensibility and irritability has also been invoked to explain from the inequality of these factors of *life* the various determinations of the whole and of health and also the variety of living species. This mode of explanation was supposed to form a basis for the natural philosophy of physiology, nosology and also zoology, but the confused hotchpotch of nonsense in which it became entangled through the uncritical use of these determinations of the Notion soon led to the abandonment in these spheres of this formalism which, however, is practised without restraint in the other sciences, particularly in physical astronomy.

Since absolute indifference may seem to be the fundamental determination of Spinoza's *substance*, we may add that this is indeed the case in so far as in both every determination of being, like every further concrete differentiation of thought and extension and so forth, is posited as vanished. If we stop short at the abstraction [of substance] then it is a matter of complete indifference what something looked like in reality before it was swallowed up in this abyss. But when substance is conceived as

indifference, it is tied up with the need for determining it and for taking this determining into consideration; it is not to remain Spinoza's substance, the sole determination of which is the negative one that everything is absorbed in it. With Spinoza, the moment of difference—attributes, thought and extension, then the modes too, the affections, and every other determination—is introduced quite empirically; it is intellect, itself a mode, which is the source of the differentiation. The relationship of the attributes to substance and to one another is not specified further than that they express the whole of substance, and their content, the order of things as extended and as thoughts, is the same. But by the determination of substance as indifference, the *difference* too, comes to be reflected on; whereas with Spinoza, the difference is an external, and more precisely a quantitative difference *only by implication*, now it is *posited* as such. The indifference does indeed, like substance, remain immanent in the differentiated moments, but only abstractly so, only *implicitly*; the difference is *not* immanent in the indifference, for as quantitative it is rather the opposite of immanence, and the quantitative indifference is rather the self-externality of the unity. Thus the difference is also not grasped qualitatively; substance is not determined as self-differentiating, not as subject. The immediate consequence with respect to the category of indifference itself is that in it the difference of quantitative or qualitative determination falls apart as we found in the explication of indifference; it is the *dissolution of measure*, in which both moments were directly posited as one.

C. TRANSITION INTO ESSENCE

Absolute indifference is the final determination of *being* before it becomes essence; but it does not attain to essence. It reveals itself as still belonging to the sphere of being through the fact that, determined as *indifferent*, it still contains difference as an *external*, quantitative determination; this is its *determinate being*, contrasted with which absolute indifference is determined as being only *implicitly* the absolute, not the absolute grasped as *actuality*. In other words, it is *external reflection* which stops short at conceiving the differences *in themselves* or in the absolute as *one and the same*, thinking of them as only indifferently distinguished, not as intrinsically distinct from one another. The further step which

requires to be made here is to grasp that this reflection of the differences into their unity is not merely the product of the *external* reflection of the subjective thinker, but that it is the very nature of the differences of this unity to sublate themselves, with the result that their unity proves to be absolute negativity, its indifference to be just as much indifferent *to itself*, to its own indifference, as it is indifferent to otherness.

But we are already familiar with this self-sublating of the determination of indifference; in the development of its positedness, this determination has shown itself to be from every aspect a contradiction. It is *in itself* the totality in which every determination of being is sublated and contained; it is thus the substrate, but at first only in the one-sided determination of the in-itself, and consequently the differences, namely, the quantitative difference and the inverse ratio of factors, are present in it only in an *external* manner. As thus the contradiction of itself and its determinedness, of its implicit determination and its posited determinateness, it is the negative totality whose determinatenesses have sublated themselves in themselves and in so doing have sublated this fundamental one-sidedness of theirs, their [merely] implicit being [*Ansichsein*]. The result is that indifference is now posited as what it in fact is, namely a simple and infinite, negative relation-to-self, its inherent incompatibility with itself, a repelling of itself from itself. The process of determining and being determined is not a transition, nor an external alteration, nor an emergence of determinations in the indifference, but is its own self-relating which is the negativity of itself, of its [merely] implicit being.

Now these repelled determinations do not possess themselves, do not emerge as self-subsistent or external determinations, but first, *as moments* belonging to the *implicit* unity, they are not expelled from it but are borne by it as the substrate and are filled solely by it; secondly, as determinations which are immanent in the *explicated* unity, they *are* only through their repulsion from themselves. The being of the determinations is no longer simply *affirmative* as in the entire sphere of being, but is now a sheer *positedness*, the determinations having the fixed character and significance of being *related* to their unity, each consequently being related to its other and with negation; this is the mark of their relativity.

Thus we see that being in general and the being or immediacy of the distinct determinatenesses, no less than the *implicit being*, has vanished and the unity is *being*, an *immediate presupposed* totality such that it is this *simple self-relation only as a result of the sublating of this presupposition*, and this presupposedness and immediate being is itself only a moment of its repelling, the original self-subsistence and self-identity *is* only as the *resulting coming together with itself*. Being, in its determining, has thus determined itself to essence, a being which, through the sublating of being, is a simple *being-with-itself*.